§2.5 #s 7–17, 21–29, 33–41, 45, 49–53, 57–63

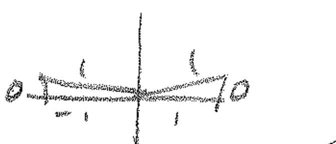
#s 7–14 Find exact solns in $[0, 2\pi)$

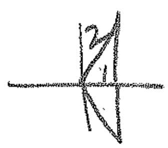
(7) $\sin(2x) - \sin x = 0$

$2\sin x \cos x - \sin x = 0$

$\sin x\,(2\cos x - 1) = 0$

$\sin x = 0 \qquad \cos x = \frac{1}{2}$





$x = 0, \pi, \cancel{2\pi} \qquad x = \frac{\pi}{3}, \frac{5\pi}{3}$

$= 0, 180^\circ \qquad 60^\circ, 300^\circ$

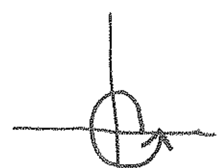
$\sin 0 - \sin 0 = 0$ ✓

$\sin 2\pi - \sin \pi = 0$ ✓

$\sin\frac{2\pi}{3} - \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = 0$ ✓

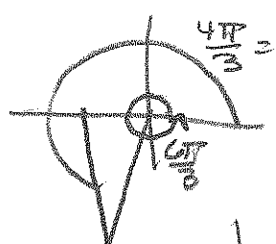
$\sin\left(2\left(\frac{5\pi}{3}\right)\right) - \sin\left(\frac{5\pi}{3}\right) =$

$\sin\left(\frac{10\pi}{3}\right) - \sin\left(\frac{5\pi}{3}\right)$

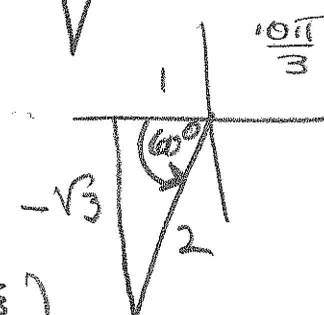


$\frac{6\pi}{3} = 2\pi$ add $\frac{4\pi}{3} = \frac{4\pi}{3} \cdot \frac{180}{\pi} = 240^\circ$

$\frac{4\pi}{3} = 240^\circ = 180^\circ + 60^\circ$



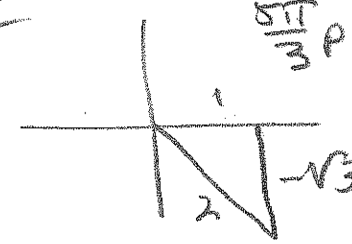
$\frac{10\pi}{3}$ pic



$\to -\frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right)$

$= 0$ ✓

$\frac{5\pi}{3}$ pic



122 §2.5 #'s 9-17, 21-29, 33-41, 45, 49-53

(9) $\cos(2x) - \cos x = 0$

$2\cos^2 x - 1 - \cos x = 0$

$2\cos^2 x - \cos x - 1 = 0$

$2u^2 - u - 1 = 0$

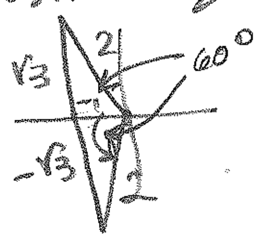
$(2u \quad 1)(u \quad 1)$

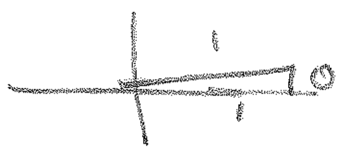
$(2u+1)(u-1)$

$2u+1=0$ $\qquad$ $u-1=0$

$u = -\frac{1}{2}$ $\qquad$ $u = 1$

$\cos x = -\frac{1}{2}$ $\qquad$ $\cos x = 1$





$x = 0$

$x = 180^\circ + 60^\circ = 240^\circ = \frac{4\pi}{3}$

$x = 180^\circ - 60^\circ = 120^\circ = \frac{2\pi}{3}$

$$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0}$$

$\cos\left(2\left(\frac{2\pi}{3}\right)\right) - \cos\left(\frac{2\pi}{3}\right)$

$= \cos\left(\frac{4\pi}{3}\right) - \cos\left(\frac{2\pi}{3}\right)$

$= -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0$ ✓

etc.

'22 § 2.5 # 5, 11–17, 21–29, 33–41, 45, 49–53

(11) $\sin(4x) = -2\sin(2x)$

$2\sin(2x)\cos(2x) + 2\sin(2x) = 0$

$2\sin(2x)\left[\cos(2x) + 1\right] = 0$

$2\sin(2x) = 0$ $\qquad$ $\cos(2x) = -1$

$\sin(2x) = 0$

$2x = \pi$

$x = \frac{\pi}{2}$

$2x = 0, \pi, 2\pi$

$\boxed{x = 0, \frac{\pi}{2}, \pi}$

$\sin(0) = -2\sin(0)$

$\sin(2\pi) = -2\sin(2\pi)$

$0 = 0$ ✓

$\sin(4\pi) = -2\sin(4\pi)$

$0 = 0$ ✓

(13) $\tan(2x) - \cot(x) = 0$

$\dfrac{2\tan(x)}{1-\tan^2(x)} - \cot(x) = 0$

$\dfrac{2\tan(x)}{\sec^2(x)} - \cot(x) = 0$

$\dfrac{2\frac{\sin x}{\cos x}}{\frac{1}{\cos^2 x}} - \dfrac{\cos x}{\sin x} = 0$

$2\dfrac{\sin x}{\cos x} \cdot \dfrac{\cos^2 x}{1} - \dfrac{\cos x}{\sin x} = 0$

$2\sin x\cos x - \dfrac{\cos x}{\sin x} = 0$

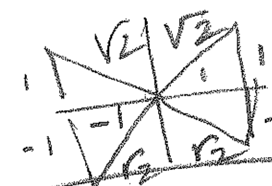
$\dfrac{2\sin^2 x\cos x}{\sin x} - \dfrac{\cos x}{\sin x} = 0$

$2\sin^2 x\cos x - \cos x = 0$

$\cos x(2\sin^2 x - 1) = 0$

$\sin x = \pm\frac{1}{\sqrt{2}}$

$\cos x = 0$



$\boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$

$\boxed{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

122 § 2.5 #s 15, 17, 21-9, 33-41, 49-53

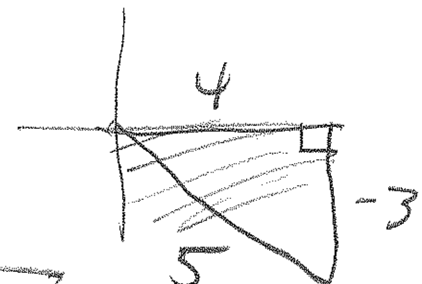
(15) Use double-angle to re-write #s 15-18

$6\sin x\cos x = 3(2\sin x\cos x) = \boxed{3\sin(2x)}$

(17) $6\cos^2 x - 3 = 3(2\cos^2 x - 1) = \boxed{3\cos(2u)}$

#s 21-24 Find sin(2u), cos(2u), tan(2u) using double-angle.

(21) $\sin u = -\frac{3}{5}$, $\frac{3\pi}{2} < u < 2\pi$



$\sin(2u) = 2\sin u\cos u$

$= 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = \boxed{-\frac{24}{25} = \sin(2u)}$

$\cos(2u) = 1 - 2\sin^2(u)$ → Nope! same mistake I made in class. I plugged in sin(2u), instead of sin(u), here

$= 1 - 2\left(\frac{24}{25}\right)^2$

$\cancel{= \frac{625 - 2(576)}{625}}$

$\cancel{= \frac{625 - 1152}{625}}$

$\cancel{= \frac{-527}{625}}$

$\cos(2u) = 1 - 2\left(-\frac{3}{5}\right)^2$

$= 1 - 2\left(\frac{9}{25}\right)$

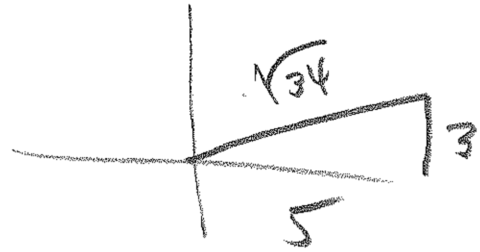
$= \frac{25 - 18}{25} = \boxed{\frac{7}{25} = \cos(2u)}$

$\therefore \tan(2u) = -\frac{24}{25} \cdot \frac{25}{7} = -\frac{24}{7}$

$\boxed{\tan(2u) = -\frac{24}{7}}$

122 §2.5 #s 23-9, 33-41, 49-53

(23) $\tan(u) = \frac{3}{5} \quad 0 < u < \frac{\pi}{2}$



$\sin(2u) = 2\sin u \cos u$

$\cancel{= 2\left(\frac{3}{\sqrt{24}}\right)\left(\frac{5}{\sqrt{24}}\right) = \frac{30}{24} = \frac{5}{4}}$ No way!

Wait! $\sqrt{34}$, not $\sqrt{24}$!

$= 2\left(\frac{3}{\sqrt{34}}\right)\left(\frac{5}{\sqrt{34}}\right) = \frac{30}{34} = \boxed{\frac{15}{17} = \sin(2u)}$

$\cos(2u) = 1 - 2\sin^2(u)$

$= 1 - 2\left(\frac{3}{\sqrt{34}}\right)^2$

$= 1 - 2\left(\frac{9}{34}\right)$

$= 1 - \frac{9}{17}$

$= \frac{17-9}{17} = \boxed{\frac{8}{17} = \cos(2u)}$

$\therefore \tan(2u) = \frac{15}{17} \cdot \frac{17}{8} = \frac{15}{8}$

$\boxed{\tan(2u) = \frac{15}{8}}$

122 § 2.5 #s 25–9, 33–41, 49–53

25 We derive the formula for $\cos(4x)$

$$\cos(4x) = \cos(2\cdot 2x) = 1-2\sin^2(2x)$$

$$= 1-2(2\sin x\cos x)^2$$

$$= 1-2(4\sin^2 x\cos^2 x)$$

$$= 1-8\sin^2 x\cos^2 x$$

Book wants final answer entirely in terms of $\cos(x)$:

$$1-8(1-\cos^2 x)\cos^2 x$$

$$= 1-8(\cos^2 x-\cos^4 x)$$

$$= \boxed{1-8\cos^2 x+8\cos^4 x}$$

$$= 8\cos^4 x-8\cos^2 x+1$$

122 §2.5 #s 29, 33–41, 49–53

(29) Re-write in terms of 1st power of cosine.

$$\tan^4(2x) = \left(\tan^2(2x)\right)^2 = \left(\frac{1-\cos(4x)}{1+\cos(4x)}\right)^2$$

$$= \frac{\cos^2(4x) - 2\cos(4x) + 1}{\cos^2(4x) + 2\cos(4x) + 1}$$

$$= \frac{\left(\frac{1+\cos(8x)}{2} - 2\cos(4x) + 1\right)(2)}{\left(\frac{1+\cos(8x)}{2} + 2\cos(4x) + 1\right)(2)}$$

$$= \frac{1+\cos(8x) - 4\cos(4x) + 2}{1+\cos(8x) + 4\cos(4x) + 2}$$

$$= \frac{\cos(8x) - 4\cos(4x) + 3}{\cos(8x) + 4\cos(4x) + 3}$$

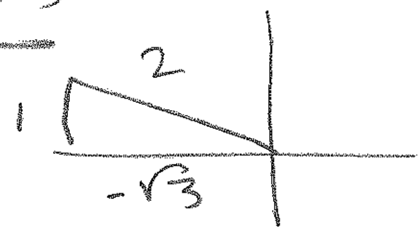
122 §2.5 #s 33-41, 49-53

#s 33-36 Use $\frac{1}{2}$-angle ids to determine exact values of sine, cosine and tangent.

(33) $\theta = 75^\circ \longrightarrow 2\theta = 150^\circ$

$\frac{u}{2} = 75^\circ \longrightarrow u = 150^\circ$ Better

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos(u)}{2}} = \sqrt{\frac{1-\cos(150^\circ)}{2}}$$



$$= \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{\sqrt{3}+2}{4}} = \boxed{\frac{\sqrt{\sqrt{3}+2}}{2} = \sin 75^\circ}$$

$$\cos(75^\circ) = \sqrt{\frac{1+\cos(150^\circ)}{2}} = \boxed{\frac{\sqrt{2-\sqrt{3}}}{2} = \cos(75^\circ)}$$

$$\tan 75^\circ = \frac{\sin 75^\circ}{\cos 75^\circ} = \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} = \sqrt{\frac{(2+\sqrt{3})^2}{4-3}}$$

I

$$= \frac{2+\sqrt{3}}{1} = \boxed{2+\sqrt{3}}$$

$\tan(75^\circ)$

122 §2.5 #s 35–41, 49–53

35. $\frac{\pi}{8} = \frac{u}{2} \Rightarrow u = \frac{\pi}{4}$

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$

$$= \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2} = \sin\frac{\pi}{8}$$

$$\cos\frac{\pi}{8} = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}} = \ldots \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\tan\frac{\pi}{8} = \sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}} = \frac{\sqrt{(2-\sqrt{2})^2}}{\sqrt{4-2}} = \frac{2-\sqrt{2}}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} - 1 = \boxed{\sqrt{2}-1 = \tan\frac{\pi}{8}}$$

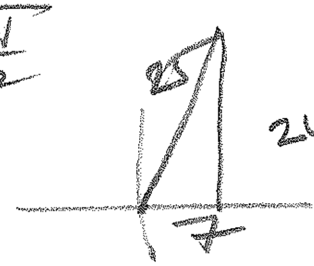
#s 37–40

(a) Find Quadrant for $\frac{u}{2}$

(b) Find $\sin\left(\frac{u}{2}\right), \cos\left(\frac{u}{2}\right), \tan\left(\frac{u}{2}\right)$

127 §2.5 #s 37-41, 49-53

37 $\cos u = \frac{7}{25}$, $0 < u < \frac{\pi}{2}$



$\Rightarrow \boxed{\frac{u}{2} \in QI}$ (a)

$\sqrt{625-49} = \sqrt{576} = 24$

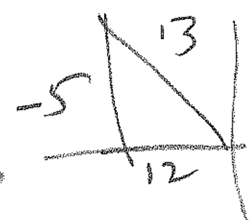
$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos u}{2}}$

$= \sqrt{\frac{1-\frac{7}{25}}{2}} = \sqrt{\frac{25-7}{50}} = \sqrt{\frac{18}{50}} = \sqrt{\frac{9}{25}} = \boxed{\frac{3}{5} = \sin\left(\frac{u}{2}\right)}$

$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1+\cos u}{2}} = \sqrt{\frac{32}{50}} = \sqrt{\frac{16}{25}} = \boxed{\frac{4}{5} = \cos\left(\frac{u}{2}\right)}$

$\Rightarrow \tan\left(\frac{u}{2}\right) = \frac{3}{5}\cdot\frac{5}{4} = \boxed{\frac{3}{4} = \tan\left(\frac{u}{2}\right)}$

39 $\tan u = -\frac{5}{12}$ $\frac{3\pi}{2} < u < 2\pi \Longrightarrow$



$\frac{3\pi}{4} < \frac{u}{2} < \pi \Longrightarrow$ $\boxed{\frac{u}{2} \in Q\,II}$

$\sin\frac{u}{2} = \sqrt{\frac{1-\cos u}{2}}$

$= \sqrt{\frac{1-\frac{12}{13}}{2}} = \sqrt{\frac{1}{26}} = \frac{1}{\sqrt{26}}$

$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{13+12}{26}} = \boxed{\frac{5}{\sqrt{26}} = \cos\left(\frac{u}{2}\right)}$

$\boxed{\tan\left(\frac{u}{2}\right) = \frac{1}{5}}$