

I think the 2.3 is likely to be a bit long for some of you. I'd be OK with cutting it about in half, although I don't see a single one of those that doesn't build some muscle.

I just have the sense that tolerance for difficulties is not very high in this class, when difficulties is the name of the game, going forward. I really want the class to transition to a cheerful hardnosed bunch, for whom nothing is too much. You get that going, NOW, and the rest of your career in college and beyond gets a whole lot easier!

But to save you some time, you may do the following problems for 2.3:

#s 1 – 6 All, 11, 15, 19, 23, 27, 31, 35, 37*, 39, 43, 45, 63, 67, 71, 73, 87, 88

* Square Both sides!

And recall, that squaring both sides often introduces extraneous solutions, the same way casting a net often introduces fish you don't want...

122 §2.3 #s $\frac{1-6}{\text{all}}$, 11-47, 63-73, 87, 88

① First, isolate the trig on one side

② $2\sin\theta + 1 = 0$ has solns $\theta = \frac{7\pi}{6} + 2n\pi$

OR $\theta = \frac{11\pi}{6} + 2n\pi$, which are general solns

③ $2\tan^2 x - 3\tan x + 1 = 0$ is quadratic type.

④ A soln that doesn't work is extraneous.

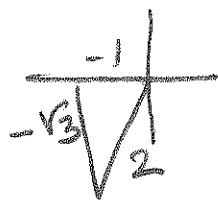
5-10 verify solns

⑤ $\tan x - \sqrt{3} = 0$



(a) $x = \frac{\pi}{3}$: $\tan \frac{\pi}{3} - \sqrt{3} = \sqrt{3} - \sqrt{3} = 0$ ✓

(b) $x = \frac{4\pi}{3}$: $\tan \frac{4\pi}{3} - \sqrt{3} = \sqrt{3} - \sqrt{3} = 0$ ✓



⑥ $\sec x - 2 = 0$



(a) $x = \frac{\pi}{3}$: $\sec \frac{\pi}{3} - 2 = 2 - 2 = 0$ ✓

(b) $x = \frac{5\pi}{3}$: $\sec \frac{5\pi}{3} - 2 = 2 - 2 = 0$ ✓



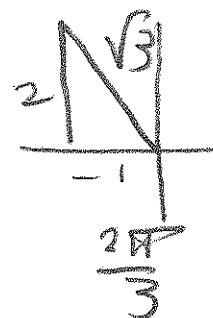
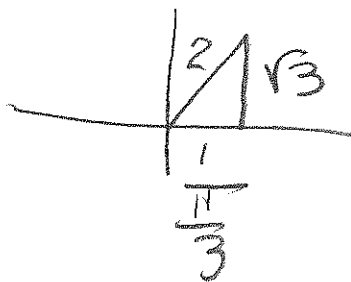
122 § 2.3 #s 11-47, 63-73, 87, 88

#s 11-24 Solve!

(11) $\sqrt{3} \csc x - 2 = 0$

$\sqrt{3} \csc x = 2$

$\csc x = \frac{2}{\sqrt{3}}$

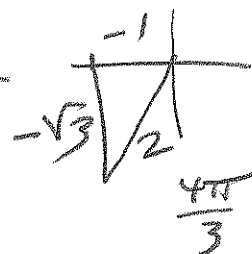
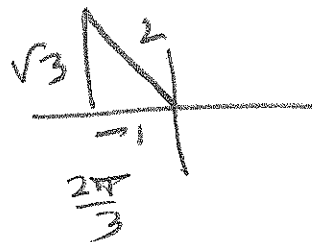


$\Rightarrow x = \frac{\pi}{3} + 2n\pi$ OR $x = \frac{2\pi}{3} + 2n\pi, n \in \mathbb{Z}$

(13) $\cos x + 1 = -\cos x$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$



$\Rightarrow x = \frac{2\pi}{3} + 2n\pi$ OR $x = \frac{4\pi}{3} + 2n\pi, n \in \mathbb{Z}$.

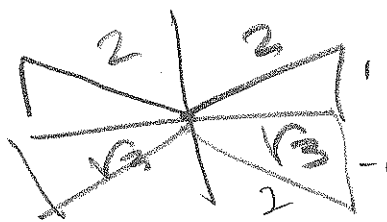
(15) $3\sec^2 x - 4 = 0$

$3\sec^2 x = 4$

$\sec^2 x = \frac{4}{3}$

$|\sec x| = \sqrt{\frac{4}{3}}$

$\sec x = \pm \frac{2}{\sqrt{3}}$



$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

π apart $\rightarrow \pi$ apart

$x = \frac{\pi}{6} + n\pi$ OR $x = \frac{5\pi}{6} + n\pi, n \in \mathbb{Z}$

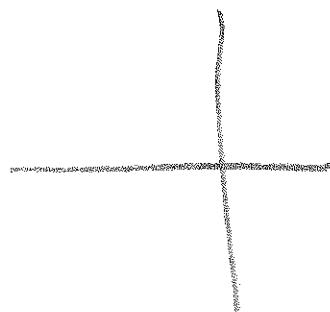
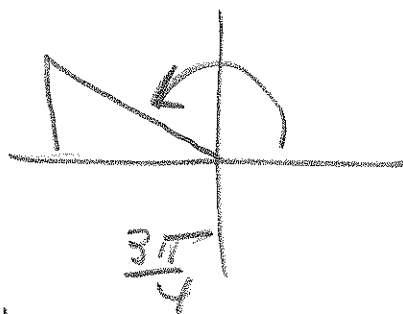
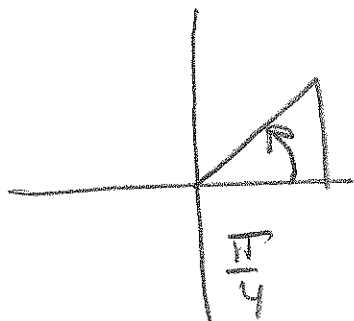
122 § 2.3 #519-47, 63-73, 87, 88

$$(9) \quad 2\sin^2(2x) = 1$$

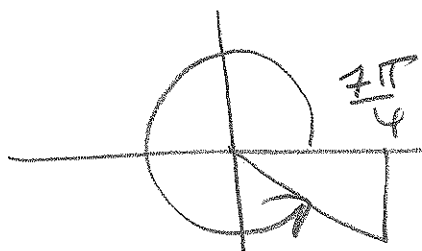
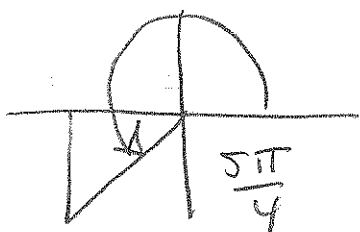
$$\sin^2(2x) = \frac{1}{2}$$

$$\sin(2x) = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{\sqrt{2}}$$

Pictures for $2x$:



$$\sin(2x) = \frac{1}{\sqrt{2}}$$



These are solutions of $\sin(2x) = \frac{1}{\sqrt{2}}$ for $2x \in [0, 2\pi]$. But we need to capture all solutions for $x \in 2\pi$, so for we can build all solutions with the $+n\pi$ & $+2n\pi$ bit.

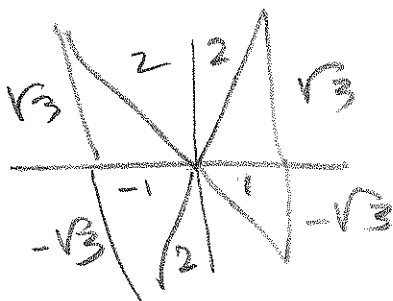
099 § 2.3 #s 17-47, 63-73, 87, 88

(17) $4\cos^2 x - 1 = 0$

$4\cos^2 x = 1$

$\cos^2 x = \frac{1}{4}$

$\cos x = \pm \frac{1}{2}$



Again, some of these pair up nicely:

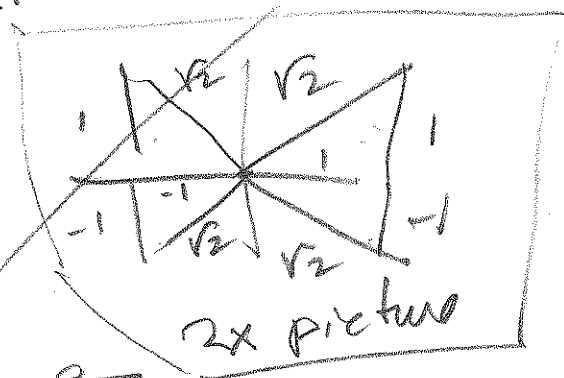
$x = \frac{\pi}{3} + n\pi$ OR $x = \frac{2\pi}{3} + n\pi, n \in \mathbb{Z}$

(19) ~~$2\sin^2(2x) = 1$ SEE NEXT PAGE~~

~~$\sin^2(2x) = \frac{1}{2}$ PAGE~~

~~$\sin(2x) = \pm \frac{1}{\sqrt{2}}$~~

~~$2x = \frac{\pi}{4}$ OR $2x = \frac{3\pi}{4}$~~

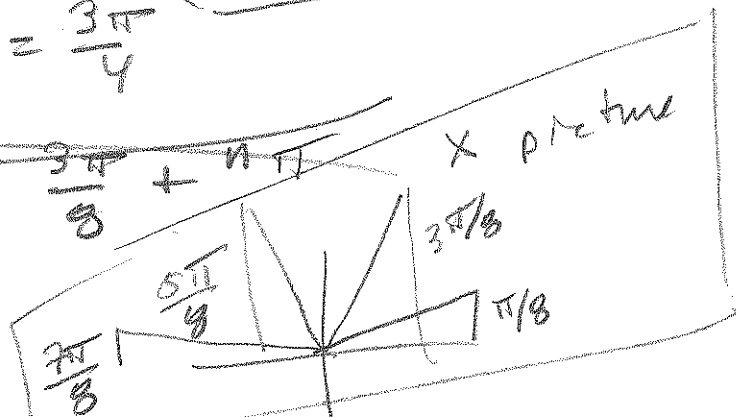


~~$x = \frac{\pi}{8} + n\pi$ OR $x = \frac{3\pi}{8} + n\pi$ X picture~~

~~Not that easy!~~

~~These aren't π apart!~~

~~$x = \frac{\pi}{8} + 2n\pi, \frac{3\pi}{8} + 2n\pi, \frac{5\pi}{8} + 2n\pi, \frac{7\pi}{8} + 2n\pi$
 $n \in \mathbb{Z}$~~



~~Not that hard!~~

~~$2x = \frac{\pi}{4} + n\pi$ OR $2x = \frac{3\pi}{4} + n\pi$~~

~~$x = \frac{\pi}{8} + n\frac{\pi}{2}$ OR $x = \frac{3\pi}{8} + \frac{n\pi}{2}$~~

122 §2.3 #s 19-47, 63-73, 87, 88

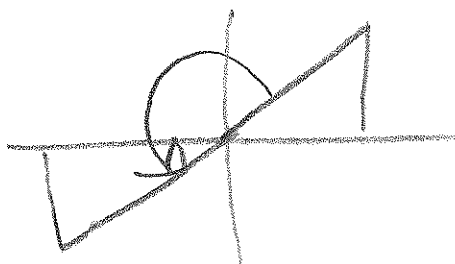
$$\text{So, } 2x = \frac{\pi}{4} + 2n\pi$$

$$2x = \frac{3\pi}{4} + 2n\pi$$

$$2x = \frac{5\pi}{4} + 2n\pi$$

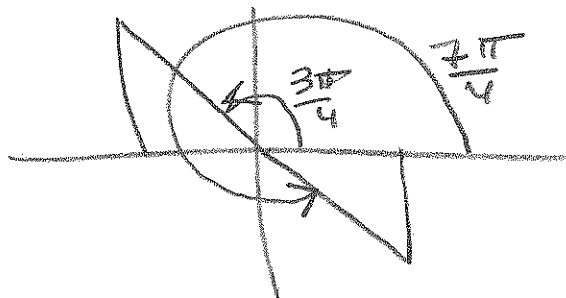
$$2x = \frac{7\pi}{4} + 2n\pi$$

We can collapse some of this by observing that $\frac{\pi}{4}$ & $\frac{5\pi}{4}$ are π radians apart.



This means that $2x = \frac{\pi}{4} + n\pi$ will capture all of them

and $2x = \frac{3\pi}{4} + n\pi$ will capture all of these guys:



" $\frac{3\pi}{4}$ & $\frac{7\pi}{4}$ are also π radians apart.

122 S 2.3 #s 19-47, 63-73, 87, 88

(19) cont'd

SO $2x = \frac{\pi}{4} + n\pi$

OR

$2x = \frac{3\pi}{4} + n\pi$

captures all solutions for $2x$

Now, just divide by 2:

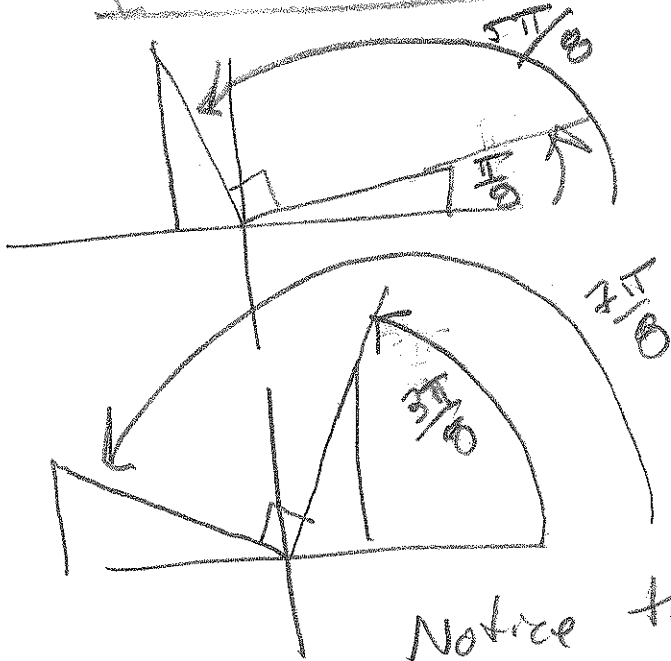
FINAL
ANS

$$2x = \frac{\pi}{8} + \frac{n\pi}{2}$$

OR

$$x = \frac{3\pi}{8} + \frac{n\pi}{2}$$

Captures all the solutions for x .

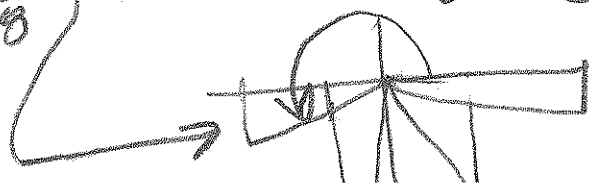


are $\frac{\pi}{2}$ apart

are $\frac{\pi}{2}$ apart

Notice that pictures for $2x$ in $[0, 2\pi]$ collapses to x in $[0, \pi]$? So we lose half of what's going on for $x \in [0, 2\pi]$

$\frac{\frac{13\pi}{4}}{2} = \frac{13\pi}{8}$



These are the angles that come from going twice around with $2x$.

122 S 2.3 #s 19-47, 63-73, 87, 88

The up-shot is that if you capture all solutions $2x$ for $\sin(2x) = \pm \frac{1}{\sqrt{2}}$, then you divide by 2 to get solutions x .

You will sometimes find book answers that are more elegant, but that comes from observing things like $\frac{3\pi}{4} - \frac{\pi}{4} = \pi$

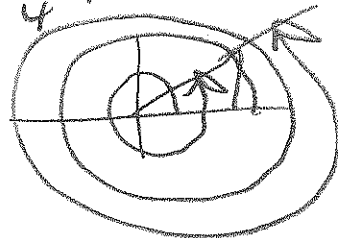
$$\text{so that } 2x = \frac{\pi}{4} + 2n\pi$$

$$\text{OR}$$
$$2x = \frac{5\pi}{4} + 2n\pi$$

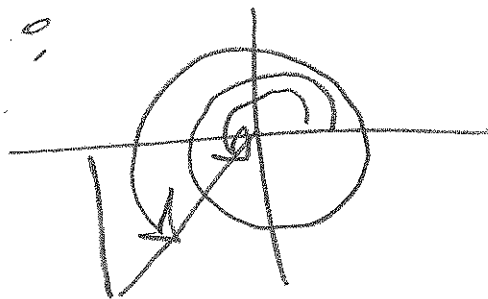
collapses down to

$$2x = \frac{\pi}{4} + n\pi$$

When n is even



When n is odd



Hope this helps!

122 S 2.3 #5 21-47, 68-73, 87, 88

(21) $\tan(3x) (\tan(x) - 1) = 0 \rightarrow$

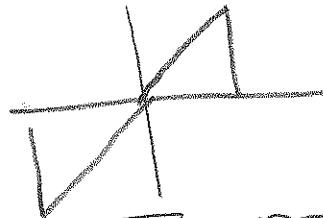
$\tan(3x) = 0$ OR $\tan x - 1 = 0$

$\tan x = 1$



$3x = 0 + n\pi = n\pi$

$x = \frac{n\pi}{3}$



$\frac{\pi}{4}, \frac{5\pi}{4}$

$\frac{\pi}{4} + 2n\pi$

OR

$\frac{5\pi}{4} + 2n\pi$

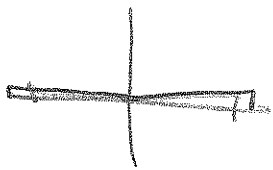
collapses to

$\frac{\pi}{4} + n\pi$

122 § 2.3 # 5 23-47, 68-73, 87, 88

(23) $\sin x (\sin x + 1) = 0$

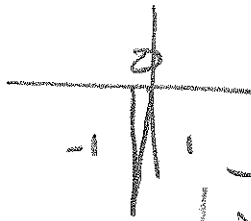
$\sin x = 0$



$x = n\pi$

(From $x = 0 + 2n\pi$
OR
 $x = \pi + 2n\pi$)

$\sin x = -1$



$x = \frac{3\pi}{2} + 2n\pi$

~~24~~ #5 25-39 Find all solutions in the interval $[0, 2\pi]$

(25) $\cos^3 x = \cos x$

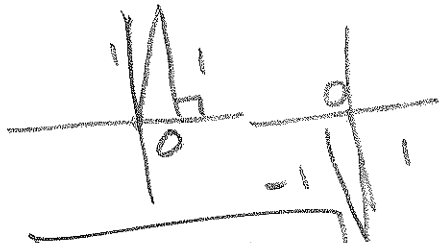
$\cos^3 x - \cos x = 0$

$\cos x (\cos^2 x - 1) = 0$

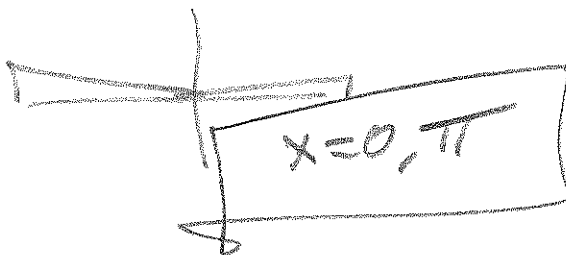
$\cos x = 0$ OR $\cos^2 x - 1 = 0$

$-\sin^2 x = 0$

$\sin x = 0$



$x = \frac{\pi}{2}, \frac{3\pi}{2}$



$x = 0, \pi$

122 § 2.3 #5 ~~27~~ 47, 68-73, 87, 88

(27) $3 \tan^3 x = \tan x$

$3 \tan^3 x - \tan x = 0$

$\tan x (3 \tan^2 x - 1) = 0$

$\tan x = 0$

OR $3 \tan^2 x - 1 = 0$

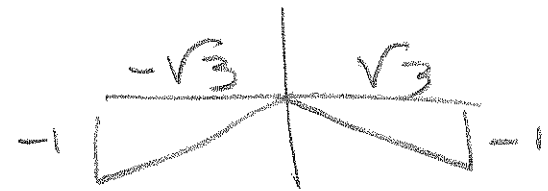
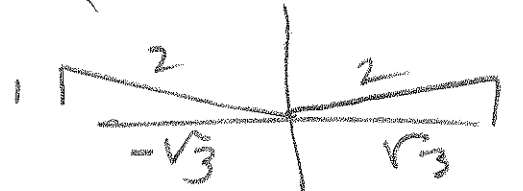
$3 \tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm \sqrt{\frac{1}{3}} = \pm \frac{1}{\sqrt{3}}$



$x = 0, \pi$
 $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$



(29) $\sec^2 x - \sec x = 2$

$u^2 - u - 2 = 0$

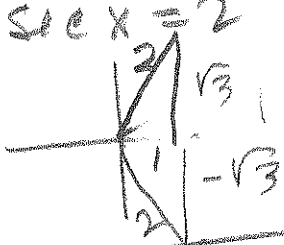
$(u-2)(u+1) = 0$

$u = 2$

$u = -1$

$\sec x = 2$

$\sec x = -1$



$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $x = \pi$

122 § 2.3 #s 31-47, 63-73, 87, 88

(31) $2\sin^2 x + \csc x = 0$

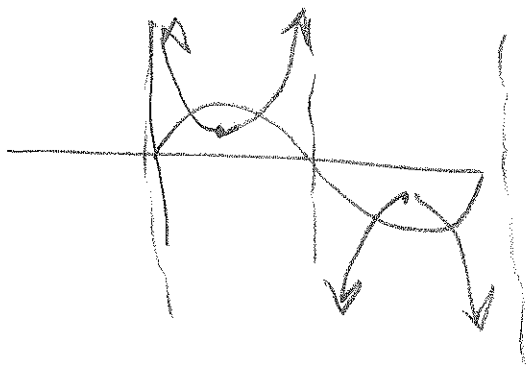
$$2\sin^2 x + \frac{1}{\sin x} = 0$$

$$\frac{2\sin^2 x + 1}{\sin x} = 0$$

$$2\sin^2 x + 1 = 0$$

$\sin^2 x = -\frac{1}{2}$ Impossible!

But doesn't it have this picture?

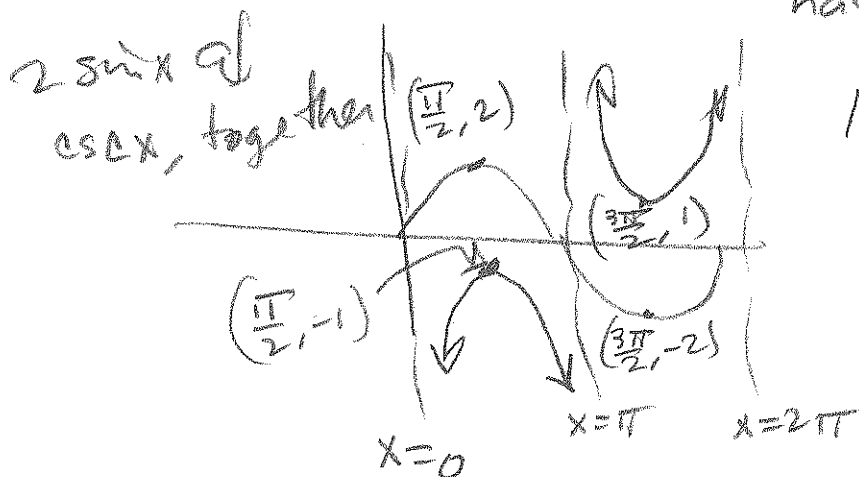


$2\sin x = \csc x$, yeah.

It has solutions

But we have

$2\sin x = -\csc x$, which has no solution



No intersections

OK.

122 § 2, 3, 5, 33-47, 63-73, 87-88

(33) $2 \cos^2 x + \cos x - 1 = 0$

$2u^2 + u - 1 = 0$

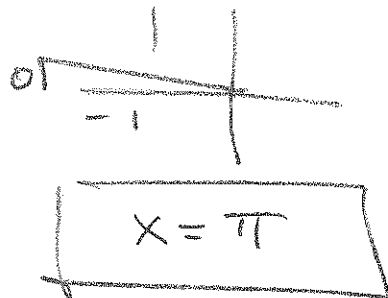
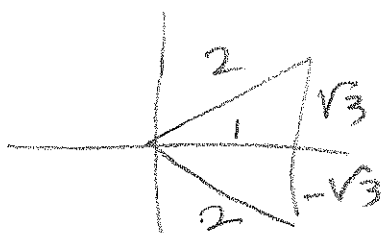
$(2u - 1)(u + 1) = 0$

$u = \frac{1}{2}$

$u = -1$

$\cos x = \frac{1}{2}$

$\cos x = -1$



$x = \frac{\pi}{3}, \frac{5\pi}{3}$

(35) $2 \sec^2 x + \tan^2 x - 3 = 0$

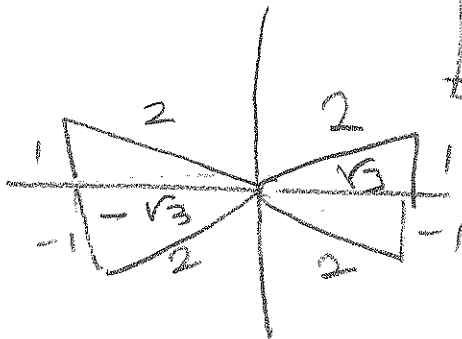
$2 \sec^2 x + \sec^2 x - 1 - 3 = 0$

$3 \sec^2 x - 4 = 0$

$3 \sec^2 x = 4$

$\sec^2 x = \frac{4}{3}$

$\sec x = \pm \sqrt{\frac{4}{3}} = \pm \frac{2}{\sqrt{3}}$



$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

122 § 2,3 #s 37-47, 63-72, 87, 88

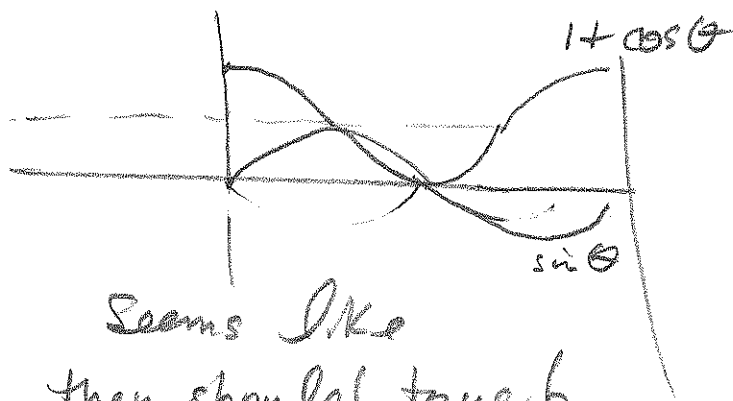
(37) $\csc x + \cot x = 1$

$$\frac{1}{\sin x} + \frac{\cos x}{\sin x} - 1 = 0$$

$$\frac{1 + \cos x - \sin x}{\sin x} = 0$$

$$1 + \cos x - \sin x = 0$$

$$1 + \cos x = \sin x$$



Seems like they should touch
 (3) $x = \frac{\pi}{2}, x = \pi$.

But the algebra isn't obvious.

EXAMPLE 6 says

try squaring both sides. Good Advice.

$$\cos x + 1 = \sin x$$

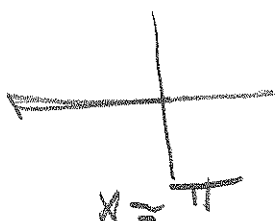
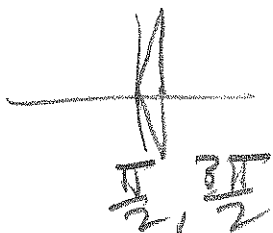
$$\cos^2 x + 2 \cos x + 1 = \sin^2 x = 1 - \cos^2 x$$

$$2 \cos^2 x + 2 \cos x = 0$$

$$2 \cos x (\cos x + 1) = 0$$

$$\cos x = 0$$

$$\cos x = -1$$



CHECK

$$\csc \frac{3\pi}{2} + \cot \frac{3\pi}{2} = 1?$$

$$-1 + 0 = 1?$$

Nope

$$\csc \frac{\pi}{2} + \cot \frac{\pi}{2} = 1?$$

$$1 + 0 = 1?$$

Yep

My picture says

Check.

$\frac{3\pi}{2}$ won't work. Let's

$$x = \frac{\pi}{2}, \pi$$

$\frac{3\pi}{2}$ is

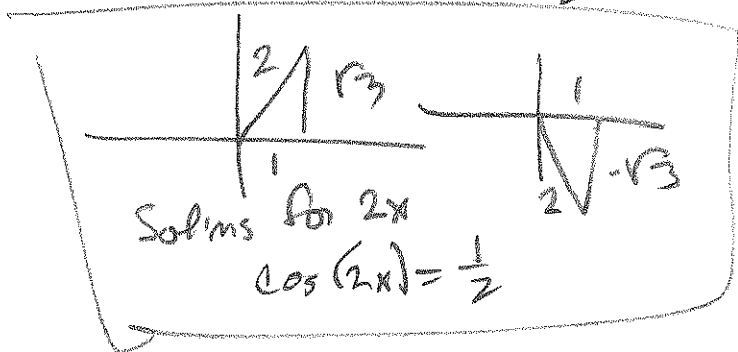
extraneous

122 8' 2.3 #5 39-47, 63-73, 87, 88

(39) $2 \cos(2x) - 1 = 0$

$$2 \cos(2x) = 1$$

$$\cos(2x) = \frac{1}{2}$$



$$2x = \frac{\pi}{3} + 2n\pi$$

OR

$$2x = \frac{5\pi}{3} + 2n\pi$$

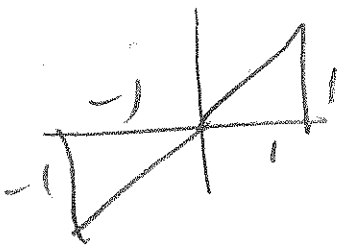
$$x = \frac{\pi}{6} + n\pi$$

OR

$$x = \frac{5\pi}{6} + n\pi$$

(41) $\tan(3x) - 1 = 0$

$$\tan(3x) = 1$$



$$3x = \frac{\pi}{4} + n\pi$$

$$x = \frac{\pi}{12} + \frac{n\pi}{3}$$

* from $3x = \frac{\pi}{4} + 2n\pi$ & $3x = \frac{5\pi}{4} + 2n\pi$
which collapse down to

$$3x = \frac{\pi}{4} + n\pi$$

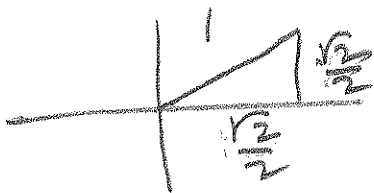
~~$$x = \frac{\pi}{12}$$~~

122 § 2,3 #s 43-47, 63-73, 87, 88

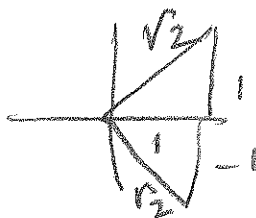
(43) $2 \cos\left(\frac{x}{2}\right) - \sqrt{2} = 0$

$$2 \cos\left(\frac{x}{2}\right) = \sqrt{2}$$

$$\cos\left(\frac{x}{2}\right) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$



OR



SO $\frac{x}{2} = \frac{\pi}{4}$ OR $\frac{7\pi}{4} \rightarrow (+2n\pi)$

$$x = \frac{\pi}{2} + 4n\pi$$

OR

$$x = \frac{7\pi}{2} + 4n\pi$$

#s 45-48 Find x-intercepts (Doing same thing.)

(45) $y = \sin\left(\frac{\pi x}{2}\right) + 1 \stackrel{SET}{=} 0 \rightarrow$

$$\sin\left(\frac{\pi x}{2}\right) = -1$$

$$\rightarrow \frac{\pi x}{2} = \frac{3\pi}{2} + 2n\pi \rightarrow$$



$$x = \frac{3\pi}{2} \cdot \frac{2}{\pi} + \frac{2n\pi}{1} \cdot \frac{2}{\pi} \rightarrow$$

$x = 3 + 4n$

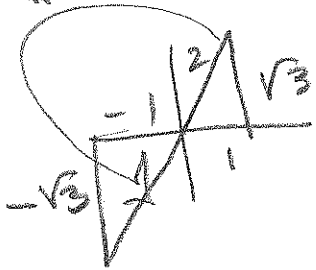
 $n \in \mathbb{Z}$

122 $\int 2.3$ #s 47, 63-73, 87-9

(47) $y = \tan^2\left(\frac{\pi x}{6}\right) - 3 \stackrel{\text{SET}}{=} 0$

$\tan^2\left(\frac{\pi x}{6}\right) = 3 \implies \frac{\pi x}{6} = \frac{\pi}{3} + n\pi$

$\tan\left(\frac{\pi x}{6}\right) = \pm\sqrt{3} \implies x = \frac{\pi}{3} \cdot \frac{6}{\pi} + \frac{n\pi}{1} \cdot \frac{6}{\pi}$



$x = 2 + 6n$

#s 63-74 Use inverse funes, as needed, to find all solutions (when things aren't "nice" in radians, degrees can really help you "see.")
Assume $x \in [0, 2\pi]$

(63) $\tan^2 x + \tan x - 12 = 0$

$u^2 + u - 12 = 0$

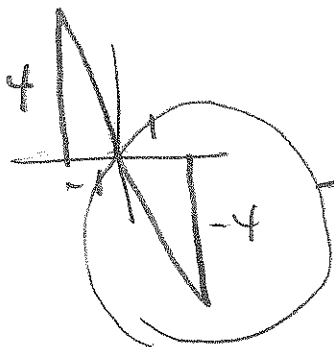
$(u+4)(u-3) = 0$

$\tan x = -4$ OR $\tan x = 3$

Wait!

Book answers suggest if it isn't clean, just leave it as

$x = \arctan(-4)$ OR $x = \arctan(3)$ an inverse trig expression



$\arctan(-4)$ is this one.
Make sure you capture the other!

63

entire
 ~~$\arctan(-4)$~~ , $\arctan(-4) + \pi$,
 $\arctan(3)$, $\arctan(3) + \pi$

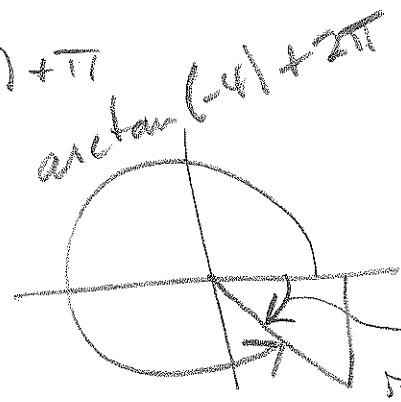
Not quite!

Wait! $\arctan(-4)$ is Not in $[0, 2\pi]$

Recall, range of $\arctan(x)$ is $(-\frac{\pi}{2}, \frac{\pi}{2})$

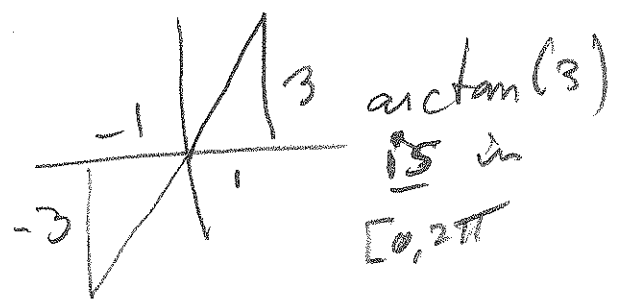
Still, our picture is good. We just need to tweak the answer to get them all in $[0, 2\pi]$

$\neq \arctan(-4) + \pi, \arctan(-4) + 2\pi$



$\arctan(-4)$ is negative!

$\arctan(-4) + \pi$
 $\arctan(-4) + 2\pi$
 $\arctan(3)$
 $\arctan(3) + \pi$
 Final Answer



$\arctan(3)$ is in $[0, 2\pi]$

122 § 2.3 #5 65-73, 87-8

(68) $\sec^2 x - 6 \tan x = -4$

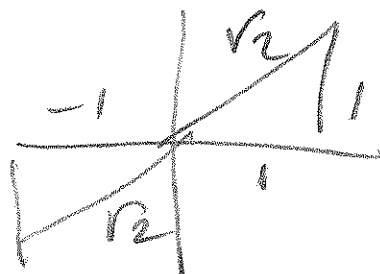
$$\tan^2 x + 1 - 6 \tan x + 4 = 0$$

$$\tan^2 x - 6 \tan x + 5 = 0$$

$$(u-5)(u-1) = 0$$

$$\tan x = 5$$

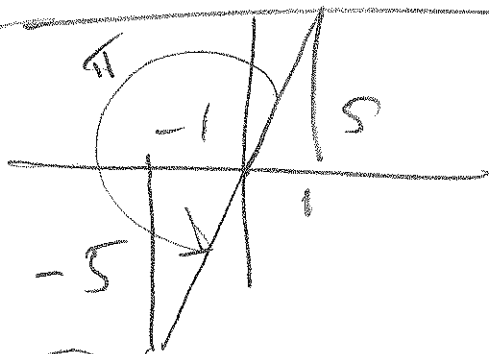
$$\tan x = 1$$



$$x = \arctan(5)$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$x = \arctan(5) + \pi$$



(67) $2\sin^2 x + 5\cos x = 4$

$$2 - 2\cos^2 x + 5\cos x - 4 = 0$$

$$-2\cos^2 x + 5\cos x - 2 = 0$$

$$2u^2 - 5u + 2 = 0$$

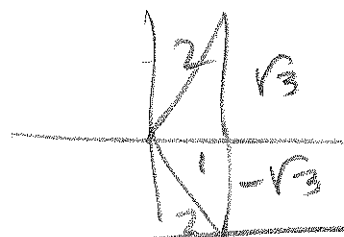
$$(2u-1)(u-2) = 0$$

$$2\cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$\cos x = 2$$

Notah!



$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

122 § 2/3 #s 69-73, 87-8

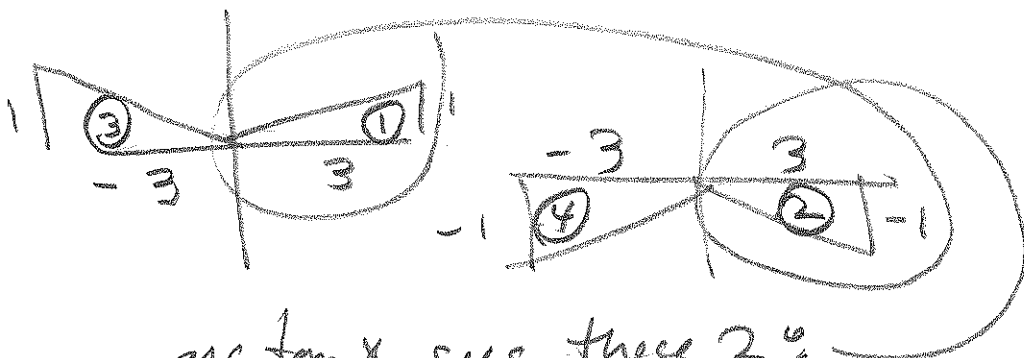
(64) $\cot^2 x - 9 = 0$

$\cot x = \pm 3$



$\tan x = \pm \frac{1}{3}$

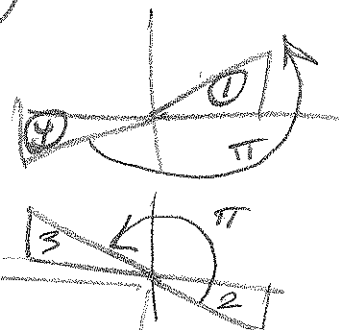
We didn't do much with $\cot^{-1}(x)$. We could work with it, or just turn this into an equivalent equation involving $\arctan(x) = \tan^{-1}(x)$



$\arctan x$ sees these 2:

Add π to 1 of get 4

Add π to 2 of get 3



So: $x = \arctan(\frac{1}{3}), \arctan(\frac{1}{3}) + \pi,$
 $\arctan(-\frac{1}{3}), \arctan(-\frac{1}{3}) + \pi$

Your answer may look different, but still be correct, e.g., $\operatorname{arccot}(3)$ is

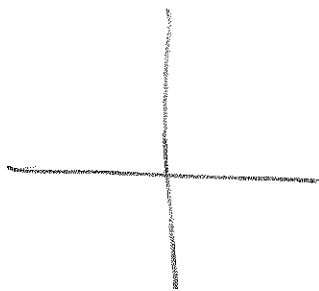
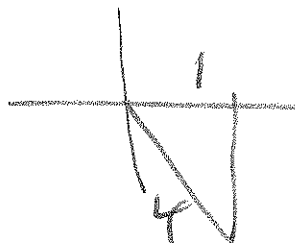
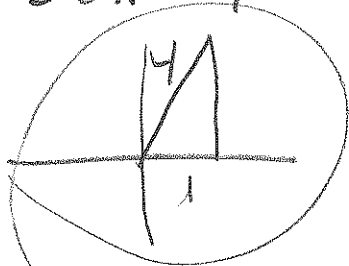
S 2,3 #s 71-3, 87-8

(71) $\sec^2 x - 4 \sec x = 0$

$$\sec x (\sec x - 4) = 0$$

$\sec x = 0$
Never!

$\sec x = 4$



\rightarrow $\text{arccsc}(x)$ sees only this one. Think back to the restriction we made on cosine in order to get this.

Heck, just turn it into a cosine question

$$\sec x = 4 \iff \cos x = \frac{1}{4} \text{ Ahhh}$$

So $x = \arccos(\frac{1}{4}), -\arccos(\frac{1}{4})$, since we have this picture for the two:



122 § 2.3 #5 73, 87 - 8

73

$$\csc^2 x + 3 \csc x - 4 = 0$$

$$u^2 + 3u - 4 = 0$$

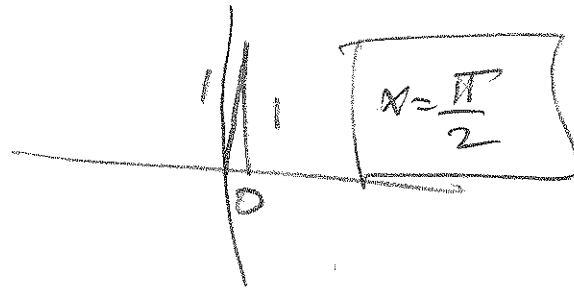
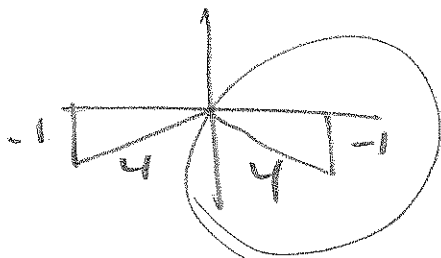
$$(u+4)(u-1) = 0$$

$$\csc x = -4$$

$$\csc x = 1$$

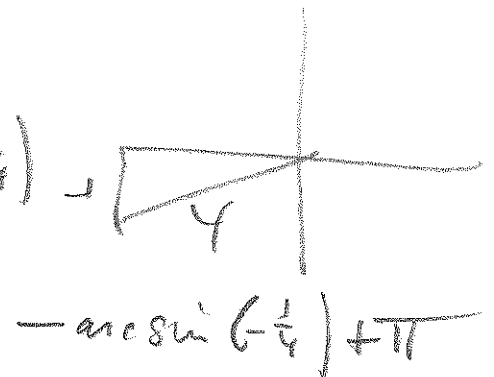
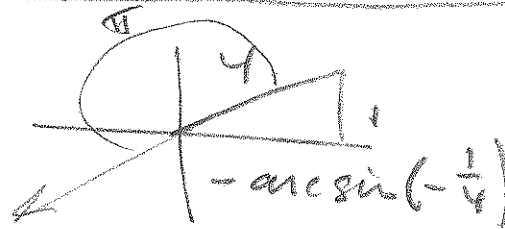
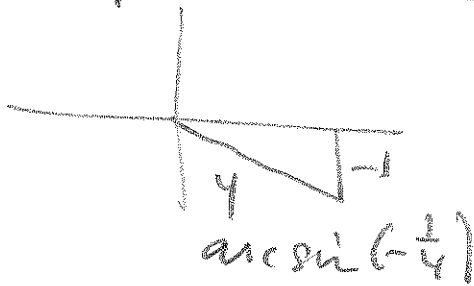
$$\sin x = -\frac{1}{4}$$

$$\sin x = 1$$



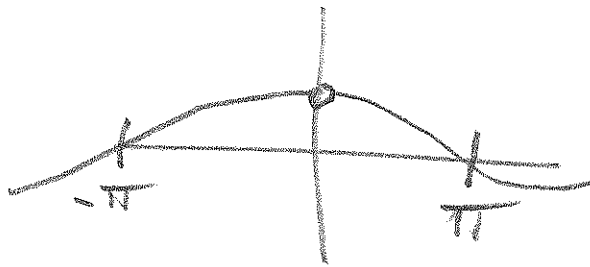
→ This is
the one
arcsin P
see 8

$$x = \arcsin\left(-\frac{1}{4}\right), -\arcsin\left(-\frac{1}{4}\right) + \pi$$



122 § 2.3 #s 87-8

(87) $f(x) = \frac{\sin x}{x}$



(a) $\mathcal{D}(f) = \mathbb{R} \setminus \{0\}$

OR $= (-\infty, 0) \cup (0, \infty)$

OR $= \{x \mid x \neq 0\}$

(b) It's symmetric about the y-axis
because it's even $\frac{\text{odd}}{\text{odd}} = \frac{-}{-} = + = \text{even}$

It has horizontal asymptote $y=0$

Eventually $x \rightarrow \infty$, but $\sin x$ rattles around between ± 1

$$\frac{\sin x}{x} \xrightarrow{x \rightarrow \infty} \frac{\text{smallish}}{\text{Gigormous}} = 0$$

(c) as $x \rightarrow 0$,

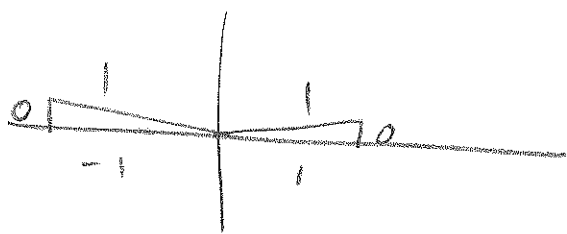
$\frac{\sin x}{x} \rightarrow 1$, but you can never quite let $x=0$!

122 § 2.3 # 87-8

(d) How many solutions does $\frac{\sin x}{x} = 0$ have in the interval $[-8, 8]$?

$\frac{\sin x}{x} = 0 \implies \sin x = 0$ So, as many solutions as $\sin x = 0$ has in $[-8, 8]$

$$x = n\pi \quad n = 0, \pm 1, \pm 2,$$



when $n = \pm 3$, we get $|\pm 3\pi| > |9| > |8|$

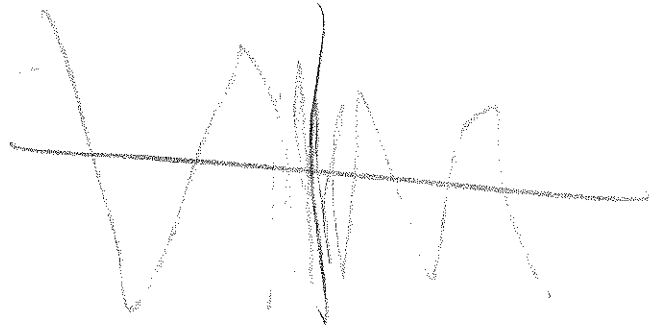
So there are 5 solutions

$$x = 0, \pm\pi, \pm 2\pi$$

122 §2.3 # 88

(88) $f(x) = \cos(\frac{1}{x})$

Hard to graph.



(a) Its Domain is

$(0, \infty) = \{x \mid x > 0\}$

EVEN!

(b) It's symmetric about the y-axis.

It has horizontal asymptote $y = 1$

as $x \rightarrow \infty$, $\frac{1}{x} \rightarrow 0$ and so $\cos(\frac{1}{x}) \rightarrow 1$.

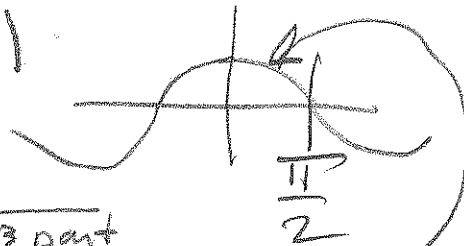
(c) As $x \rightarrow 0^+$ or $x \rightarrow 0^-$, it basically oscillates an infinite # of times!

(d) $\cos(\frac{1}{x}) = 0$ has a greatest solution

$\cos(\frac{1}{x}) = 0$ has an infinite # of solutions

solutions

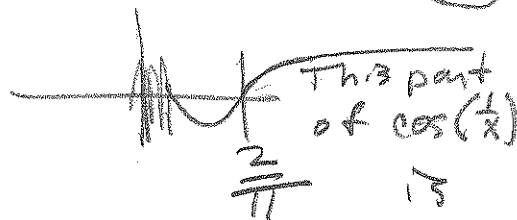
$\cos(x)$



$\cos(\frac{1}{x})$

Max solutions

$x = \frac{2}{\pi}$



This part of $\cos(\frac{1}{x})$ is

this part of $\cos(x)$