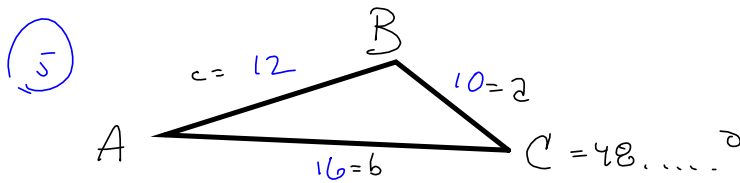


$$c^2 = a^2 + b^2 - 2ab \cos C$$



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$12^2 = 16^2 + 16^2 - 2(16)(16) \cos C$$

$$2(16)(16) \cos C = 10^2 + 16^2 - 12^2 = 100 + 256 - 144$$

$$320 \cos C = 212$$

$$\cos C = \frac{212}{320} = \frac{106}{160} = \frac{53}{80}$$

$$C = \arccos\left(\frac{53}{80}\right) \approx 48.50918314^\circ \approx C$$

```
cos-1(53/80)
48.50918314
```

$$\frac{\sin B}{16} = \frac{\sin C}{12} \Rightarrow \sin B = \frac{16 \sin C}{12} = \frac{4 \sin C}{3} = \frac{4}{3} \sin(48.50918314^\circ)$$

$$\Rightarrow B = \sin^{-1}(\text{previous}) \approx 87.13401602^\circ \approx B$$

```
cos-1(53/80)
48.50918314
sin(Ans)*4/3
.9987492178
sin-1(Ans)
87.13401602
```

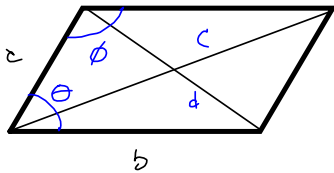
$$\Rightarrow A = 180^\circ - B - C = 180^\circ - 87.13^\circ - 48.51^\circ$$

$$\approx 44.35680084^\circ \approx A$$

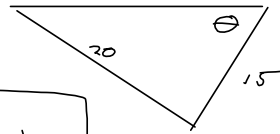
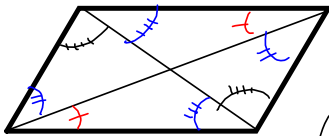
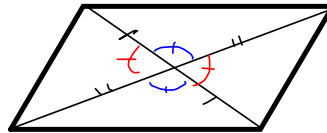
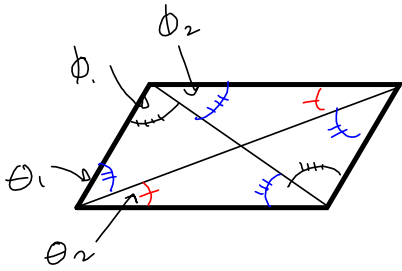
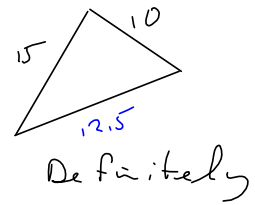
```
sin(Ans)*4/3
.9987492178
sin-1(Ans)
87.13401602
180-Ans-48.50918314
44.35680084
```

$A \approx 44.36^\circ$   
 $B \approx 87.13^\circ$   
 $C \approx 48.51^\circ$

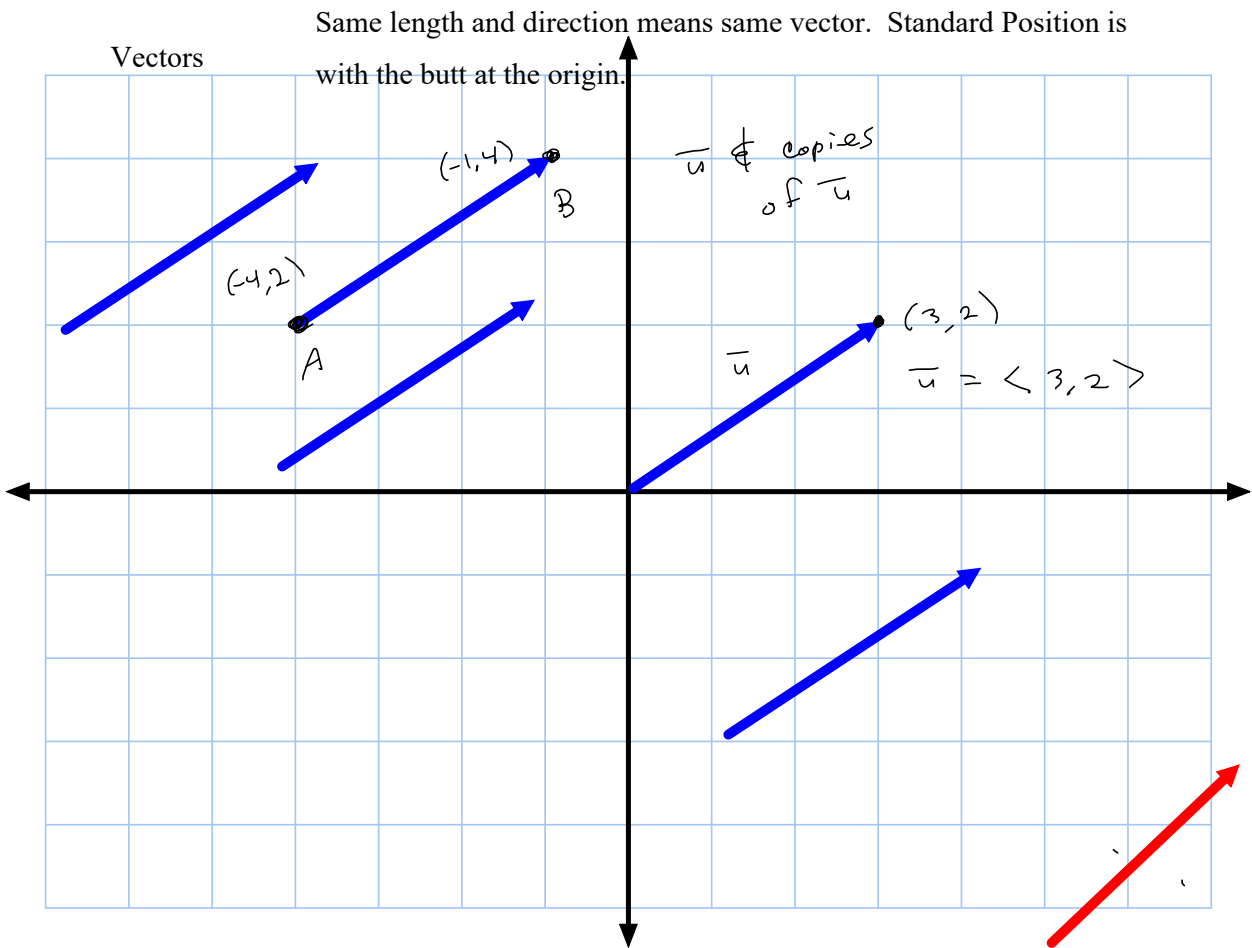
The diagonals meet at their midpoints,  
because of CONGRUENT triangles.



(29)  $a = 15$   
 $c = 25$   
 $d = 20$   
 $b = ?$   
 $\theta = ?$   
 $\phi = ?$



#4 Heron's



$\vec{AB}$  = Directed line segment from A to B  
is equivalent to  $\vec{u}$

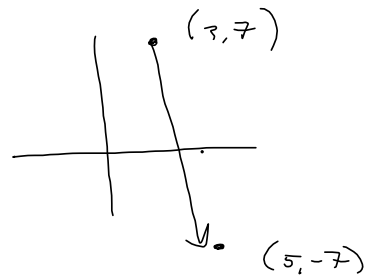
$A = (-4, 2), B = (-1, 4)$

$\vec{AB} = \langle -1 - (-4), 4 - 2 \rangle = \langle 3, 2 \rangle = u$  ! See?

Subtract the "from" from the "to" to write the vector from A to B

$A = (3, 7), B = (5, -7)$

$\Rightarrow u = \vec{AB} = \langle 2, -14 \rangle$   
 $= \langle 5 - 3, -7 - 7 \rangle$

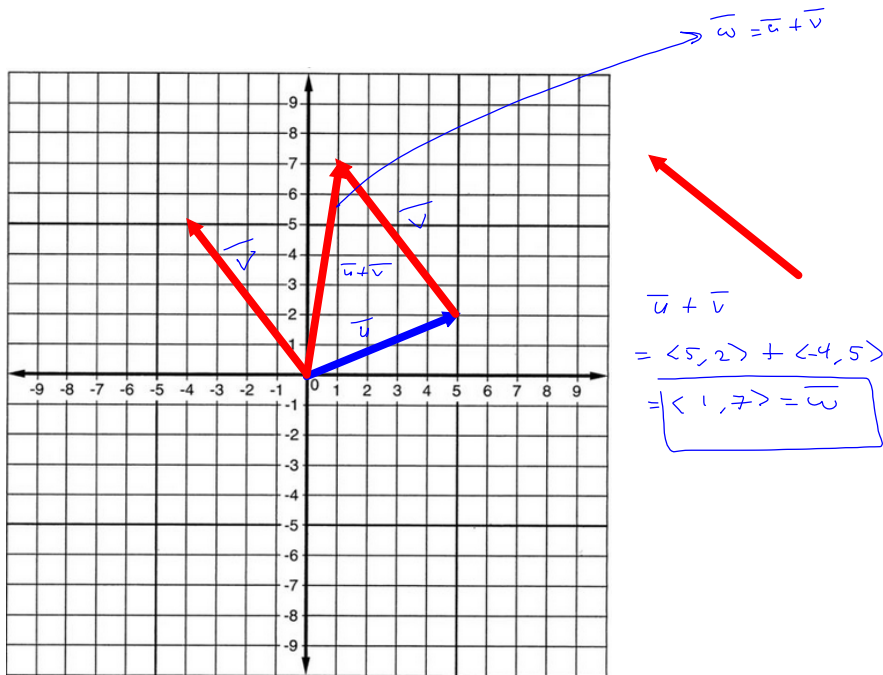


Adding Vectors

$$\vec{u} = \langle u_1, u_2 \rangle \quad \vec{v} = \langle v_1, v_2 \rangle \quad \vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$$

$\swarrow$  1<sup>st</sup> component/entry       $\nwarrow$  2<sup>nd</sup> component

Geometric Addition:  
Nose to Tail



$\vec{u} \cdot \vec{v}$  = Dot Product

$$\vec{u} \cdot \vec{v} = \langle 5, 2 \rangle \cdot \langle -4, 5 \rangle = (5)(-4) + (2)(5) = -10$$

$$= u_1 v_1 + u_2 v_2$$

$$\vec{u} \cdot \vec{u} = 5^2 + 2^2 = 25 + 4 = 29 = \|\vec{u}\|^2$$

= Square of the length/magnitude of  $\vec{u}$ ,

Length =  $\sqrt{\vec{u} \cdot \vec{u}}$

