Dot Product $\bar{u} = \langle u_1, u_2 \rangle$, $\bar{v} = \langle v_1, v_2 \rangle$

$\Rightarrow \bar{u} \cdot \bar{v} = u_1 v_1 + u_2 v_2$

Note $\bar{u} \cdot \bar{u} = u_1 u_1 + u_2 u_2 = u_1^2 + u_2^2$

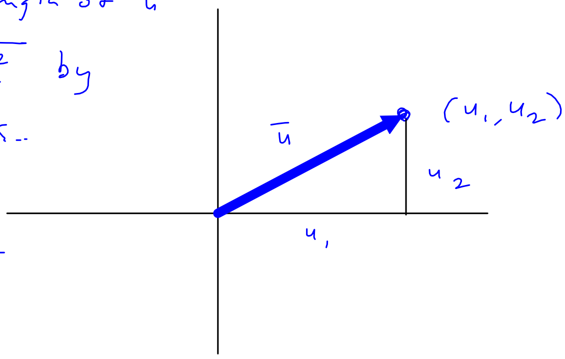
$\|\bar{u}\|$ = length of $\bar{u}$

$= \sqrt{u_1^2 + u_2^2}$ by

Pythagorus..

So,....

$\|\bar{u}\|^2 = \bar{u} \cdot \bar{u}$



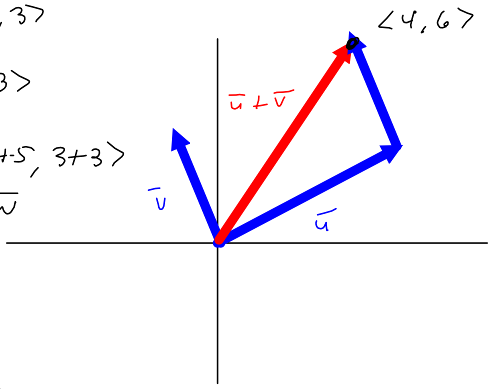
Addition of vectors: Nose to tail.

$\bar{v} = \langle -1, 3 \rangle$

$\bar{u} = \langle 5, 3 \rangle$

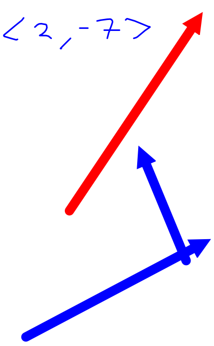
$\Rightarrow \bar{u} + \bar{v} = \langle -1+5, 3+3 \rangle$

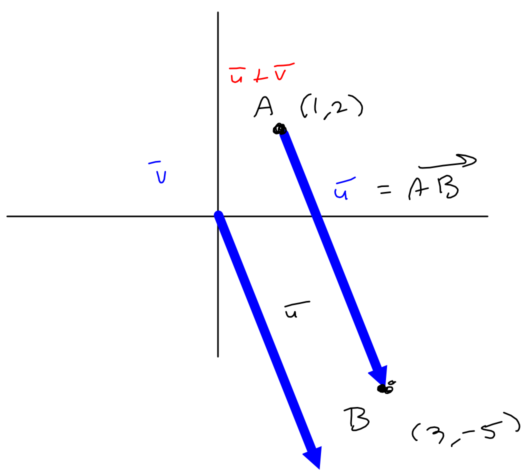
$= \langle 4, 6 \rangle = \bar{w}$



Directed Line segment from $A(1,2)$ & $B = (3,-5)$

is the vector $\bar{u} = \overrightarrow{AB} = \langle 3-1, -5-2 \rangle = \langle 2, -7 \rangle$

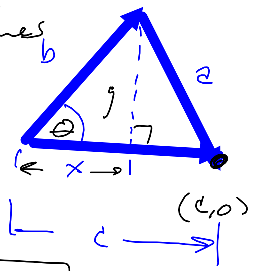




Angle between 2 vectors & Dot Product

$c^2 = a^2 + b^2 - 2ab\cos C$

Law of cosines pic.



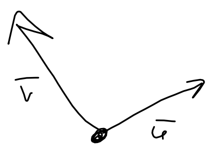
It turns out

See pp 310, 312 for derivation of Law of Cosines & Angle between vectors, in the OLD EDITION.

sigh

$$\boxed{\cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}}$$

by manipulating law of cosines.

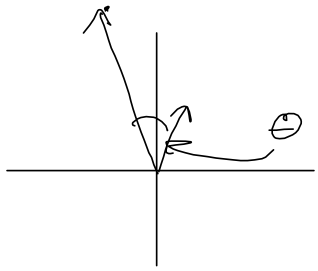


what direction does the sled go?

It's the RESULTANT, $\bar{u} + \bar{v}$

What's the angle between

$\bar{u} = \langle 1, 2 \rangle$ & $\bar{v} = \langle -1, 4 \rangle$?



$$\cos\theta = \frac{\bar{u} \cdot \bar{v}}{\|\bar{u}\|\|\bar{v}\|}$$

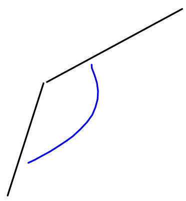
$$= \frac{(1)(-1) + (2)(4)}{\sqrt{1^2+2^2}\sqrt{(-1)^2+4^2}}$$

$$= \frac{-1+8}{\sqrt{5}\sqrt{17}} = \frac{7}{\sqrt{85}} = \cos\theta$$

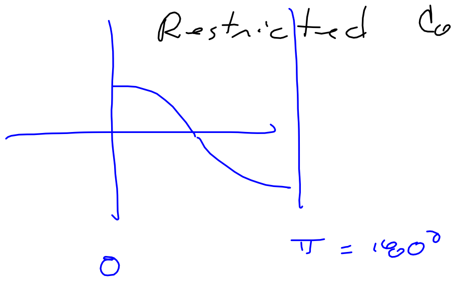
```
log(11)/log(7)
        1.232274406
cos⁻¹(8/√(85))
        29.80502878
cos⁻¹(7/√(85))
        40.60129465
```

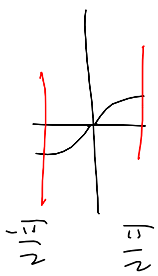
$\approx 40.60129465^\circ$

The angle between 2 vectors is NEVER more than 180°. You can TRUST your $\cos^{-1}$ key on your calculator



Restricted Cosine.

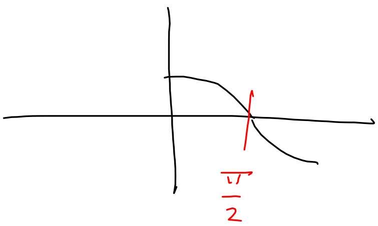




Restricted sine.
(For $\sin^{-1}$

$\bar{u} = \langle 3, 15 \rangle$, $\langle -1, 5 \rangle = \bar{v}$

Check for "orthogonality"



$\cos\theta = 0$ when $\theta = \frac{\pi}{2}$.

FACT: $\bar{u} \cdot \bar{v} = 0 \Longrightarrow$

$\theta = 0$

$$\cos\theta = \frac{\bar{u} \cdot \bar{v}}{\|\bar{u}\| \|\bar{v}\|} = 0$$

ONLY when $\bar{u} \cdot \bar{v} = 0$.

$\bar{u} \cdot \bar{v} = -3 + 75 \neq 0$

NO.

$\bar{u} = 2\bar{i} - 2j = \langle 2, -2 \rangle$

$\bar{v} = -\bar{i} - \bar{j} = \langle -1, -1 \rangle$

$\bar{u} \cdot \bar{v} = -2 + 2 = 0$.

Yes

$\bar{i} = \langle 1, 0 \rangle$, $\bar{j} = \langle 0, 1 \rangle$

$\langle 3, -7 \rangle = 3\bar{i} - 7\bar{j}$

81. Work A tractor pulls a log 800 meters, and the tension in the cable connecting the tractor and log is approximately 15,691 newtons. The direction of the force is 35° above the horizontal. Approximate the work done in pulling the log.

WORK = FORCE · DISTANCE

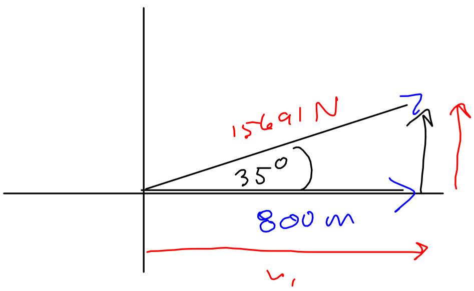
FORCE 1 Newton = $\frac{kg-m}{s^2}$ (units)

1 JOULE = $\frac{kg-m^2}{s^2}$

$\|\vec{u}\| = 15{,}691$ N

Work = horizontal component* of force times distance

u_2* because motion was horizontal.



$\|\vec{u}\| = \langle u_1, u_2 \rangle$

$\frac{u_2}{15691} = \sin 35°$

$\frac{u_1}{15691} = \cos 35°$

$u_1 = 15691 \cos 35°$

= horizontal component of the force.

Work done = F · D = $(15691 \cos 35°)$ Newtons $(800\ m)$

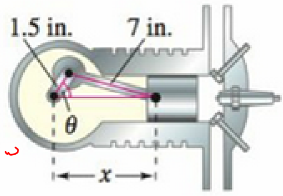
$\approx (12853.31473\ N)(800\ m)$

$\approx 10{,}282{,}651.78$ N-m → Joules!

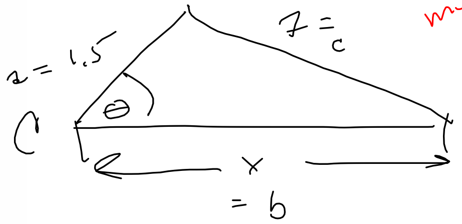
```
     29.80502878
cos⁻¹(7/√(85))
     40.60129465
15691cos(35)
     12853.31473
Ans*800
     10282651.78
```

56. Engine Design

An engine has a seven-inch connecting rod fastened to a crank (see figure).



Toni's mean. A+.



$c^2 = a^2 + b^2 - 2ab\cos\theta$

(a) Use the Law of Cosines to write an equation giving the relationship between x and θ.

(a) $\boxed{49 = 2.25 + x^2 - 2(1.5)(x)\cos\theta}$

(b) Write x as a function of θ. (Select the sign that yields positive values of x.)

(c) Use a graphing utility to graph the function in part (b).

(d) Use the graph in part (c) to determine the total distance the piston moves in one cycle.

(b) $x^2 = 49 - 2.25 + 3x\cos\theta$

$= 46.75 + 3x\cos\theta$

$\Rightarrow \quad x^2 - (3\cos\theta)x - 46.75 = 0$

$a = 1,\ b = 3\cos\theta,\ c = -46.75$

$b^2 - 4ac = 9\cos^2\theta - 4(1)(-46.75)$

$= 9\cos^2\theta + 187$

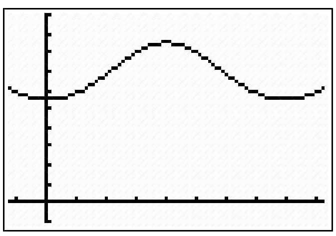
$46.75 \times 4 = 187.00$

$\Rightarrow \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-3\cos\theta \pm \sqrt{9\cos^2\theta + 187}}{2}$

Take the positive:

$> \sqrt{9\cos^2\theta} = 3|\cos\theta|$

$x(\theta) = x = \dfrac{-3\cos\theta + \sqrt{9\cos^2\theta + 187}}{2}$



To find the total distance traveled by the piston, I think all you need is to measure the total change in x through one cycle. So, I'm thinking

(High - Low)*2 in the graph.