122 §2.1 #s 1-6 (ALL), 7-49, 53-58 (ALL), 69

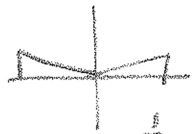
Fill in the blank.

(1) $\frac{\sin u}{\cos u} = \underline{\tan u}$ (2) $\frac{1}{\csc u} = \underline{\sin u}$ (3) $\frac{1}{\tan u} = \cot u$

(4) $\sec(\frac{\pi}{2} - u) = \underline{\csc u}$ (5) $1 + \underline{\cot^2 u} = \csc^2 u$ (3) $\cot(-u) = \underline{-\cot u}$

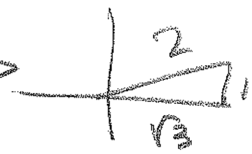
#s 7-14 Use given values to find all six trigs (if possible)

(7) $\sin x = \frac{1}{2}$, $\cos x = \frac{\sqrt{3}}{2}$



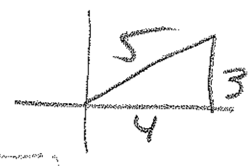
$\sin x = \frac{1}{2}$

$\cos x = \frac{\sqrt{3}}{2}$

$\Rightarrow$ 

$\boxed{\tan x = \frac{1}{\sqrt{3}}, \cot x = \sqrt{3}, \csc x = 2, \sec x = \frac{2}{\sqrt{3}}}$

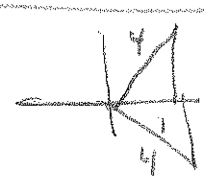
(9) $\cos(\frac{\pi}{2} - x) = \frac{3}{5} = \sin x$, $\cos x = \frac{4}{5}$



cofunctions

$\boxed{\tan x = \frac{3}{4}, \cot x = \frac{4}{3}, \csc x = \frac{5}{3}, \sec x = \frac{5}{4}}$

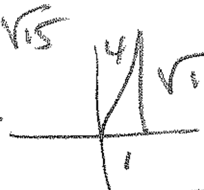
(11) $\sec x = 4$, $\sin x > 0$



$\sec x = 4$

& $\sin x > 0$

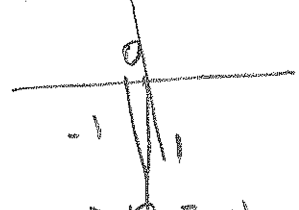
$\sqrt{4^2 - 1^2} = \sqrt{15}$

$\Rightarrow$ 

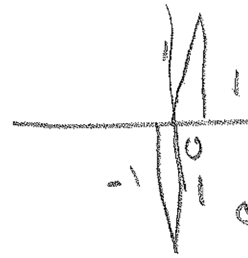
$\boxed{\sin x = \frac{\sqrt{15}}{4}, \cos x = \frac{1}{4}, \tan x = \sqrt{15}, \cot x = \frac{1}{\sqrt{15}}, \csc x = \frac{4}{\sqrt{15}}}$

(13) $\underline{\sin\theta = -1}$, $\cot\theta = 0$

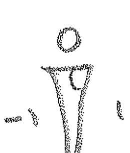
only one possibility



$\sin\theta = -1$



$\cot\theta = 0$



$\boxed{\cos\theta = 0, \tan\theta \nexists, \csc\theta = -1, \sec\theta \nexists,}$

122 § 2.1 #s 15-49, 53-58 ALL, 69

#s 15-20 Match trig expression with one of the following

(a) $\csc x$ (b) -1 (c) 1 (d) $\sin x \tan x$

(e) $\sec^2 x$ (f) $\sec^2 x + \tan^2 x$

(15) $\sec x \cos x = \frac{1}{\cos x} \cos x = 1$ (c)

(17) $\sec^4 x - \tan^4 x = (\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$

$= (1 + \tan^2 x - \tan^2 x)(1 + \tan^2 x + \tan^2 x)$

$= (1)(1 + \tan^2 x + \tan^2 x)$ = oh. $= (\sec^2 x + \tan^2 x)$ (f)

(19) $\frac{\sec^2 x - 1}{\sin^2 x} = \frac{\tan^2 x}{\sin^2 x} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\sin^2 x} = \frac{\sin^2 x}{\cos^2 x \sin^2 x} = \frac{1}{\cos^2 x}$

$= \sec^2 x$ (e)

#s 21-28 Factor & Simplify

(21) $\tan^2 x - \tan^2 x \sin^2 x = \tan^2 x (1 - \sin^2 x)$

$= \tan^2 x (\cos^2 x) = \frac{\sin^2 x}{\cos^2 x} \cos^2 x = \boxed{\sin^2 x}$

(23) $\frac{\sec^2 x - 1}{\sec x - 1} = \frac{(\sec x - 1)(\sec x + 1)}{\sec x - 1} = \boxed{\sec x + 1}$

122 §2.1 #s 25-49, $\underset{\text{ALL}}{\underline{53-58}}$, 69

(25) $1-2\cos^2 x+\cos^4 x$ Let $u=\cos x$. Then

$1-2u^2+u^4$. Let $v=u^2$. Then

$1-2v+v^2 = v^2-2v+1 = (v-1)^2$

$= (u^2-1)^2 = ((u-1)(u+1))^2 = (\cos x-1)^2(\cos x+1)^2$
meh

meh

Hmmmm $(\cos^2 x-1)^2 = (-\sin^2 x)^2 = \boxed{\sin^4 x}$

Me Like

(27) $\cot^3 x+\cot^2 x+\cot x+1$ Let $u=\cot x$

$= u^3+u^2+u+1 = u^2(u+1)+u+1$

$= (u+1)(u^2+1) = (\cot x+1)(\cot^2 x+1) =$

$= \boxed{(\cot x+1)(\csc^2 x)} = \csc^2 x\cot x+\csc^2 x$

$= \dfrac{1}{\sin^2 x}\,\dfrac{\cos x}{\sin x}+\dfrac{1}{\sin^2 x}$ meh

Looks as "simple" as we can make it.
In "real life," i.e. Calculus, it would depend on what you were trying to accomplish.

#s 29-32 Factor the trig expressions

(29) $3\sin^2 x-5\sin^2 x-2 = 3u^2-5u-2$

$= (3u \quad)(u \quad) = (3u+1)(u-2) = \cancel{3\sin x}$

$= \boxed{(3\sin x+1)(\sin x-2)}$

122 §3.1 #s 31–49, 53–58, 69

31 $\cot^2 x + \csc x - 1 = (\csc^2 x - 1) + \csc x - 1$

$= \csc^2 x + \csc x - 2 = u^2 + u - 2 = (u+2)(u-1)$

$= \boxed{(\csc x + 2)(\csc x - 1)}$

#s 33–4 Multiply & Simplify

33 $(\sin x + \cos x)^2 = (u+v)^2 = u^2 + 2uv + v^2$

$= \sin^2 x + 2\sin x \cos x + \cos^2 x$ = again, it would depend on your goals, as to what the definition of "simplified" would be, but a standard move is replacing $\sin^2 x$ with $1-\cos^2 x$ or something similar, like $\cos^2 x$ with $1-\sin^2 x$.

$= 1 - \cos^2 x + 2\sin x \cos x + \cos^2 x$

$= \boxed{2\sin x \cos x + 1}$ OR

$= \sin^2 x + 2\sin x \cos x + (1 - \sin^2 x)$

$= 2\sin x \cos x + 1$ again.

#s 35–44 Use identities to simplify the expression.

35 $\cot\theta \sec\theta = \dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{1}{\cos\theta} = \dfrac{1}{\sin\theta} = \boxed{\csc\theta}$

122 §2.1 #s 37–49, $\underset{\text{ALL}}{\underline{53-58}}$, 69

37 $\sin\phi(\csc\phi - \sin\phi) = \sin\theta\csc\phi - \sin^2\phi$

$= \dfrac{\sin\phi}{\sin\phi} - \sin^2\phi = 1 - \sin^2\phi = \boxed{\cos^2\phi}$

39 $\dfrac{1-\sin^2 x}{\csc^2 x - 1} = \dfrac{\cos^2 x}{\cot^2 x} = \dfrac{\cos^2 x}{\left(\dfrac{\cos^2 x}{\sin^2 x}\right)} = \cos^2 x\left(\dfrac{\sin^2 x}{\cos^2 x}\right)$

$= \boxed{\sin^2 x}$

41 $\cos\left(\frac{\pi}{2} - x\right)\sec x = \sin x \sec x = \sin x\left(\dfrac{1}{\cos x}\right)$

$= \boxed{\tan x}$

43 $\sin\beta\tan\beta + \cos\beta = \sin\beta\,\dfrac{\sin\beta}{\cos\beta} + \cos\beta$

$= \dfrac{\sin^2\beta}{\cos\beta} + \dfrac{\cos\beta}{1}\cdot\dfrac{\cos\beta}{\cos\beta} = \dfrac{\sin^2\beta + \cos^2\beta}{\cos\beta} = \dfrac{1}{\cos\beta}$

$= \boxed{\sec\beta}$

#s 45–48 Add/Subtract and simplify

LCD $=$ $(1-\cos x)(1+\cos x)$ $= 1-\cos^2 x$

45 $\dfrac{1}{1+\cos x} + \dfrac{1}{1-\cos x} =$

$= \dfrac{1}{1+\cos x}\cdot\dfrac{1-\cos x}{1-\cos x} + \dfrac{1}{1-\cos x}\cdot\dfrac{1+\cos x}{1+\cos x}$

$= \dfrac{1-\cos x + (1+\cos x)}{1-\cos^2 x} = \dfrac{2}{\sin^2 x} = \boxed{2\csc^2 x}$

122 §2.1 #s 47-9, ~~53-58~~ (ALL), 69

(47) $\tan x - \dfrac{\sec^2 x}{\tan x} = \tan x - \left(\dfrac{\tan^2 x + 1}{\tan x}\right)$

$= \tan x - \left(\dfrac{\tan^2 x}{\tan x} + \dfrac{1}{\tan x}\right) = \tan x - (\tan x + \cot x)$

$= \tan x - \tan x - \cot x = \boxed{-\cot x}$

#s 49-50 rewrite so it's not in fractional form

(49) $\dfrac{\sin^2 x}{1-\cos x} = \dfrac{1-\cos^2 x}{1-\cos x} = \dfrac{(1-\cos x)(1+\cos x)}{1-\cos x}$

$= \boxed{1+\cos x}$ A lot like complex #s ?

$\dfrac{2+3i}{5-7i} = \left(\dfrac{2+3i}{5-7i}\right)\left(\dfrac{5+7i}{5+7i}\right) = \dfrac{10+14i+15i+21i^2}{5^2-(7i)^2}$

$= \dfrac{10+29i-21}{25-7^2i^2} = \dfrac{-11+29i}{25+49} = \dfrac{-11+29i}{74}$

$= -\dfrac{11}{74} + \dfrac{29}{74}i = a+bi$. Any complex # can be written this way.

122 §2.1 #s 57-8, 69

These substitutions come up in Calculus, because in some situations, $\sqrt{a^2-x^2}$ is a pain in the neck, but $a\sin\theta$ is nice. You WILL SEE

$\sqrt{a^2-x^2} \Longrightarrow x = a\cos\theta$ OR $x = a\sin\theta$, depending on which helps you more.

$\sqrt{x^2-a^2} \Longrightarrow x = a\sec\theta$ OR $x = a\csc\theta$, whichever helps more.

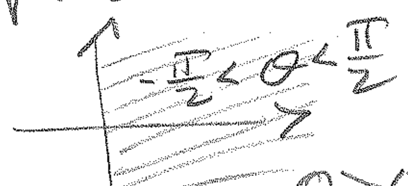
$\sqrt{a^2x^2+b^2} \Longrightarrow x = \frac{b}{a}\tan\theta$ ...

All because the trigs are easier to manipulate than these big, messy square root thingies—

#s 57-8 Same as 53-56, but now θ is in a different range: $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$

57 $3-\sqrt{9-x^2}$ & $x=3\sin\theta \longrightarrow$

$$3-\sqrt{9-x^2} = 3-\sqrt{9-9\sin^2\theta} = 3-3\sqrt{1-\sin^2\theta}$$

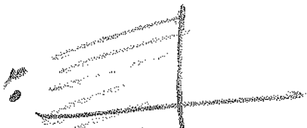


$$= 3-3\sqrt{\cos^2\theta} = 3-3|\cos\theta|$$

makes $\cos\theta > 0$

$$= \boxed{3-3\cos\theta}$$

What if the question read "$\frac{\pi}{2}<\theta<\frac{3\pi}{2}$"? Then we'd be here:



and $\cos\theta<0$ & so $|\cos\theta| = -\cos\theta$!

122 § 2.1 #s $\underset{\text{ALL}}{\underline{53-58}}$, 69

#s 53-56 Use trig substitution to write as trig func. HUGE for Calculus II. __Assume__ $0 < \theta < \frac{\pi}{2}$

(53) $\sqrt{9-x^2}$ & $x = 3\cos\theta \Longrightarrow$

$\sqrt{9-x^2} = \sqrt{9-9\cos^2\theta} = \sqrt{9(1-\cos^2\theta)}$

$= 3\sqrt{1-\cos^2\theta} = 3\sqrt{\sin^2\theta} = 3|\sin\theta|$ Easy step to miss!

$= \boxed{3\sin\theta}$ since $0 < \theta < \frac{\pi}{2} \Longrightarrow \sin\theta > 0$

(54) $\sqrt{49-x^2}$ & $x = 7\sin\theta \Longrightarrow$

$\sqrt{49-x^2} = \sqrt{49-(7\sin\theta)^2} = \sqrt{49-49\sin^2\theta} = \sqrt{49(1-\sin^2\theta)}$

$= 7\sqrt{1-\sin^2\theta} = 7\sqrt{\cos^2\theta} = 7|\cos\theta| = 7\cos\theta$, since

$0 < \theta < \frac{\pi}{2} \Longrightarrow \cos\theta > 0$.

(55) $\sqrt{x^2-4}$ & $x = 2\sec\theta \Longrightarrow$

$\sqrt{(2\sec\theta)^2-4} = \sqrt{4\sec^2\theta-4} = \sqrt{4(\sec^2\theta-1)}$

$= 2\sqrt{\sec^2\theta-1} = 2\sqrt{\tan^2\theta} = 2|\tan\theta| = 2\tan\theta$,

since $0 < \theta < \frac{\pi}{2} \Longrightarrow \tan\theta > 0$.

(56) $\sqrt{9x^2+25}$ & $3x = 5\tan\theta \Longrightarrow x = \frac{5}{3}\tan\theta \Longrightarrow$

$\sqrt{9x^2+25} = \sqrt{9(\frac{5}{3}\tan\theta)^2+25} = \sqrt{9\cdot\frac{25}{9}\tan^2\theta+25}$

$= 5\sqrt{\tan^2\theta+1} = 5|\sec\theta| = 5\sec\theta$!

122 §2.1 #s 58, 69

58 $-5\sqrt{3} - \sqrt{100 - x^2}$ & $x = 10\cos\theta \longrightarrow$

$-5\sqrt{3} - \sqrt{100-x^2} = -5\sqrt{3} - \sqrt{10^2 - 10^2\cos^2\theta}$

$= -5\sqrt{3} - 10\sqrt{1 - \cos^2\theta} = -5\sqrt{3} - 10\sqrt{\sin^2\theta}$

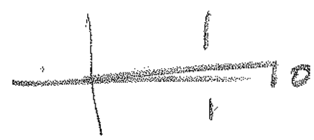
$= -5\sqrt{3} - 10\,|\sin\theta|$

oops! All cool stuff, but I misread the question!

#s 57-8 Write the equation as a <u>trig</u> equation. Then solve for $\sin\theta$ & $\cos\theta$

57 $3 = \sqrt{9 - x^2}$ & $x = 3\sin\theta$ $\longrightarrow 3 = \sqrt{9 - 9\sin^2\theta}$

$\Longrightarrow 3 = 3\sqrt{1 - \sin^2\theta} = 3\,|\cos\theta| = 3\cos\theta$



$\Longrightarrow 3 = 3\cos\theta \Longrightarrow \boxed{\begin{array}{l}\cos\theta = 1 \\ \sin\theta = 0\end{array}}$

58 $-5\sqrt{3} = \sqrt{100 - x^2}$ & $x = 10\cos\theta$ appears to be poorly posed

$$\left.\begin{array}{l} -5\sqrt{3} < 0 \\ \sqrt{100 - x^2} \geq 0 \end{array}\right\} \text{Impossible!}$$

"$\sqrt{}$" is the "principal" or <u>nonnegative</u> square root!

122 §2.1 #s 58, 69

58 cntd. If we go thru this, anyway, we end up with:

$$-5\sqrt{3} = \sqrt{100 - 100\cos^2\theta} = 10\sqrt{1-\cos^2\theta}$$

$$= 10\sqrt{\sin^2\theta} = \underline{10|\sin\theta|} \text{ and}$$

$10|\sin\theta|$ will NEVER equal $-5\sqrt{3}$!

Book answer, they IGNORE this uncomfortable fact! Just call $10|\sin\theta| = 10\sin\theta$

& obtain $10\sin\theta = -5\sqrt{3} \Longrightarrow$

$$\boxed{\sin\theta = -\frac{\sqrt{3}}{2} \text{ \& } \cos\theta = \frac{1}{2}}$$

But I HATE that author is as fuzzy as an 099 student on how to handle square roots!

122 §2.1 ~~#~~69

69 use $u = a\tan\theta$ & assume $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ and $a > 0$, to simplify

$$\sqrt{a^2+u^2} = \sqrt{a^2 + a^2\tan^2\theta}$$

$$= \sqrt{a^2(1+\tan^2\theta)} = \sqrt{a^2}\sqrt{1+\tan^2\theta}$$

$$= |a|\sqrt{\sec^2\theta} \underset{a>0}{=} a\,|\sec^2\theta\,| \underset{-\frac{\pi}{2}<\theta<\frac{\pi}{2}}{=} \boxed{a\sec\theta}$$

This one, they handled "$\sqrt{\ }$" just fine.