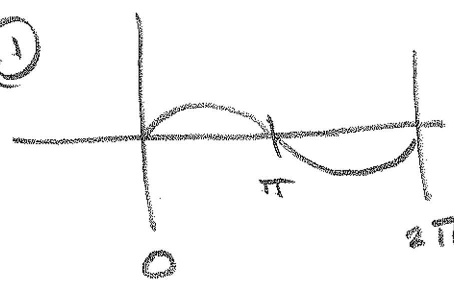
MAT 122 §2.4 IV #s 57-8-9, 80B, 85, 91, 97-8

#s 57-64 Prove the identity.

(57) $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

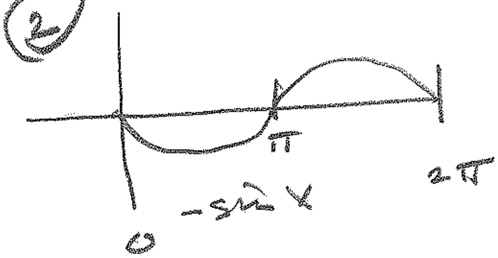
$\sin\left(\frac{\pi}{2} - x\right) = \sin\left(-\left(x - \frac{\pi}{2}\right)\right) = -\sin\left(x - \frac{\pi}{2}\right)$, since sine is odd. This is a right shift of $\frac{\pi}{2}$ of −sine:

(1)



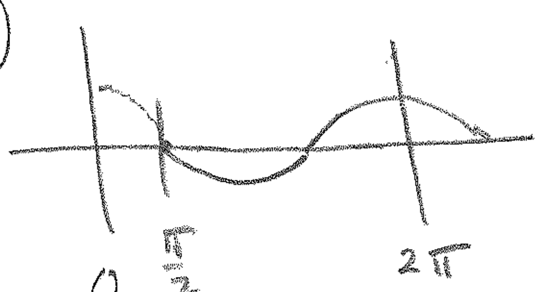
$\sin x$

(2)



$-\sin x$

(3)



$-\sin\left(x - \frac{\pi}{2}\right)$

That sure <u>Looks</u> like cosine! This is <u>not</u> proof, but an api

To <u>PROVE</u> we use angle sum identity:

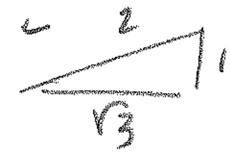
$$\sin\left(\frac{\pi}{2} - x\right) = \sin\frac{\pi}{2}\cos(-x) + \cos\frac{\pi}{2}\sin(-x)$$
$$= (1)(\cos(-x)) + (0)(\sin(-x))$$
$$= \cos(-x) = \cos x,$$ because cosine is even.

58 $\sin\left(\frac{\pi}{2}+x\right) = \cos x$. Seems pretty obvious that $\sin\left(x+\frac{\pi}{2}\right)$ is left shift by $\frac{\pi}{2}$ & it lines up with cosine. But lets prove it:

$$\sin\left(x+\frac{\pi}{2}\right) = \sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2}$$

$$= (\sin x)(0) + (\cos x)(1) = \cos x \blacksquare$$

59 $\sin\left(\frac{\pi}{6}+x\right) = \frac{1}{2}(\cos x + \sqrt{3}\sin x)$!?



$$\sin\left(x+\frac{\pi}{6}\right) = \sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6}$$

$$= (\sin x)\left(\frac{\sqrt{3}}{2}\right) + (\cos x)\left(\frac{1}{2}\right)$$

$$= \frac{1}{2}\left[\sqrt{3}\sin x + \cos x\right] \blacksquare$$

$\boxed{80B}$ The equation of a standing wave is obtained by adding the displacements of waves moving in opposite directions. Assume each wave has an amplitude A, period T, and wavelength λ ("Lambda")

Given
$$y_1 = A\cos\left(2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)\right) \text{ and}$$
$$y_2 = A\cos\left(2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)\right), \text{ then show that}$$

122 §2.4 II #s 85, 91, 97–8

85 Let $f(h) = \dfrac{\sin\left(\frac{\pi}{3}+h\right) - \sin\left(\frac{\pi}{3}\right)}{h}$ and

$$g(h) = \frac{\left(\cos\frac{\pi}{3}\right)(\sin h)}{h} - \frac{\left(\sin\frac{\pi}{3}\right)(1-\cos h)}{h}$$

(a) Domains of f & g are both

$\mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$

$h = 0$ is only bad apple.

(b) We complete the table:

~~h~~	~~.5~~	~~.2~~	~~.1~~	~~.05~~	~~.02~~	~~.01~~
~~$f(h)$~~	~~.017449~~	~~.0174490~~	~~.0174501~~	~~.0174502~~	~~.0174503~~	~~.01745035~~
~~$g(h)$~~	~~.99983~~	~~.9998324~~	~~.99983~~	~~.999833~~	~~.9998333~~	~~.99983295~~

I'd rather see them JUST look at $h = .01, .001, .0001$

$\dfrac{\sin h}{h} \xrightarrow{h \to 0} 1$ and $\dfrac{1-\cos h}{h} \xrightarrow{h \to 0} 0$ as $h \to 0$

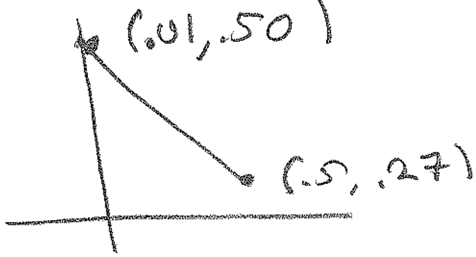
These limits are PROVED in calculus I because they're needed

Not sure what they're trying to do to us, because I'm in degrees mode on my calculator!

122 §2.4 II #s 85, 91, 97-8

h	$f(h)$	$g(h)$
.5	.267392	.2673923
.2	.410359	.41035908
.1	.455902	.45590189
.05	.4781456	.4781456
.02	.4913066̅8	.4913067033
.01	.49566154	.4956615743
		⋮
		0.5

(c)



(d) f & g are identical

$$\lim_{h\to 0} f(h) = \lim_{h\to 0} g(h) = \frac{1}{2} = .5$$

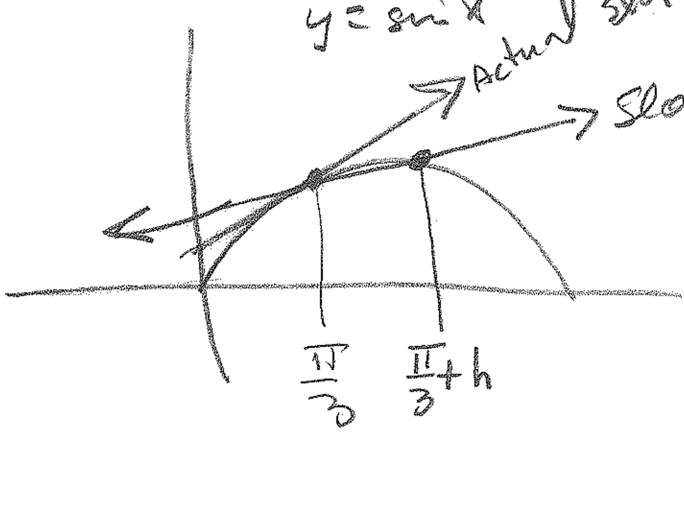
Here's a picture:

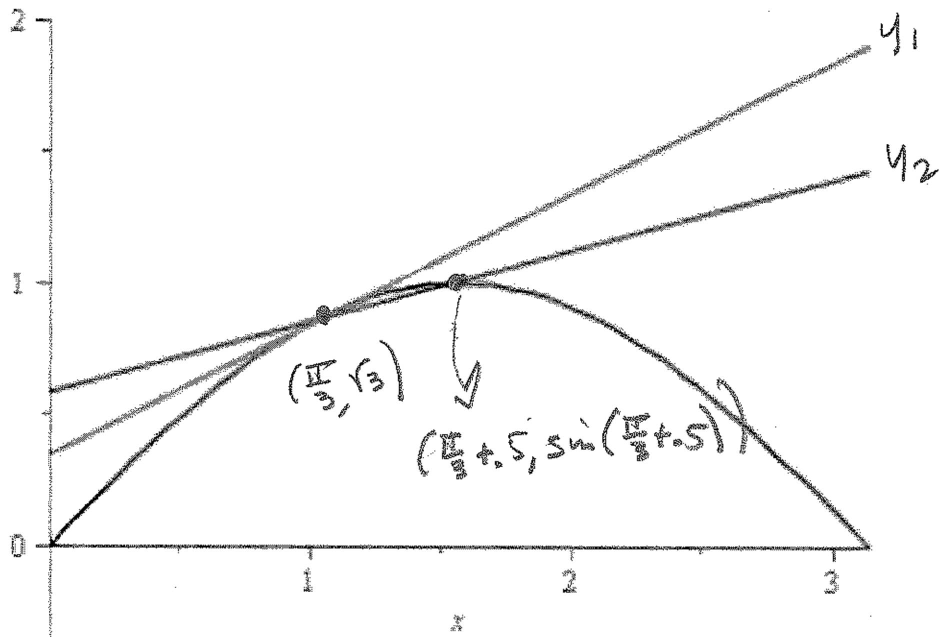
y = sin x

Actual slope @ $x=\frac{\pi}{3}$ is 0.5, by our work!

Slope of this line is $\frac{f(\frac{\pi}{3}+h) - f(\frac{\pi}{3})}{h}$

for $f(x) = \sin x$





y_1 is line between $(\frac{\pi}{3}, \sqrt{3})$ & $(\frac{\pi}{3}+0.1, \sin(\frac{\pi}{3}+.01))$

Its slope m is the value of $f(0.01)$ from the table. Very close to $m = .5$.

y_2 is line between $(\frac{\pi}{3}, \sqrt{3})$ and $(\frac{\pi}{3}+.5, \sin(\frac{\pi}{3}+.5))$

Our work is saying that as you take your second point closer and closer to $(\frac{\pi}{3}, \sqrt{3})$, the slope of the line between the points is getting closer and closer to 0.5.

By "coincidence," $\cos(\frac{\pi}{3}) = .5$

Note: sine hits its high point @ $x = \frac{\pi}{2}$

There, its slope is zero. Coincidentally, $\cos(\frac{\pi}{2}) = 0$. Or is it coincidence?

'122 §2.4 II #s 85, 91, 97-8.

In Calc I, you will PROVE that $\cos x$ tells you the slope of $\sin x$ at every x. It boils down to this proof, taken largely from EXAMLE 8:

$\sin(x+h) = \sin x \cos h + \cos x \sin x$, so

$$\frac{\sin(x+h)-\sin(x)}{h} = \text{slope between } x \text{ \& } x+h$$

$$\left.\begin{aligned}
&= \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} \\
&= \frac{\sin x \cos h - \sin x + \cos x \sin h}{h} \\
&= \frac{\sin x (\cos h - 1) + \cos x \sin h}{h} \\
&= \sin x \left(\frac{\cos h - 1}{h}\right) + \cos x \left(\frac{\sin h}{h}\right)
\end{aligned}\right\} \text{You guys can do this!}$$

The $\frac{\cos h - 1}{h}$ vanishes! The $\frac{\sin h}{h}$ approaches 1, and the slope of $\sin x$ approaches the value of $\cos x$!

122 § 2.4 IV #s 91, 97-8

#s 91-94 Use the following formulas, from #s 89 & 90 to write the trig expression in the form

(a) $\sqrt{a^2+b^2}\ \sin(B\theta + C)$

(b) $\sqrt{a^2+b^2}\ \cos(B\theta - C)$

Formulas from 89 & 90:

89 $a\sin(B\theta) + b\cos(B\theta) = \sqrt{a^2+b^2}\ \sin(B\theta + C)$, where $C = \arctan\left(\frac{b}{a}\right)$ & $a > 0$

90 $a\sin(B\theta) + b\cos(B\theta) = \sqrt{a^2+b^2}\ \cos(B\theta - C)$, where $C = \arctan\left(\frac{a}{b}\right)$ & $b > 0$

91 $\sin\theta + \cos\theta$, $a = 1$, $b = 1$, $B = 1$ $\Longrightarrow$

(a)
$$\sin\theta + \cos\theta = \sqrt{1^2+1^2}\ \sin(\theta + \arctan(1)) = \boxed{\sqrt{2}\ \sin\left(\theta + \frac{\pi}{4}\right)}$$

(b)
$$\sin\theta + \cos\theta = \sqrt{2}\cos(\theta - \arctan(1)) = \boxed{\sqrt{2}\ \cos\left(\theta - \frac{\pi}{4}\right)}$$

Notice the sine in a & the cosine in b are $\frac{\pi}{2}$ out-of-phase?

122 S&P IX #s 97-98

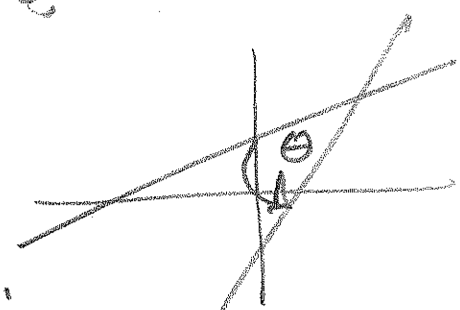
(97)

Assume that $y_1 = m_1 x + b_1$ &

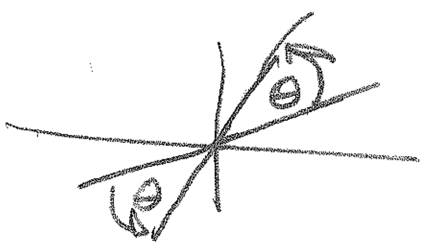
$y_2 = m_2 x + b_2$ are lines. Dub

Assume m_1 & m_2 are positive

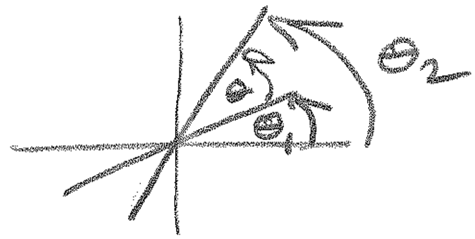
Derive a formula for the angle Θ between the two lines.



Position them with intersection at origin



tan



Notice $\Theta = \Theta_2 - \Theta_1$?

Hmmm: $\tan \Theta_1 = m_1$

$\tan \Theta_2 = m_2$

$\tan(\Theta_2 - \Theta_1) = \tan \Theta$

$$\tan(\Theta_2 - \Theta_1) = \frac{\tan \Theta_2 - \tan \Theta_1}{1 + \tan \Theta_2 \tan \Theta_1} = \boxed{\frac{m_2 - m_1}{1 + m_2 m_1}} !$$

Cool!

$$\boxed{\Theta = \arctan\left(\frac{m_2 - m_1}{1 + m_2 m_1}\right)} !$$

'22 S 2nd II

$m = 1$

45°

$m = \sqrt{3}$

60°

(97) $y = x \,, \quad y = \sqrt{3}\, x$

$m_1 = 1, \quad m_2 = \sqrt{3} \implies$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}(1)} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$\therefore$ so $\theta = \arctan\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) \approx .2617994$ radians OR $\boxed{15^\circ}$

LOGIC: $\theta_1 = 45^\circ$, $\theta_2 = 60^\circ \implies \theta = 15^\circ$ ✓

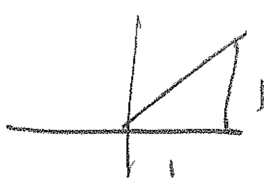
(98) $y = x$ $\therefore$ $y = \frac{1}{\sqrt{3}} x$

$m_1 = 1 \quad m_2 = \frac{1}{\sqrt{3}}$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1} = \frac{\frac{1}{\sqrt{3}} - 1}{1 + \left(\frac{1}{\sqrt{3}}\right)(1)} = \frac{\frac{1}{\sqrt{3}} - 1}{\frac{1}{\sqrt{3}} + 1}$$

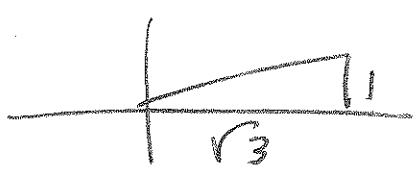
$$= \frac{\frac{1 - \sqrt{3}}{\sqrt{3}}}{\frac{1 + \sqrt{3}}{3}} = \frac{1 - \sqrt{3}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{1 + \sqrt{3}} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}} \implies$$

$$\theta = \arctan\left(\frac{1 - \sqrt{3}}{1 + \sqrt{3}}\right) = \boxed{-15^\circ}$$



$m = 1$

45°



$m = \frac{1}{\sqrt{3}}$

30°