$a+bi$ $z = 3+2i$ $i=\sqrt{-1}$, $i^2=-1$

$Re(z)$ $Im(z)=2$

$(3+2i)(5-7i) = 15-35i+10i-14i^2$

$= 15-25i+14 = 29-25i$

Write $\frac{3+2i}{5-7i}$ in the form $a+bi$.

The conjugate, $\bar{z}$, of $z=a+bi$ is $\bar{z}=a-bi$

Note $z+\bar{z} = a+bi+a-bi = 2a = 2Re(z)$.

& $z\bar{z} = \boxed{(a+bi)(a-bi)} = a^2-abi+abi-b^2i^2$

$\boxed{= a^2+b^2}$

$(bi)(bi) = bbii = b^2i^2$

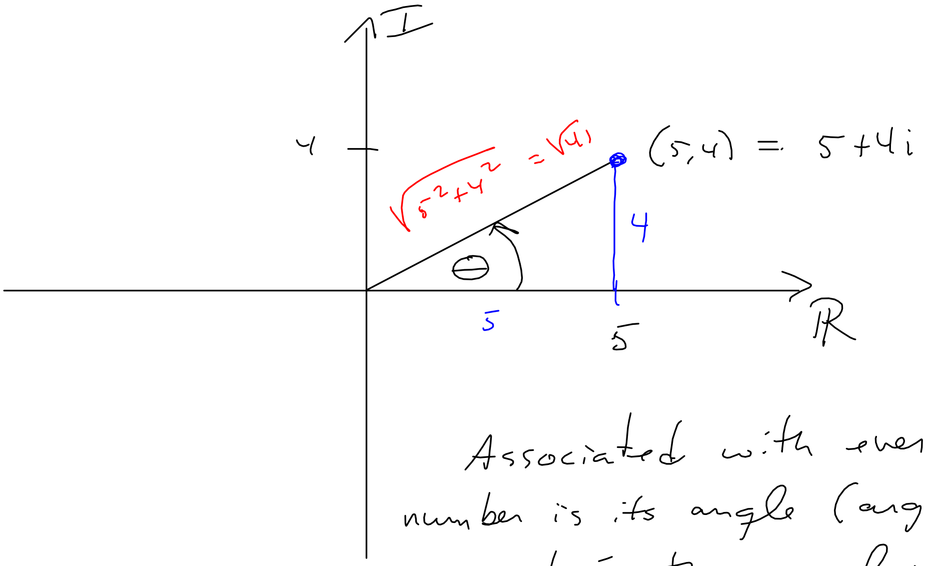
$-b^2i^2 = +b^2$

$\boxed{\left(\frac{3+2i}{5-7i}\right)\left(\frac{5+7i}{5+7i}\right)} = \frac{15+21i+10i+14i^2}{5^2+7^2}$

$= \frac{15+31i-14}{25+49} = \frac{1+31i}{74} = \boxed{\frac{1}{74}+\frac{31}{74}i}$

Standard Form.

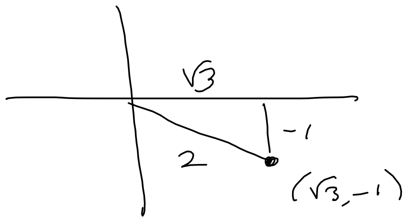
The complex plane



Associated with every complex number is its angle (argument) θ, measured in the usual way

For $5+4i$, $\theta = \arcsin\left(\frac{4}{\sqrt{41}}\right) = \arctan\left(\frac{4}{5}\right) = \arccos\left(\frac{5}{\sqrt{41}}\right)$ is nice, because inverse trigs are sweet in QI.

Other quadrants, you need a picture & a brain, and understand reference angles



$\arctan\left(-\frac{1}{\sqrt{3}}\right) = -30^\circ$

Depending on θ's need to be positive, you'd do $360^\circ - 30^\circ = 330^\circ$.

$\sqrt{3} - i$ has length $|\sqrt{3} - i| = 2$

and its angle is $330^\circ = \frac{11\pi}{6}$

Consider:

$$2\left(\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right)$$

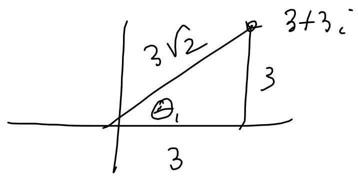
$$= 2\left(\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right) = \sqrt{3} - i \,!$$

$\rightarrow$ TRIGONOMETRIC FORM OF THE COMPLEX Number. Very Handy.

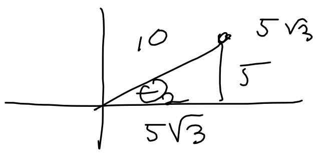
MULTIPLYING 2 Complex #s:

Multiply length. ADD angles.

Do a 45° & 30° example.



$\sqrt{18} = 3\sqrt{2}$



$$(3+3i)(5\sqrt{3}+5i) = 15\sqrt{3} + 15i + 15i\sqrt{3} + 15i^2$$

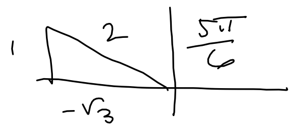
SAME! $= 15\sqrt{3} - 15 + (15 + 15\sqrt{3})i$

Using trig form:

$$\left(3\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right)\left(10\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right)$$

$$= (3\sqrt{2})(10)\left(\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\right)$$

$$= 30\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$$



$$\frac{\pi}{4}\cdot\frac{3}{3} + \frac{\pi}{6}\cdot\frac{2}{2} = \frac{5\pi}{12}$$

$$\cos\frac{5\pi}{12} = \cos\left(\frac{1}{2}\cdot\frac{5\pi}{6}\right) = \sqrt{\frac{1+\cos\frac{5\pi}{6}}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$

$$= \frac{\sqrt{2-\sqrt{3}}}{2} \quad \& \quad \sin\frac{5\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

This gives $30\sqrt{2}\left(\frac{\sqrt{2-\sqrt{3}}}{2} + i\,\frac{\sqrt{2+\sqrt{3}}}{2}\right)$ Trig Form

$= 15\sqrt{3} - 15 + (15 + 15\sqrt{3})\,i$ Rectangular Form

$30\cdot\mathrm{sqrt}(2)\cdot\left(\cos\left(\frac{5\cdot\mathrm{Pi}}{12}\right) + I\cdot\sin\left(\frac{5\cdot\mathrm{Pi}}{12}\right)\right)$ 10.98076211 + 40.98076209 I

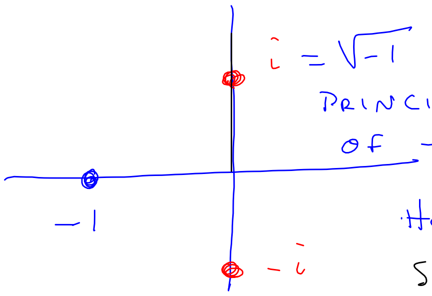
$evalf(15\cdot\mathrm{sqrt}(3) - 15 + (15 + 15\cdot\mathrm{sqrt}(3))\cdot I)$ 10.98076212 + 40.98076212 I

See? They're the same!!!!

THE BIG APPLICATION: COMPUTER GRAPHICS AND ROOTS OF COMPLEX NUMBERS.

$x^2 = -1$

$x^2 + 1 = 0 \implies x = \pm i$ = The 2 $\underbrace{\text{roots}}_{}$ of unity (Square)



$i = \sqrt{-1}$ = PRINCIPAL SQUARE ROOT of -1, SIGNIFIED BY $\sqrt{}$.

How'd we get the 2nd Square Root?

SQUARE ROOTS OF -1 : $\sqrt{-1} = i$
& $-i$

$$-1 = 1(\cos\pi + i\sin\pi)$$

$$i = 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

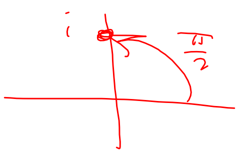
Add $\frac{2\pi}{2}$ to get $-i$:

Because it's a square root. Index 2.

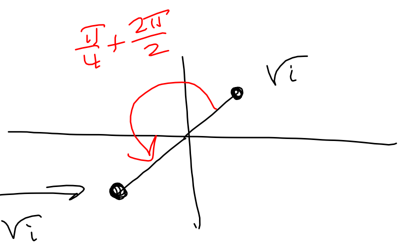
$$\frac{\pi}{2} + \frac{2\pi}{2} = \frac{3\pi}{2}$$

$$-i = 1\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$$

$$\sqrt{i} = 1\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right) \quad !$$



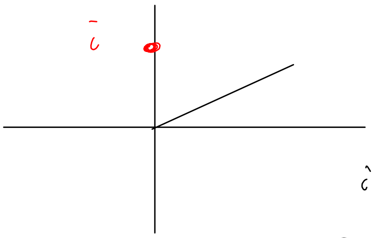
How about the OTHER square root of i?
Add $\frac{2\pi}{2}$ to the $\frac{\pi}{4}$



Not $\sqrt{i}$ but one of its square roots

What about the cube roots of i?

$\sqrt[3]{i}$



$i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

$\sqrt[3]{i} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

& there are 2 more 3^{rd} roots of i:

$\cos\left(\frac{\pi}{6}+\frac{2\pi}{3}\right) + i\sin\left(\frac{\pi}{6}+\frac{2\pi}{3}\right)$

$\cos\left(\frac{\pi}{6}+\frac{4\pi}{3}\right) + i\sin\left(\frac{\pi}{6}+\frac{4\pi}{3}\right)$

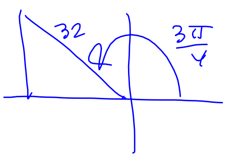
DeMoivre's Theorem:

The n n^{th} roots of $z = r(\cos\theta + i\sin\theta)$

$r^{\frac{1}{n}}\left(\cos\left(\frac{\theta}{n}+\frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta}{n}+\frac{2k\pi}{n}\right)\right)$

$k = 0, 1, 2, \ldots, n-1$

Find ALL 5^{th} roots of $32\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = z$



$\sqrt[5]{z} = 2\left(\cos\frac{3\pi}{20} + i\sin\frac{3\pi}{20}\right)$

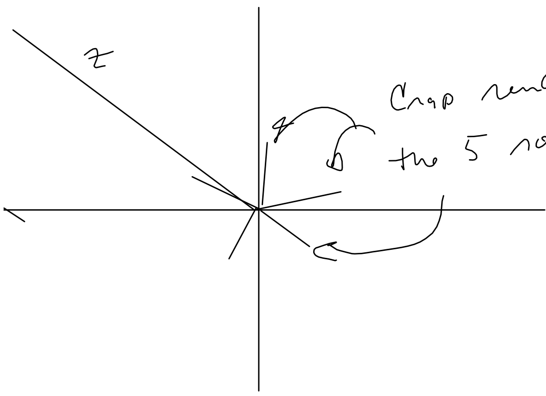
INCREMENT: $\frac{2\pi}{5} = \frac{8\pi}{20}$

Scratch: $\frac{3\pi+8\pi}{20} = \frac{11\pi}{20}$ $\qquad 2\left(\cos\frac{11\pi}{20} + i\sin\frac{11\pi}{20}\right)$

$\frac{11\pi+8\pi}{20} = \frac{19\pi}{20}$ $\qquad 2\left(\cos\frac{19\pi}{20} + i\sin\frac{19\pi}{20}\right)$

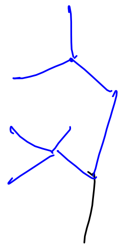
$\frac{19\pi+8\pi}{20} = \frac{27\pi}{20}$ $\qquad 2\left(\cos\frac{27\pi}{20} + i\sin\frac{19\pi}{20}\right)$

$27+8 = 35 \rightarrow \frac{35\pi}{20} \rightarrow = \frac{7\pi}{4}$ $\qquad 2\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$



Crap rendering of the 5 roots

Random walks.



Great Images,
Simple Code

For k = 0 to n-1
Do
Demoivre's for k
REPEAT