$3x^2 - x - 4 = (3x-4)(x+1)$

$a=3,\ b=-1,\ c=-4$

$\Rightarrow b^2 - 4ac = (-1)^2 - 4(3)(-4)$

$= 1 + 48 = 49$ NOTE $7^2 \Rightarrow$

2 rational sol'ns, i.e., factors in the high school way.

$49 > 0 \Rightarrow$ 2 distinct real sol'ns.

$= 0 \Rightarrow$ 1 real (repeated) sol'n.

$< 0 \Rightarrow$ 2 nonreal sol'ns.

} Real Sol'ns

In addition, we observed that the discriminant is a perfect square, which gives 2 real solutions that are also rational.

What does "rational" mean in algebra. Yes, no radicals (→ are constructible) or transcendental #s, like *e* or π. (→ Not constructible)

It means the number can be written as a fraction, i.e., a quotient of integers.

A "constructible" number is one which is a root of a polynomial with rational coefficients.

$\sqrt{3}$ $(x-\sqrt{3})(x+\sqrt{3}) = x^2 - 3$

Blazing Saddles.

Quadratic in form equations:

$$x^4 + 3x^2 + 2 = 0$$

Factor wrt x^2:

$$(x^2+2)(x^2+1) = 0$$

$\Rightarrow x^2 + 2 = 0$

$a = 1, b = 0, c = 2$

$b^2 - 4ac = 0^2 - 4(1)(2) = -8$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{0 \pm 2i\sqrt{2}}{2(1)}$$

$= \pm i\sqrt{2} = \pm\sqrt{2}\, i$

$2 \,|\, 8$, $2 \,|\, 4$, 2

$\sqrt{8} = 2\sqrt{2}$

$\sqrt{-8} = 2i\sqrt{2}$

$x^2 - (\sqrt{2}\,i)^2 = x^2 - (-2)$

$= x^2 + 2 = 0$

$(x - \sqrt{2}\,i)(x + \sqrt{2}\,i) = 0 \Rightarrow x = \pm\sqrt{2}\,i$

$x^2 + 2 = 0$

$x^2 = -2$

$\sqrt{x^2} = \sqrt{-2}$

$|x| = \sqrt{-2} = i\sqrt{2}$

$x = +i\sqrt{2}$

$x = -i\sqrt{2}$

$x = \pm\sqrt{-2} = \pm i\sqrt{2} = x$

Make a u-substitution $u = x^2$:

Then $u^2 + 3u + 2 = 0 \rightarrow$

$(u+2)(u+1) = 0$

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,

$u + 2 = 0$ OR $u + 1 = 0$

$x^2 + 2 = 0$ OR $x^2 + 1 = 0$

⋮

$x = \pm i\sqrt{2}$ $\quad x = \pm i$

CPT: CONJUGATE PAIRS THEOREM. $f(a+bi)=0 \Longrightarrow f(a-bi)=0$ if the coefficients of f are real.

Solve $4x^3 - 3x - 26 = 0$

Homework: Use grapher for x-intercepts.

Test: Rational Zeros Theorem:

If $f\left(\frac{a}{b}\right)=0$, then a is a factor of the constant term & b " " " " the n^{th}-degree term (also known as the "leading coefficient").

$2 \lfloor 26$, 13

$\frac{a}{b}$'s: a: 26

b: 4

$\pm 1, \pm 2, \pm 13, \pm 26$

$\pm \frac{1}{2}, \pm \frac{2}{2}, \pm \frac{13}{2}, \pm \frac{26}{2}$ → Already used.

$\pm \frac{1}{4}, \pm \frac{2}{4}, \pm \frac{13}{4}, \pm \frac{26}{4}$

Teacher loves you, so on test, expect the nice one(s) to work, like $\pm 1, \pm 2$

$4x^3 - 3x - 26$

Try x=1

1	4	0	-3	-26
		4	4	1
	4	4	1	-25 ≠ 0 No

Missing the place-holder for $0x^2$ term

Fixed:

1	4	0	-3	-26	
		4	4	1	
	4	4	1	-25	≠ 0 No

-1	4	0	-3	-26	
		-4	4	-1	
	4	-4	1	Nope	

2	4	0	-3	-26	
		8	16	26	
	4	8	13	0	Sweet!

This says $4x^3 - 3x - 26 = (x-2)(4x^2 + 8x + 13)$

Depressed Polynomial. is of degree 2. Clobber it!

& $a = 4,\ b = 8,\ c = 13$

$\Rightarrow b^2 - 4ac = 8^2 - 4(4)(13)$

$= 64 - 208$

$= -144$

$2 \,|\, 144$, $2 \,|\, 72$, $2 \,|\, 36$, $2 \,|\, 18$, $3 \,|\, 9$, 3

$\sqrt{-144} = 2 \cdot 2 \cdot 3 \cdot i\sqrt{1}$

$= 12i$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-8 \pm 12i}{2(4)} = \dfrac{4(-2 \pm 3i)}{2(4)}$

$= \dfrac{-2 \pm 3i}{2}$. This says $4x^3 - 3x - 26$

$4(x-2)\left(x - \left(\frac{-2+3i}{2}\right)\right)\left(x - \left(\frac{-2-3i}{2}\right)\right) = 4x^3 + \dots$

PONG!

I can make it look like how I built it

$(x-2)(2)\left(x - \left(\frac{2+3i}{2}\right)\right)(2)\left(x - \left(\frac{-2-3i}{2}\right)\right)$

4 = 2·2 multiplication is commutative.

$= (x-2)(2x - (-2+3i))(2x - (-2-3i))$

$= (x-2)(2x + 2 - 3i)(2x + 2 + 3i)$

How I built it: $(x-2)\cdot(2\cdot x + (2 - 3\cdot I))\cdot(2\cdot x + (2 + 3\cdot I))$

We are told that the given polynomial has -3 + *i* as a zero. Find the rest.

$g(x) = 3x^3 + 17x^2 + 24x - 10 \qquad -3 + i$

i.e., $g(-3+i) = 0$.

Divide by $(x-(-3+i))$:

scratch:

$(-3+i)(8+3i)$
$= -24 - 9i + 8i + 3i^2$
$= -24 - i - 3$
$= -27 - i$

$-3+i$	3	17	24	-10
		$-9+3i$	$-27-i$	10
$-3-i$	3 (x^2)	$8+3i$ (x^1)	$-3-i$ (c)	0 (r) Sweet!
		$-9-3i$	$3+i$	
	3 (x^1)	-1 (a)	0 (r) Sweet!	

This says $g(x) = (x-(3+i))(x-(3-i))(3x-1)$

$(-3-i)(-3+i) = (-3)^2 + (1)^2$
$= 9+1 = 10$

$(a+bi)(a-bi) = a^2+b^2$