122 §1.6 #s 1-8, 15-37, 57-64 ALL, 69, 70, 85

(1) Tangent, cotangent and cosecant are odd, so their graphs are symmetric w.r.t. the origin

(2) Tangent, cotangent, secant and cosecant have vertical asymptotes

(3) For graphs of secant & cosecant, make graphs of their reciprocal functions, cosine and sine, resp.

(4) For $g(x)\sin(x)$, $g(x)$ is called the dampening function.

(5) $y = \tan(x)$ has period π.

(6) $D(\cot(x)) = \{x \mid x \neq n\pi, n \in \mathbb{Z}\} =$

$= \{x \mid x \neq 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots\}$

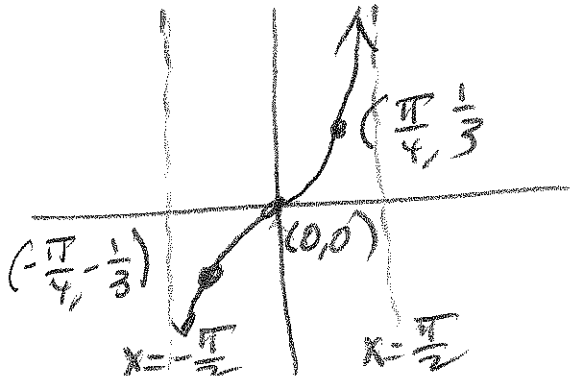
(7) $R(\sec(x)) = \{y \mid |y| \geq 1\} = \{y \mid y \geq 1 \text{ or } y \leq -1\}$

$= (-\infty, -1] \cup [1, \infty)$

(8) Period of $y = \csc(x)$ is $T = 2\pi$.

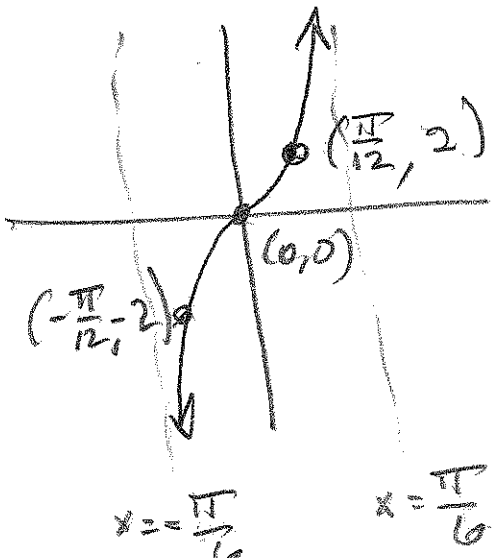
#s 15-38 Sketch the graph

(15) $y = \frac{1}{3}\tan(x)$



122 §1.6 #s 17-37, 57-64 ALL, 69, 70, 85

(17) $y = -2\tan(3x)$



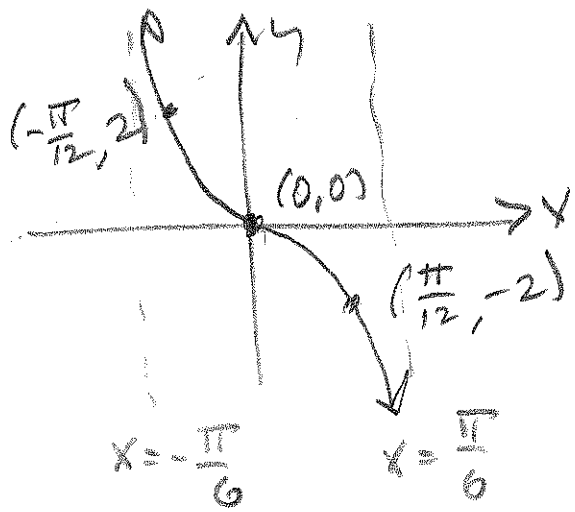
$(x,y) \longmapsto (\frac{1}{3}x, -2y)$

$\frac{\frac{\pi}{2}}{3} = \frac{\pi}{6}$

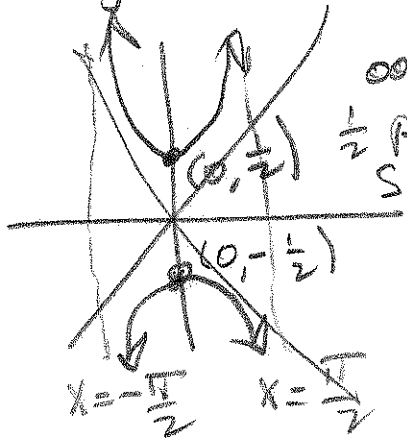
$\frac{\frac{\pi}{4}}{3} = \frac{\pi}{12}$

$x = -\frac{\pi}{6}$ $\quad x = \frac{\pi}{6}$

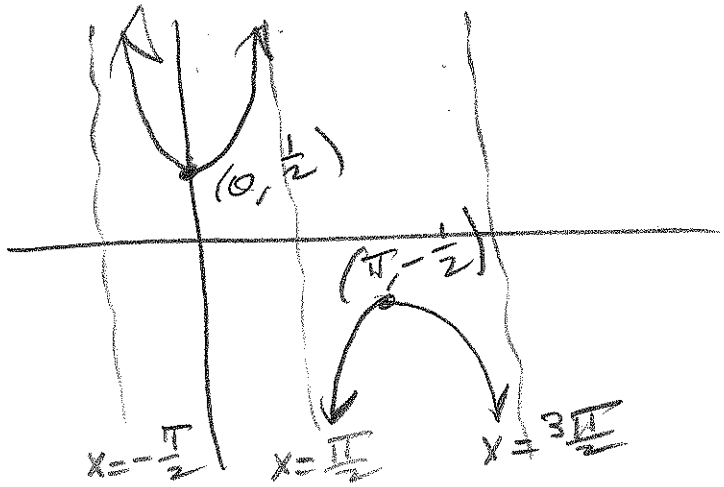
oops! This is $y = +2\tan(3x)$ want $-2\tan(3x)$:=



(19) $y = -\frac{1}{2}\sec(x)$ $\quad (x,y) \longmapsto (x, -\frac{1}{2}y)$

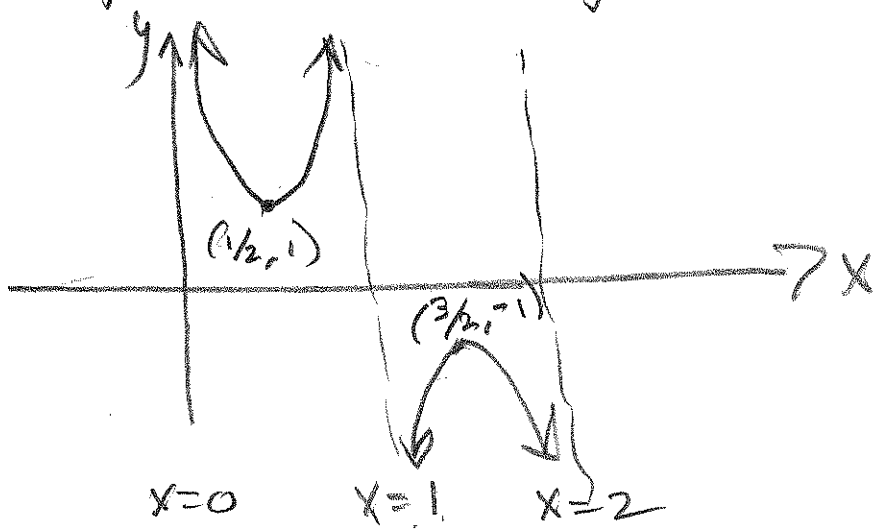


oops! ½ period! Scrunched!? What?!



122 §1.6 #521-37, 57-64ALL, 69, 70, 85

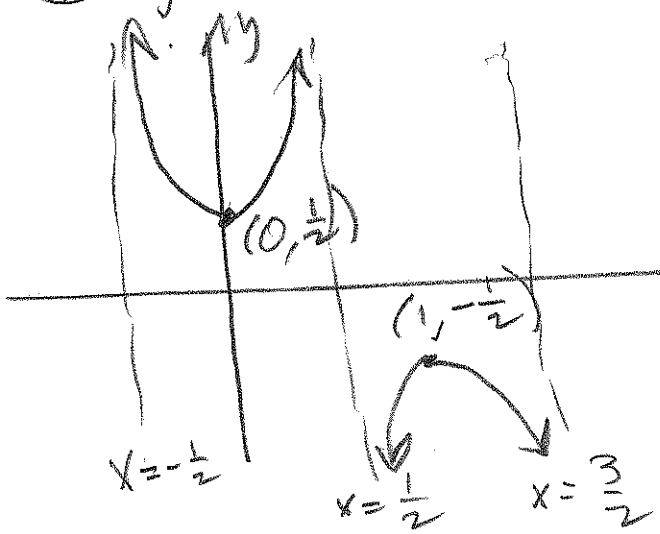
(21) $y = \csc(\pi x)$ $\quad (x,y) \longmapsto \left(\frac{1}{\pi}x, y\right)$



$\frac{\pi}{\pi} = 1$

$\frac{2\pi}{\pi} = 2$

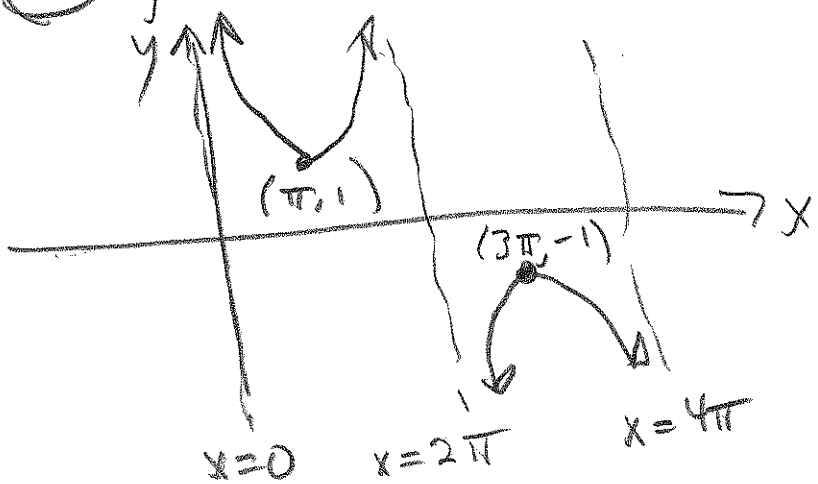
(23) $y = \frac{1}{2}\sec(\pi x)$ $\quad (x,y) \longmapsto \left(\frac{1}{\pi}x, \frac{1}{2}y\right)$



$\frac{-\frac{\pi}{2}}{\pi} = -\frac{1}{2}$

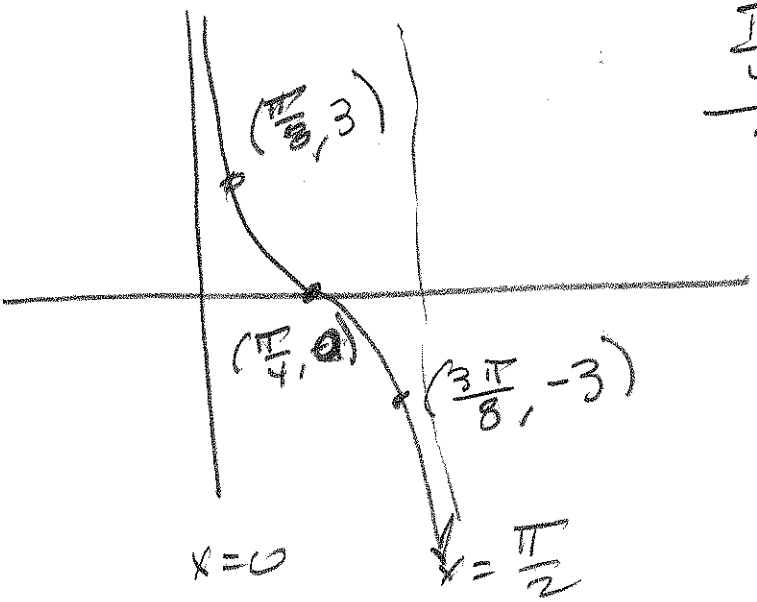
(25) $y = \csc\left(\frac{1}{2}x\right)$ $\quad (x,y) \longmapsto (2x, y)$

2π



122 §1.6 #s 27-37, 57-64 All, 69, 70, 85

(27) $y = 3\cot(2x)$ $\quad (x, y) \longmapsto \left(\frac{1}{2}x, 3y\right)$

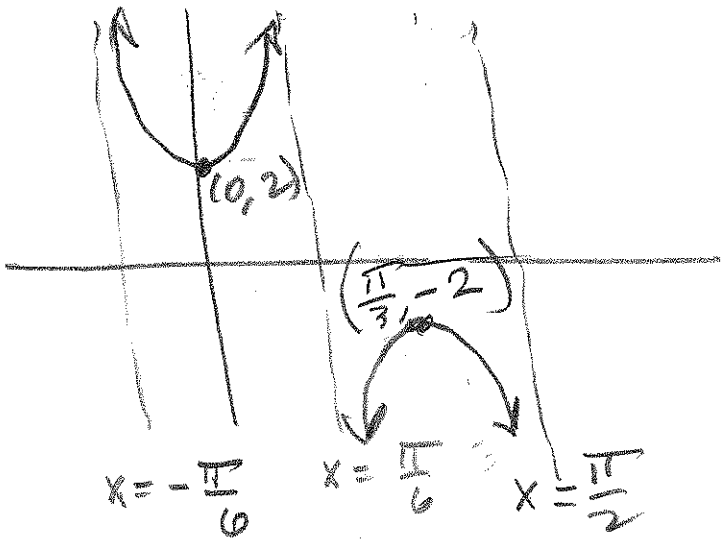


$\frac{\frac{\pi}{4}}{2} = \frac{\pi}{8}$

$\frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$

$\pi \cdot \frac{1}{2} = \frac{\pi}{2}$

(29) $y = 2\sec(3x)$ $\quad (x, y) \longmapsto \left(\frac{1}{3}x, 2y\right)$



$-\frac{\pi}{2} \cdot \frac{1}{3} = -\frac{\pi}{6}$

$\frac{3\pi}{2} \cdot \frac{1}{3} = \frac{\pi}{2}$

$\pi \cdot \frac{1}{3} = \frac{\pi}{3}$

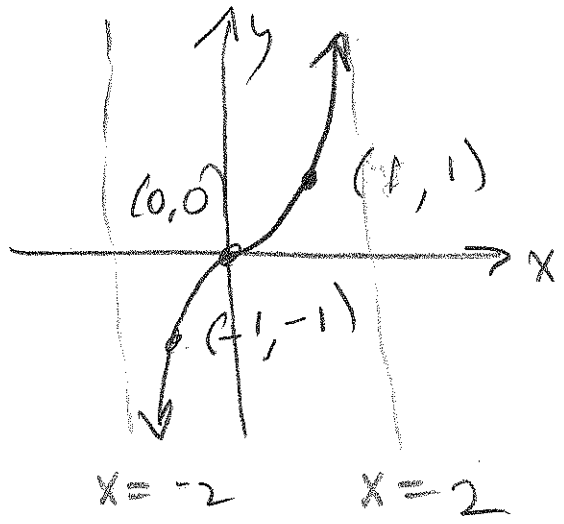
122 § 1.6 #s 31-37, 57-64 All, 69, 70, 85

(31) $y = \tan\left(\frac{\pi}{4}x\right)$

$(x, y) \longmapsto \left(\frac{4}{\pi}x, y\right)$

$-\frac{\pi}{2} \cdot \frac{4}{\pi} = -2$

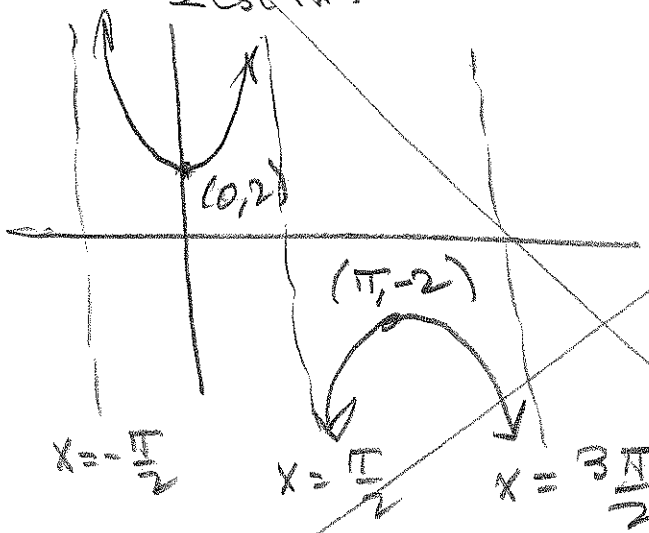
$\frac{\pi}{4} \cdot \frac{4}{\pi} = 1$

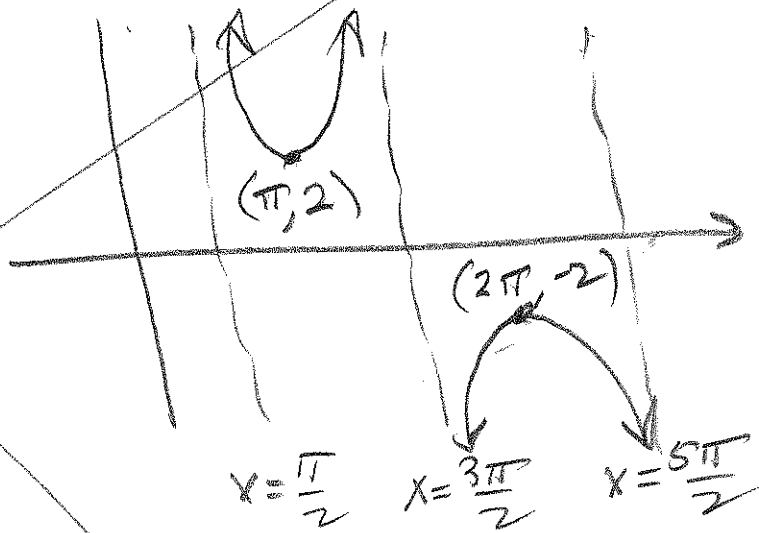


(33) $y = 2\csc(x - \pi)$

$2\csc(x)$ $(x, y) \longmapsto (x, 2y)$

$2\csc(x - \pi)$ $(x, y) \longmapsto (x + \pi, y)$





Beautiful job on $y = 2\sec(x - \pi)$. Unfortunately, that wasn't asked. NOT a graph of $2\csc(x - \pi)$

$y = 2\csc(x - \pi)$

$-\frac{\pi}{2} + \pi = +\frac{\pi}{2}$

$\frac{\pi}{2} + \pi = \frac{3\pi}{2}$

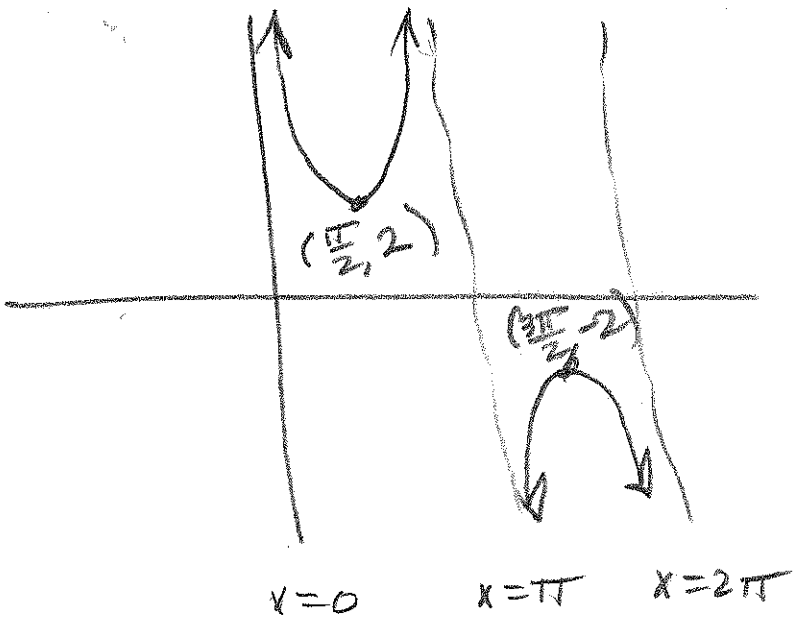
122 §1.6 #s 35, 37, 57-64 ALL, 69, 70, 85

(33) $y = 2\csc(x - \pi)$

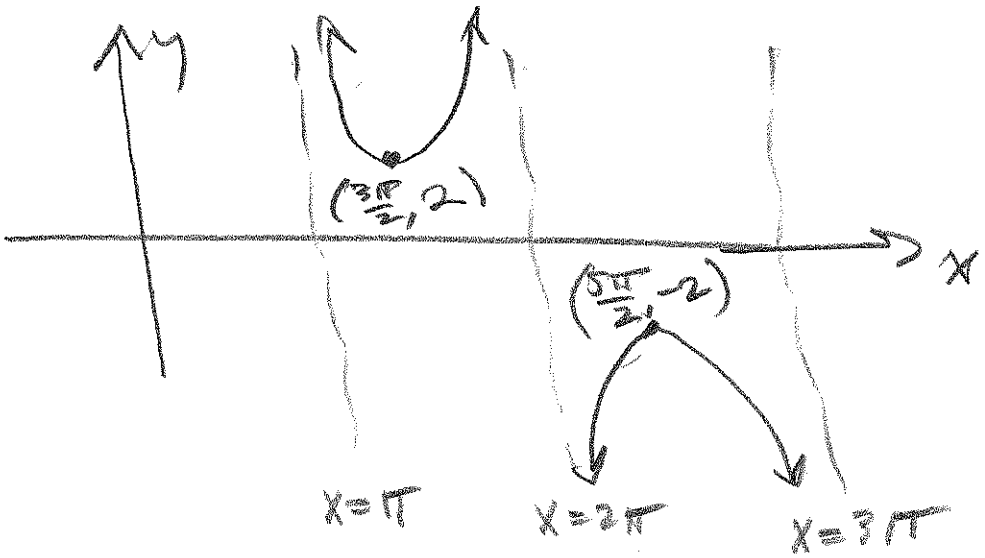
$2\csc(x)$ $(x, y) \longmapsto (x, 2y)$

$2\csc(x - \pi)$ $(x, y) \longmapsto (x + \pi, y)$

$y = 2\csc(x)$



$y = 2\csc(x - \pi)$

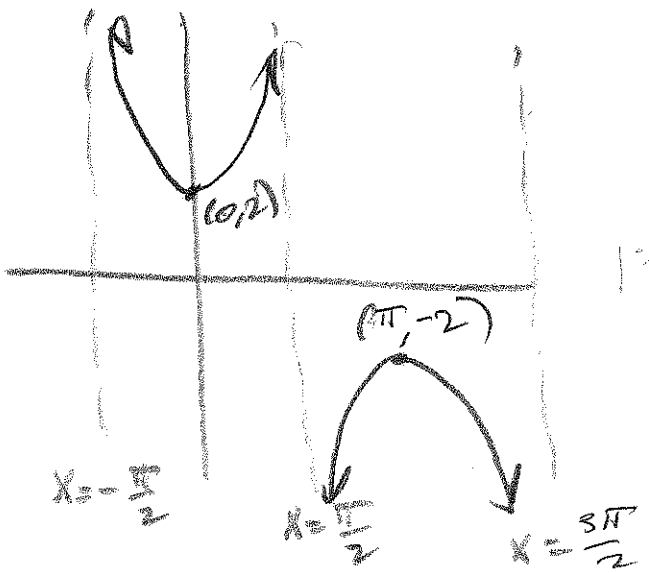


122 §1.6 #s 35, 37, 57–64 All, 69, 70, 85

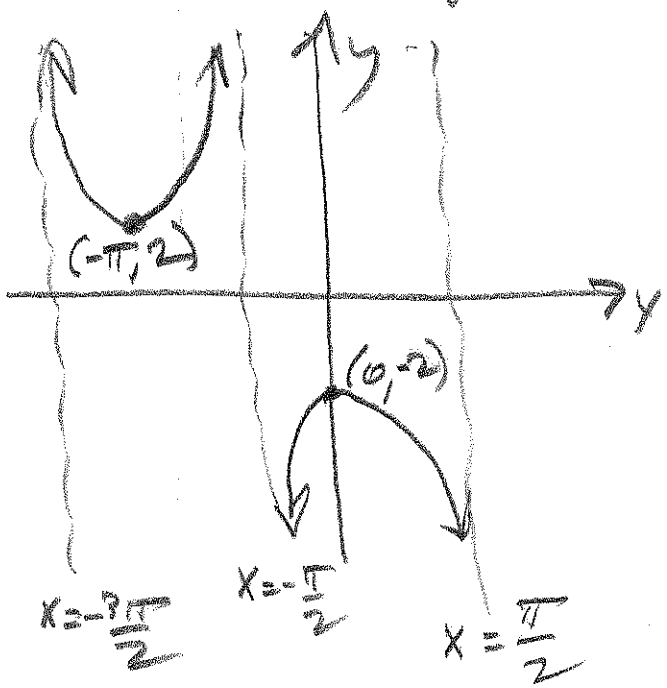
(35) $y = 2\sec(x+\pi)$

$2\sec(x)$ $(x,y) \longmapsto (x,2y)$

$2\sec(x+\pi)$ $(x,y) \longmapsto (x-\pi, y)$



$y = 2\sec(x)$



$y = 2\sec(x+\pi)$

(37) $y = \frac{1}{4}\csc\left(x+\frac{\pi}{4}\right)$

$(x,y) \longmapsto \left(x-\frac{\pi}{4}, \frac{1}{4}y\right)$

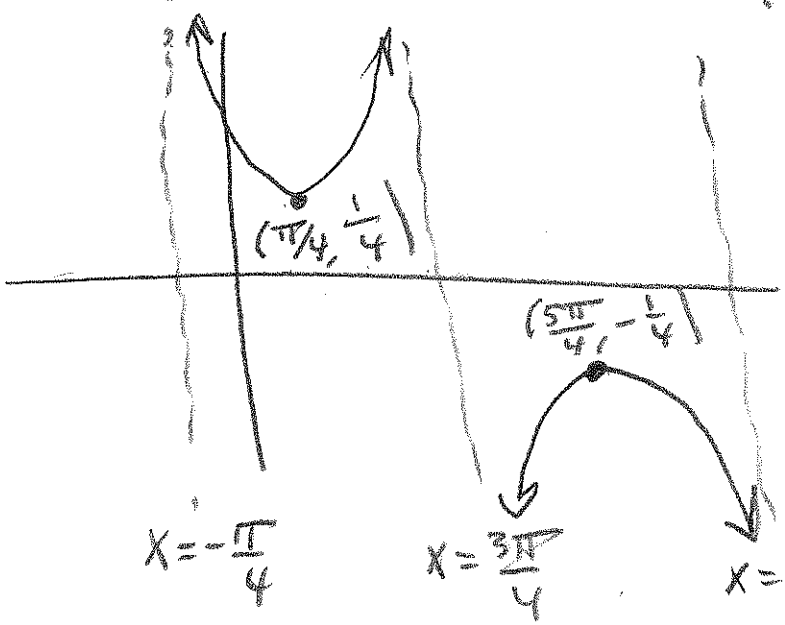
$\left(\frac{\pi}{2}, 1\right) \longmapsto \left(\frac{\pi}{4}, \frac{1}{4}\right)$

$\left(\frac{3\pi}{2}, -1\right) \longmapsto \left(\frac{5\pi}{4}, -\frac{1}{4}\right)$

$x = 0 \longrightarrow x = -\frac{\pi}{4}$

$x = \pi \longrightarrow x = \frac{3\pi}{4}$

$x = 2\pi \longrightarrow x = \frac{7\pi}{4}$



122 §1.6 #s 57-64 ALL, 69, 70, 85

#s 57-64 Determine odd/even/neither

(57) $f(x) = \sec(x)$

$f(-x) = \sec(-x) = \frac{1}{\cos(-x)} = \frac{1}{\cos(x)} = \sec(x)$

$\Rightarrow$ EVEN.

(59) $f(x) = \cot(x)$ is odd b/c $\tan(x)$ is odd and I can remember that. Also, just off odd sine & even cosine, we get

$\tan \cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{+}{+} = -$ is odd.

(61) $f(x) = x + \tan(x) = \text{odd} + \text{odd} = \text{odd}$

$\left(f(-x) = -x + \tan(-x) = -(x + \tan(x)) = -f(x) \right)$

(62) $f(x) = x^2 - \sec(x)$

$f(-x) = (-x)^2 - \sec(-x) = x^2 - \sec(x) = f(x)$

$\Rightarrow$ EVEN!

122 §4.6 #s 63, 64, 69, 70, 85

(63) $g(x) = x \csc(x)$

$g(-x) = (-x)(\csc(-x)) = +x\csc(x) = g(x)$

$(-)(-) = +$ (EVEN)

$(+)(-) = -$ (ODD)

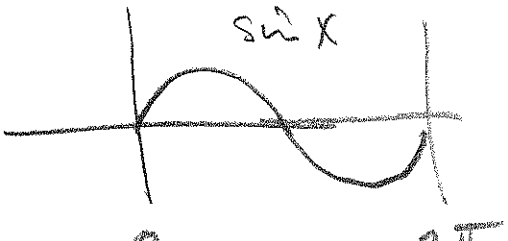
(64) $g(x) = x^2 \cot(x)$

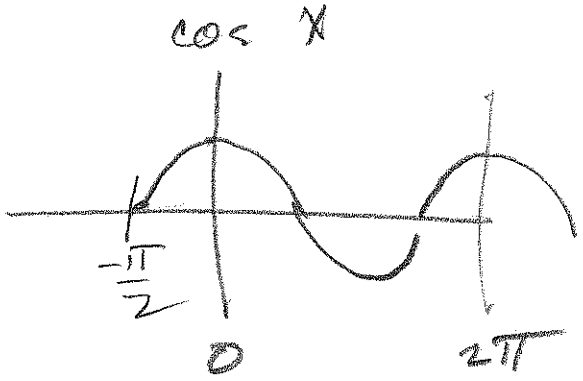
$g(-x) = (-x)^2 \cot(-x) = x^2(-\cot(x)) = -x^2\cot(x)$

$= -g(x)$ ODD

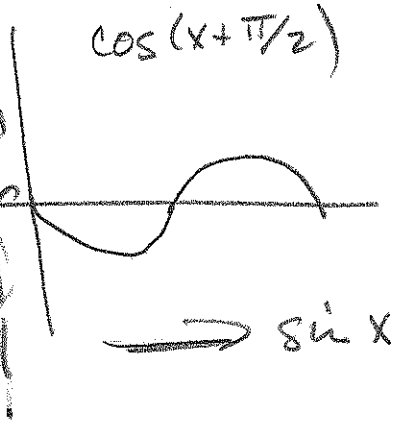
#s 69–72 Graph f & g. Conjecture.

(69) $f(x) = \sin x + \cos\left(x + \frac{\pi}{2}\right)$, $g(x) = 0$





They cancel each other out when you add them!

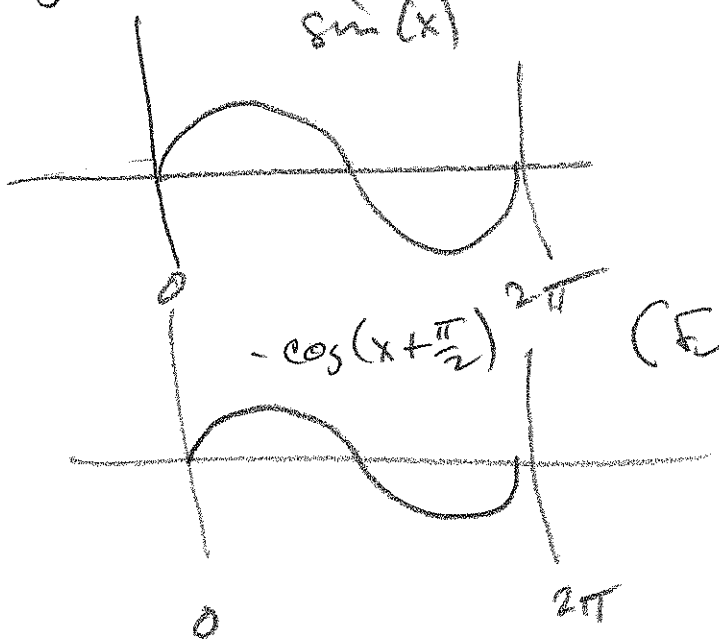


$\Rightarrow \sin x + \cos\left(x + \frac{\pi}{2}\right) = f(x) = g(x) = 0$

122 Sec. #s 70, 85

(70) $f(x) = \sin x - \cos\left(x + \frac{\pi}{2}\right)$, $g(x) = 2\sin(x)$

By previous work:



They add to get $g(x) = 2\sin(x)$, i.e.

$$\sin x - \cos\left(x + \frac{\pi}{2}\right) = 2\sin(x)$$

$f = g$

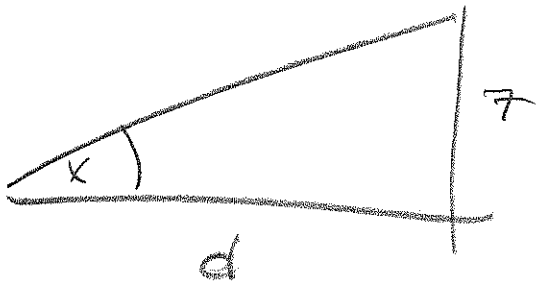
122 S 1.6 #s ~~70~~ 85

(85) Plane flies at an altitude of 7 mi ~~[illegible]~~

Radar antenna is distance d away horizontally.

Let x = angle between oncoming plane & antenna

Model d as function of angle x, $0 < x < \pi$



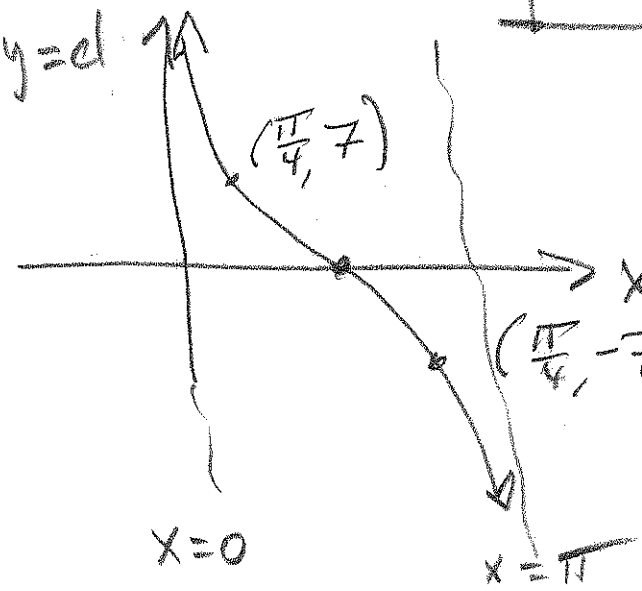
$\frac{7}{d} = \tan x \Longrightarrow$

$\frac{7}{\tan x} = d \longrightarrow$

d = distance, in miles.

x = angle (radians)

$d = 7\cot(x)$



$(\frac{\pi}{4}, -7)$ is plane already past and pulling away