122 §2.4 I #s 1-9 ALL, 11-25, 31, 35-39, 47, 48, 49, 53, 54

1) $\sin(u-v) = \sin u \cos v - \cos u \sin v$

2) $\cos(u+v) = \cos u \cos v - \sin u \sin v$

3) $\tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$

4) $\sin(u+v) = \sin u \cos v + \cos u \sin v$

5) $\cos(u-v) = \cos u \cos v + \sin u \sin v$

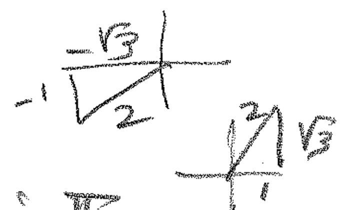
6) $\tan(u-v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

7) $\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

(a)

$$= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} - \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} = \boxed{\frac{1-\sqrt{3}}{2\sqrt{2}}}$$

(b) $\cos\frac{\pi}{4} + \cos\frac{\pi}{3} = \boxed{\frac{1}{\sqrt{2}} + \frac{1}{2}} = \dfrac{\sqrt{2}+2}{2\sqrt{2}}$



8) (a) $\sin\left(\frac{7\pi}{6} - \frac{\pi}{3}\right) = \sin\frac{7\pi}{6}\cos\frac{\pi}{3} - \cos\frac{7\pi}{6}\sin\frac{\pi}{3}$

$$= \frac{1}{\sqrt{3}} \cdot \frac{1}{2} - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{1}{2\sqrt{3}} + \frac{3}{4}} = \frac{2+3\sqrt{3}}{4\sqrt{3}}$$

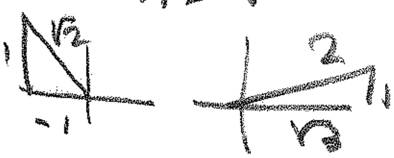
(b) $\sin\frac{7\pi}{6} - \sin\frac{\pi}{3} = \boxed{\frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{2}} = \dfrac{2-3}{2\sqrt{3}} = -\dfrac{1}{2\sqrt{3}} = \dfrac{-\sqrt{3}}{6}$

Nope. Should be -1/2, dummy!
Good eye, Logan.

122 §2.4 I #s 9-25, 31, 35-9, 47, 48, 49, 53, 54

(9) (a) $\sin(135^\circ - 30^\circ) = \sin(135^\circ)\cos(30^\circ) - \cos(135^\circ)\sin(30^\circ)$

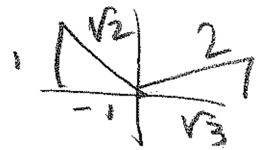
$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \boxed{\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$



(b) $\sin 135^\circ - \cos 30^\circ = \boxed{\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}} = \frac{2-\sqrt{6}}{2\sqrt{2}}$

#s 11-26 Find <u>exact</u> values of sine, cosine, and tangent.

(11) $\frac{11\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6} = \theta = u + v$



$$\sin\left(\frac{3\pi}{4} + \frac{\pi}{6}\right) = \sin\frac{3\pi}{4}\cos\frac{\pi}{6} + \cos\frac{3\pi}{4}\sin\frac{\pi}{6}$$

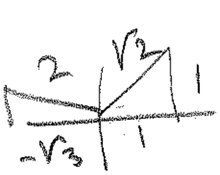
$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \boxed{\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}} = \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

(13) $\frac{17\pi}{12} = \frac{20\pi}{12} - \frac{3\pi}{12} = \frac{10\pi}{6} - \frac{\pi}{4} = \frac{5\pi}{3} - \frac{\pi}{4}$

Book Did $\frac{27\pi}{12} - \frac{10\pi}{12} = \frac{9\pi}{4} - \frac{5\pi}{6}$ Same Diff.

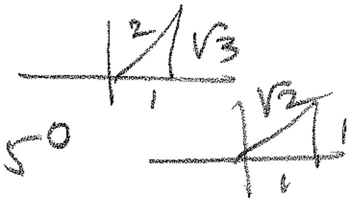
The one I did I found sooner on my own.

$$\sin\left(\frac{17\pi}{12}\right) = \sin\left(\frac{9\pi}{4}\right)\cos\left(\frac{5\pi}{6}\right) - \cos\left(\frac{9\pi}{4}\right)\sin\left(\frac{5\pi}{6}\right)$$



$$= \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

122 §2.4 II #s 15-25, 31, 35-9, 47-8-9, 53-4



15 $105^\circ = 60^\circ + 45^\circ$

$$\sin(60^\circ+45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \boxed{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

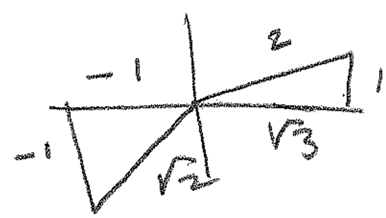
$$\cos(60^\circ+45^\circ) = \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \boxed{\frac{1-\sqrt{3}}{2\sqrt{2}}}$$

$$\tan(60^\circ+45^\circ) = \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ}$$

$$= \frac{\frac{\sqrt{3}}{1} + \frac{1}{1}}{1 - \frac{\sqrt{3}}{1} \cdot \frac{1}{1}} = \boxed{\frac{\sqrt{3}+1}{1-\sqrt{3}}}$$

17 $195^\circ = 225^\circ - 30^\circ$ (I can "see" $150^\circ + 45^\circ$)



$$\sin 195^\circ = \sin 225^\circ \cos 30^\circ - \cos 225^\circ \sin 30^\circ$$

$$= \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \boxed{\frac{-\sqrt{3}+1}{2\sqrt{2}}}$$

$$\cos 195^\circ = \cos 225^\circ \cos 30^\circ + \sin 225^\circ \sin 30^\circ$$

$$= \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \boxed{\frac{-\sqrt{3}-1}{2\sqrt{2}}}$$

$$\tan(195^\circ) = \tan(225^\circ - 30^\circ) = \frac{\tan 225^\circ - \tan 30^\circ}{1 + \tan 225^\circ \tan 30^\circ}$$

$$= \frac{\frac{1}{1} - \frac{1}{\sqrt{3}}}{1 + (1)\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}+1} = \boxed{\frac{\sqrt{3}-1}{\sqrt{3}+1}}$$

122 § 2.4 I #s 19-25, 31, 35-9, 47-8-9, 53-4

(19) $\frac{13\pi}{12} = \frac{10\pi}{12} + \frac{3\pi}{12} = \frac{5\pi}{6} + \frac{\pi}{4}$

$$\sin\left(\frac{5\pi}{6} + \frac{\pi}{4}\right) = \sin\frac{5\pi}{6}\cos\frac{\pi}{4} + \cos\frac{5\pi}{6}\sin\frac{\pi}{4}$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \boxed{\frac{1-\sqrt{3}}{2\sqrt{2}}}$$

$$\cos\left(\frac{5\pi}{6} + \frac{\pi}{4}\right) = \cos\left(\frac{5\pi}{6}\right)\cos\frac{\pi}{4} - \sin\frac{5\pi}{6}\sin\frac{\pi}{4}$$

$$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \boxed{\frac{-\sqrt{3}-1}{2\sqrt{2}}}$$

$$\tan\left(\frac{5\pi}{6} + \frac{\pi}{4}\right) = \frac{\tan\frac{5\pi}{6} - \tan\frac{\pi}{4}}{1 - \tan\frac{5\pi}{6}\tan\frac{\pi}{4}} = \frac{-\frac{1}{\sqrt{3}} - \frac{1}{1}}{1 - \left(-\frac{1}{\sqrt{3}}\right)\left(\frac{1}{1}\right)}$$

$$= \frac{\frac{-1-\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \frac{\frac{-1-\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{-1-\sqrt{3}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}+1} = \frac{-1-\sqrt{3}}{\sqrt{3}(\sqrt{3}+1)}$$

$$= \frac{-(1+\sqrt{3})}{\sqrt{3}(\sqrt{3}+1)} = -\frac{1}{\sqrt{3}}$$ Hmmmmmm Not checking

$\frac{\tan u + \tan v}{1 - \tan u \tan v}$ Oh Got sign wrong.

$$\frac{\frac{-1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{-1+\sqrt{3}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}+1} = \boxed{\frac{\sqrt{3}-1}{\sqrt{3}+1}}$$

122 §2.4 I #s 21-25, 31, 35-9, 47-8-9, 53-4

21 $-\frac{13\pi}{12} = -\frac{3\pi}{12} - \frac{10\pi}{12} = -\frac{\pi}{4} - \frac{5\pi}{6} = -\left(\frac{\pi}{4} + \frac{5\pi}{6}\right)$

I'll do $\frac{13\pi}{12}$ & handle the "$-$" at the end w/ odd/even.

$\sin\left(\frac{\pi}{4} + \frac{5\pi}{6}\right) =$ See work for #19!

ODD

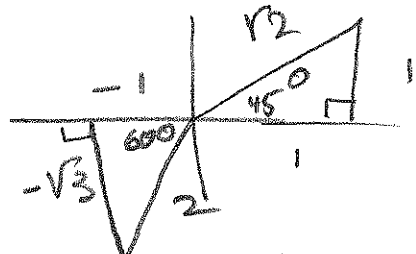
$= \frac{-\sqrt{3}-1}{2\sqrt{2}}$, so $\sin\left(-\left(\frac{\pi}{4} + \frac{5\pi}{6}\right)\right) = \boxed{\frac{\sqrt{3}+1}{2\sqrt{2}}}$

$\cos\left(\frac{\pi}{4} + \frac{5\pi}{6}\right) = \cos\left(-\left(\frac{\pi}{4} + \frac{5\pi}{6}\right)\right) = \boxed{\frac{-\sqrt{3}-1}{2\sqrt{2}}}$ by #19

EVEN

$\tan\left(-\left(\frac{\pi}{4} + \frac{5\pi}{6}\right)\right) = -\tan\left(\frac{\pi}{4} + \frac{5\pi}{6}\right) = \boxed{-\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)}$

ODD



23 $285^0 = 240^0 + 45^0$

$\sin(240^0 + 45^0) = \sin 240^0 \cos 45^0 + \cos 240^0 \sin 45^0$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \boxed{\frac{-\sqrt{3}=1}{2\sqrt{2}}}$

$\cos(240^0 + 45^0) = \cos 240^0 \cos 45^0 - \sin 240^0 \sin 45^0$

$= \left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \boxed{\frac{-1+\sqrt{3}}{2\sqrt{2}}}$

$\tan(240^0 + 45^0) = \frac{\tan 240^0 + \tan 45^0}{1 - \tan 240^0 \tan 45^0} = \frac{\frac{\sqrt{3}}{1} + \frac{1}{\sqrt{2}}}{1 - \left(\frac{\sqrt{3}}{1}\right)\left(\frac{1}{\sqrt{2}}\right)} = \frac{\frac{\sqrt{6}+1}{\sqrt{2}}}{1 - \frac{\sqrt{3}}{\sqrt{2}}}$

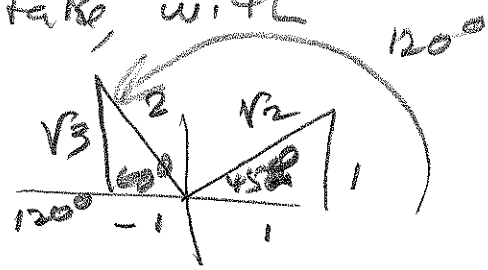
$= \frac{\sqrt{6}+1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}-\sqrt{3}} = \frac{\sqrt{6}+1}{\sqrt{2}-\sqrt{3}}$ Doesn't Check.

2.4 I #s 23-25, 31, 35-7, 47-8-9, 53, 54

23 $\tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v} = \dfrac{\tan(240°) + \tan(45°)}{1 - \tan(240°)\tan(45°)}$

$$= \frac{\sqrt{3}+1}{1-(\sqrt{3})(1)} = \boxed{\frac{\sqrt{3}+1}{1-\sqrt{3}}}$$

There's where I made the mistake, with $\frac{1}{\sqrt{2}}$, there, instead of a '1'.



25 $-165°$; $165° = 120° + 45°$

$\sin(120°+45°) = \sin 120° \cos 45° + \cos 120° \sin 45°$

ODD

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{3}-1}{2\sqrt{2}} = \sin 165°$, so

$\sin(-165°) = -\sin(165°) = -\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$ OR $\boxed{\frac{1-\sqrt{3}}{2\sqrt{2}}}$

$\cos(120°+45°) = \cos 120° \cos 45° - \sin 120° \sin 45°$

EVEN

$= \left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \boxed{\frac{-1-\sqrt{3}}{2\sqrt{2}}} = \cos 165° = \cos(-165°)$

$\tan(120°+45°) = \dfrac{\tan 120° + \tan 45°}{1 - \tan 120° \tan 45°} = \dfrac{-\frac{\sqrt{3}}{1} + \frac{1}{1}}{1-\left(\frac{\sqrt{3}}{-1}\right)\left(\frac{1}{1}\right)}$

ODD

$= \dfrac{-\sqrt{3}+1}{1+\sqrt{3}} = \tan(+165°)$ $\Longrightarrow$

$\Longrightarrow$ $\tan(-165°) = \boxed{\dfrac{\sqrt{3}-1}{\sqrt{3}+1}}$

122 §2.4 II #s 27-34, 47-8-9, 53-4

#s 27-34 Write as sine/cosine/tangent of a single angle.

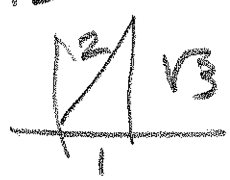
(31) $\dfrac{\tan 45^\circ - \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} = \tan(45^\circ + 30^\circ) = \boxed{\tan(75^\circ)}$

#s 35-40 Find exact value of the expression

(35) $\sin\frac{\pi}{12}\cos\frac{\pi}{4} + \cos\frac{\pi}{12}\sin\frac{\pi}{4}$

$= \sin\left(\frac{\pi}{12} + \frac{\pi}{4}\right) = \sin\left(\frac{3\pi + \pi}{12}\right) = \sin\left(\frac{4\pi}{12}\right)$

$= \sin\left(\frac{\pi}{3}\right) = \boxed{\frac{\sqrt{3}}{2}}$

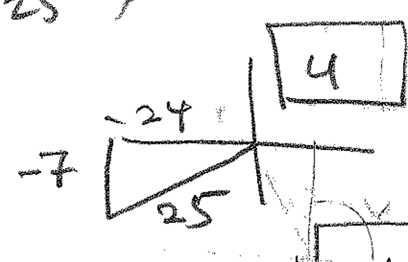


(37) $\sin 120^\circ \cos 60^\circ - \cos 120^\circ \sin 60^\circ$

$= \sin(120^\circ - 60^\circ) = \sin 60^\circ = \boxed{\frac{\sqrt{3}}{2}}$

#s 47-52 Given $\sin u = -\frac{7}{25}$ & $\cos v = -\frac{4}{5}$,

and u & v are in QIII

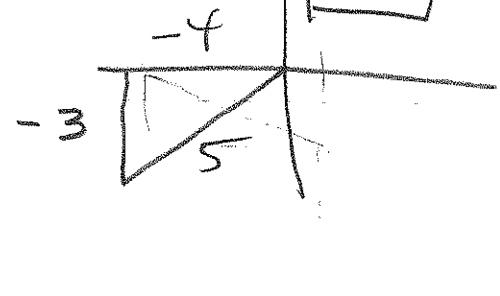


(47) $\cos(u+v) =$

$\cos u \cos v - \sin u \sin v$

$= \left(-\frac{24}{25}\right)\left(-\frac{4}{5}\right) - \left(-\frac{7}{25}\right)\left(-\frac{3}{5}\right)$

$= \frac{96 - 21}{125} = \frac{75}{125} = \boxed{\frac{3}{5}}$



Note: u+v is in QI!

122 §2.4 I #s 48-9, 53-4

(48) sin(u+v) = sin u cos v + cos u sin v

$$= \left(-\frac{7}{25}\right)\left(-\frac{4}{5}\right) + \left(-\frac{24}{25}\right)\left(-\frac{3}{5}\right) = \frac{28+72}{125} = \frac{100}{125} = \boxed{\frac{4}{5}}$$

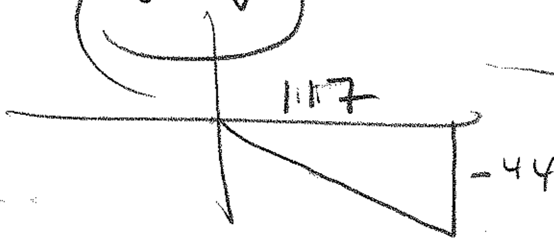
(49) $$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v} = \frac{\frac{7}{24} - \frac{3}{4}}{1 + \left(\frac{7}{4}\right)\left(\frac{3}{4}\right)}$$

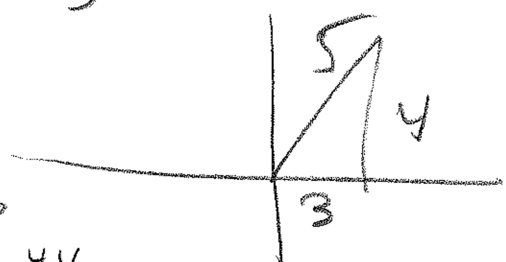
Nope!

$$= \frac{\frac{7-18}{24}}{\frac{16+21}{16}} = -\frac{11}{\cancel{24}_{\cancel{6}\,3}} \cdot \frac{\cancel{16}^{\cancel{4}\,2}}{37} = -\frac{22}{121} = -\frac{2}{11}$$

u+v

U−V





Something wrong with my tan(u+v)

$$\frac{-\frac{11}{24}}{1 + \frac{21}{96}} = \frac{-\frac{11}{24}}{\frac{96+21}{96}} = -\frac{11}{24} \cdot \frac{96}{117} = -\frac{11}{24} \cdot \frac{32}{39} =$$

Hmmmmmmm

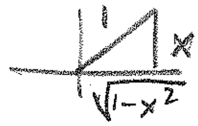
Still not working out.

Wait. It is! I was still looking @ u+v in Q1. u−v is in QIV

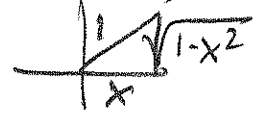
$$\left(-\frac{11}{24}\right)\left(\frac{32}{39}\right) = \frac{(-11)(8)}{(6)(39)} = \frac{(-11)(4)}{(3)(39)} = \boxed{\frac{-44}{117}}$$

'22 §2.4 I #s 53-4

arcsin x



arccos x

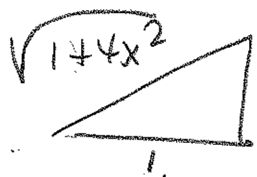


53 $\sin(\arcsin x + \arccos x)$

$= \sin(\arcsin x)\cos(\arccos x) + \cos(\arcsin x)\sin(\arccos x)$

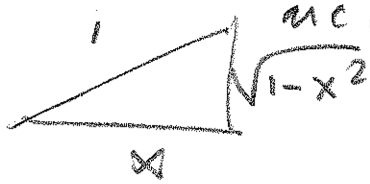
$= (x)(x) + \sqrt{1-x^2}\sqrt{1-x^2} = x^2 + 1 - x^2 = \boxed{1}$!

54 $\sin(\arctan(2x) - \arccos(x))$



$= \sin(\arctan(2x))\cos(\arccos(x)) - \sin(\arccos(x))\cos(\arctan(2x))$

$= \left(\frac{2x}{\sqrt{4x^2+1}}\right)\left(\frac{x}{1}\right) - \left(\frac{\sqrt{1-x^2}}{1}\right)\left(\frac{1}{\sqrt{4x^2+1}}\right)$



$= \boxed{\frac{2x^2 - \sqrt{1-x^2}}{\sqrt{4x^2+1}}}$