122 §1.5 #'s 1-21, 39-45, 51-61, 65, 83, 85, 87

(1) One period of sine or cosine is called one <u>cycle</u>.

(2) The <u>amplitude</u> of sine or cosine is one half the distance between max & min values

(3) For the function $y = a\sin(bx - c) = \underline{a\sin\left(b\left(x - \frac{c}{b}\right)\right)}$,

$\frac{c}{b}$ represents the <u>phase</u> <u>shift</u>

How you should <u>always</u> look @ it.

I say "horizontal" shift, but "phase" sounds like you're smart.

(4) For $y = d + a\cos(bx - c)$, d is the <u>vertical</u> <u>shift</u>.

~~(5)~~ #s 5-18 Find period, T, and amplitude, A.

(5) $y = 2\sin(5x) \Longrightarrow$ $\boxed{A = 2}$

T: $5x = 2\pi$, when $\boxed{x = \frac{2\pi}{5} = T}$

§1.5 #s 6–21, 39–45, 51–61, 65, 83, 85, 87

(6) $y = 3\cos(2x) \Rightarrow \boxed{A = 3,}\ T = \frac{2\pi}{2} = \boxed{2 = T}$

(7) $y = \frac{3}{4}\cos\left(\frac{x}{2}\right) \longrightarrow \boxed{A = \frac{3}{4},}\ T = \frac{2\pi}{\frac{1}{2}} = \boxed{4\pi = T}$

(9) $y = \frac{1}{2}\sin\left(\frac{\pi}{3}x\right) \longrightarrow \boxed{A = \frac{1}{2},}\ T = \frac{2\pi}{\frac{\pi}{3}} = \boxed{6 = T}$

(11) $y = -4\sin(x) \Longrightarrow \boxed{A = -4,\ T = 2\pi}$

(13) $y = 3\sin(10x) \Longrightarrow \boxed{A = 3,}\ T = \frac{2\pi}{10} = \boxed{\frac{\pi}{5} = T}$

(15) $y = \frac{5}{3}\cos\left(\frac{4x}{5}\right) \Longrightarrow \boxed{A = \frac{5}{3},}\ T = \frac{2\pi}{\frac{4}{5}} = (2\pi)\left(\frac{5}{4}\right) = \boxed{\frac{5\pi}{2} = T}$

(17) $y = \frac{1}{4}\sin(2\pi x) \Longrightarrow \boxed{A = \frac{1}{4},}\ T = \frac{2\pi}{2\pi} = \boxed{1 = T}$

#s 19–30 Describe the graphs of f & g; the relationship between them, rather. Consider Amplitude, period and shifts.

(19) $f(x) = \sin x$
$g(x) = \sin(x - \pi)$
$\Longrightarrow$ g(x) is a shift to the right of π units of f(x). All else identical.

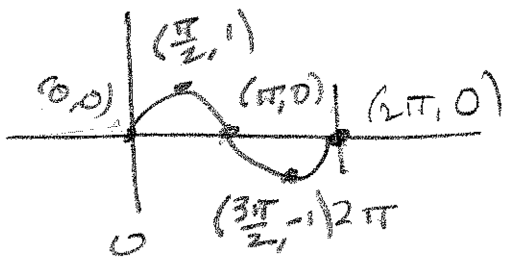
(21) $f(x) = \cos(2x)$
$g(x) = -\cos(2x)$
$\longrightarrow$ g(x) is a reflection of f(x) over/thru the x-axis.

p. 122 § 1.5 #s 39–45, 51–61, 65, 83, 85, 87

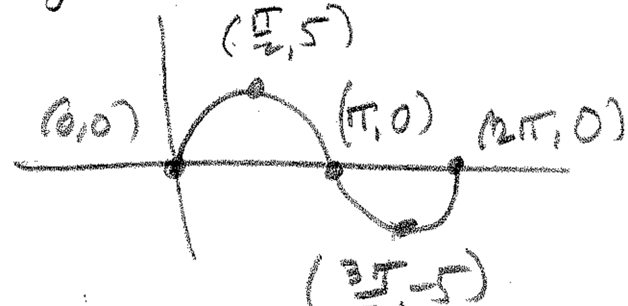
#s 39–60 Sketch ONE period...

(39) $y = 5 \sin x = g(x)$

$f(x) = \sin x$

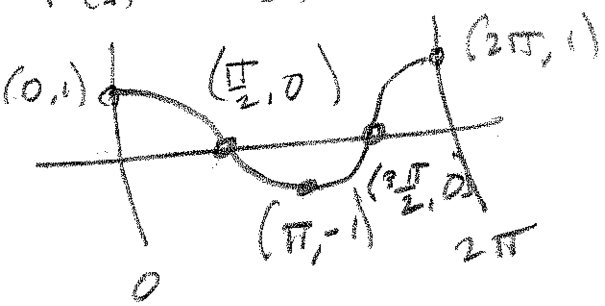


$g(x) = 5 f(x) = 5 \sin x$



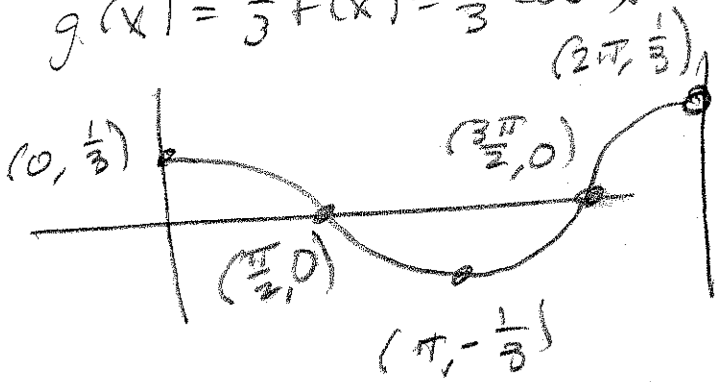
(41) $y = \frac{1}{3} \cos x = g(x)$

$f(x) = \cos x$



$(x, y) \longmapsto (x, \frac{1}{3} y)$

$g(x) = \frac{1}{3} f(x) = \frac{1}{3} \cos x$

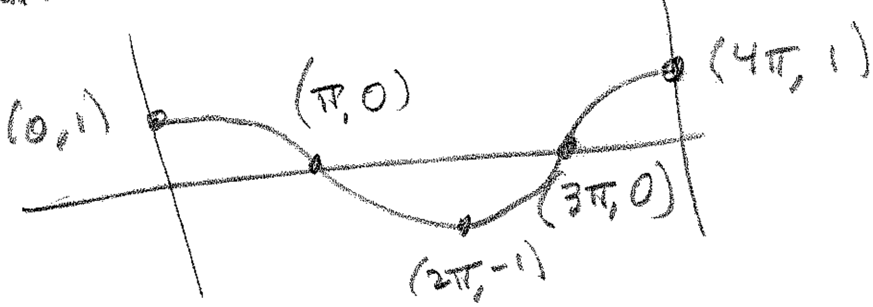


(43) $y = \cos(\frac{1}{2} x) = g(x)$

See $\cos x$, above $f(x) = \cos x$

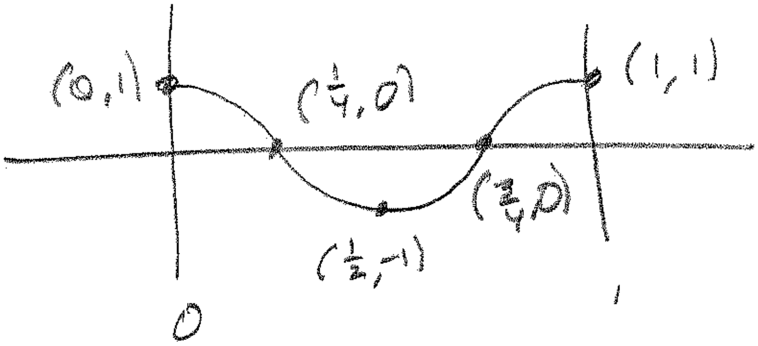
$g(x) = \cos(\frac{1}{2} x) = f(\frac{1}{2} x)$

$(x, y) \longmapsto (2x, y)$



12.1 §1.8 #s 45, 51-61, 65; 83, 88; 8?

45 $y = \cos(2\pi x)$ $\quad (x, y) \longmapsto \left(\frac{x}{2\pi}, y\right)$



$$\frac{\frac{\pi}{2}}{2\pi} = \frac{\pi}{2} \cdot \frac{1}{2\pi} = \frac{1}{4}$$

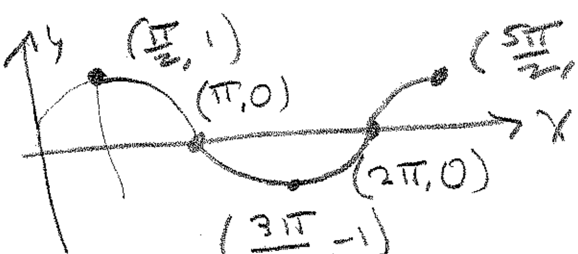
$$\frac{\pi}{2\pi} = \frac{1}{2}$$

$$\frac{\frac{3\pi}{2}}{2\pi} = \frac{3}{4}$$

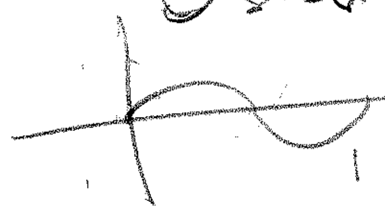
$$\frac{2\pi}{2\pi} = 1$$

~~47~~ 51 $y = \sin\left(x - \frac{\pi}{2}\right)$ $\quad (x, y) \longmapsto \left(x + \frac{\pi}{2}, y\right)$

$$2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$$



1 $\sin(x)$

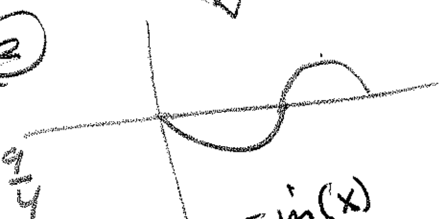


53 $x = -\sin\left(\frac{2\pi}{3}x\right) + 2$

$-y \quad \frac{3}{2\pi}x \quad y+2$

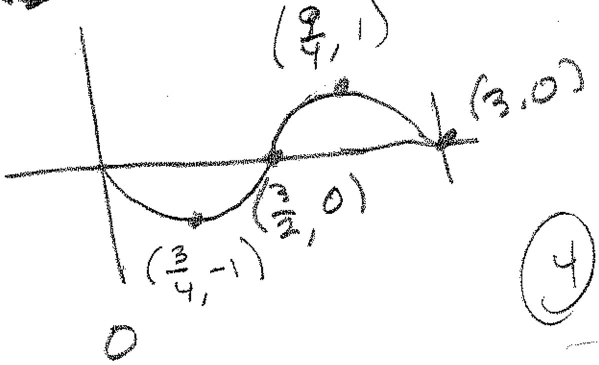
$$\frac{2\pi}{\frac{2\pi}{3}} = 3 = T$$

2

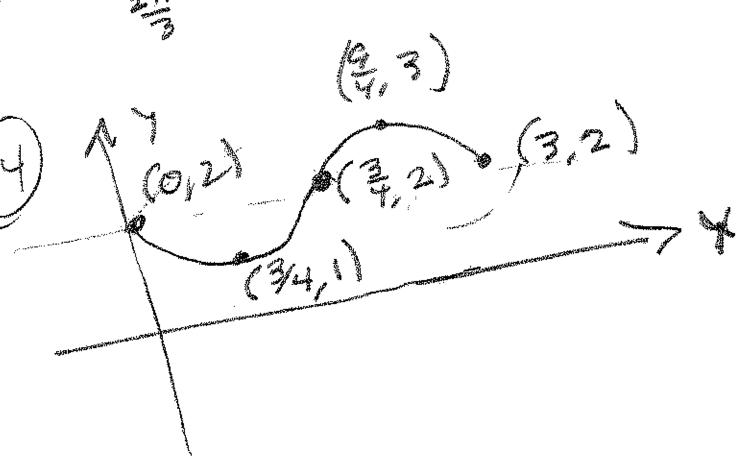


3 $-\sin\left(\frac{2\pi}{3}x\right)$

$$\frac{\frac{3\pi}{2}}{\frac{2\pi}{3}} = \frac{3\pi}{2} \cdot \frac{3}{2\pi} = \frac{9}{4}$$



4

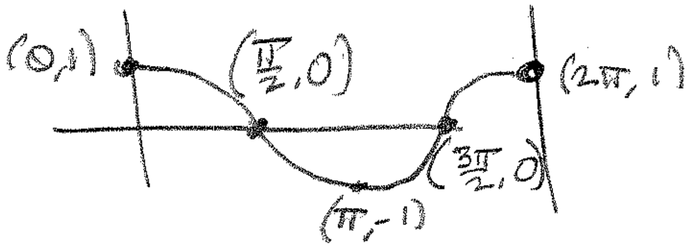


§ 1.5 #s 55-61, 65, 83, 85, 87

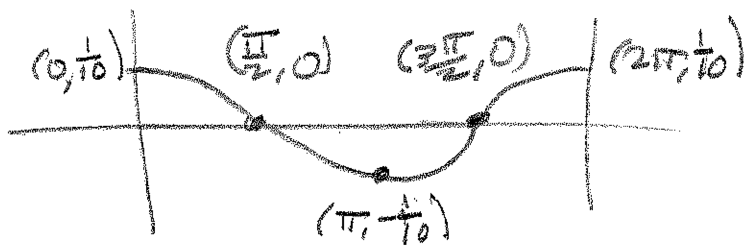
(55) $y = 2 + \frac{1}{10}\cos(60\pi x)$

$= \frac{1}{10}\cos(60\pi x) + 2 = g(x)$

(1) $f(x) = \cos x$

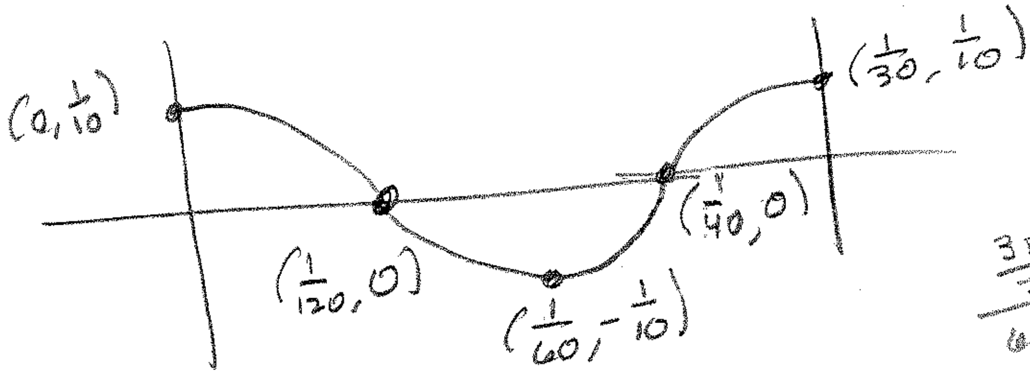


(2) $\frac{1}{10}\cos(x) = \frac{1}{10}f(x)$



(3) $\frac{1}{10}\cos(60\pi x) = \frac{1}{10}f(60\pi x)$

$\frac{2\pi}{60\pi} = \frac{1}{30}$



$\frac{\frac{\pi}{2}}{60\pi} = \frac{1}{120}$

$\frac{\frac{3\pi}{2}}{60\pi} = \frac{1}{40}$

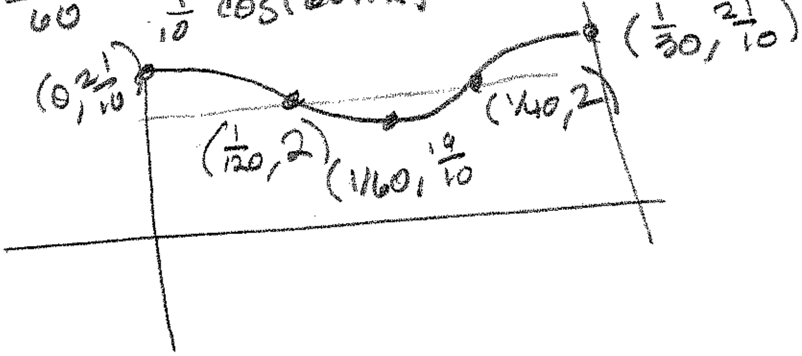
$\frac{\pi}{60\pi} = \frac{1}{60}$

$\frac{1}{10} + 2 = \frac{21}{10}$

$\frac{1}{60} + 2 = \frac{121}{60}$

$-\frac{1}{10} + 2 = \frac{19}{10}$

$\frac{1}{10}\cos(60\pi x) + 2 = \frac{1}{10}f(60\pi x) + 2 = g(x)$

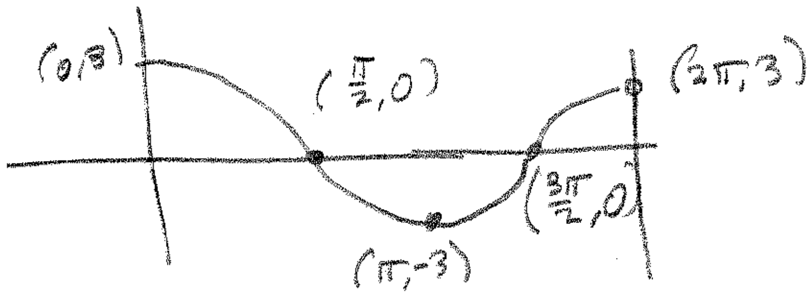


$\frac{\frac{1}{30}+\frac{1}{60}}{2} = \frac{\frac{3}{60}}{2} = \frac{3}{120} = \frac{1}{40}$

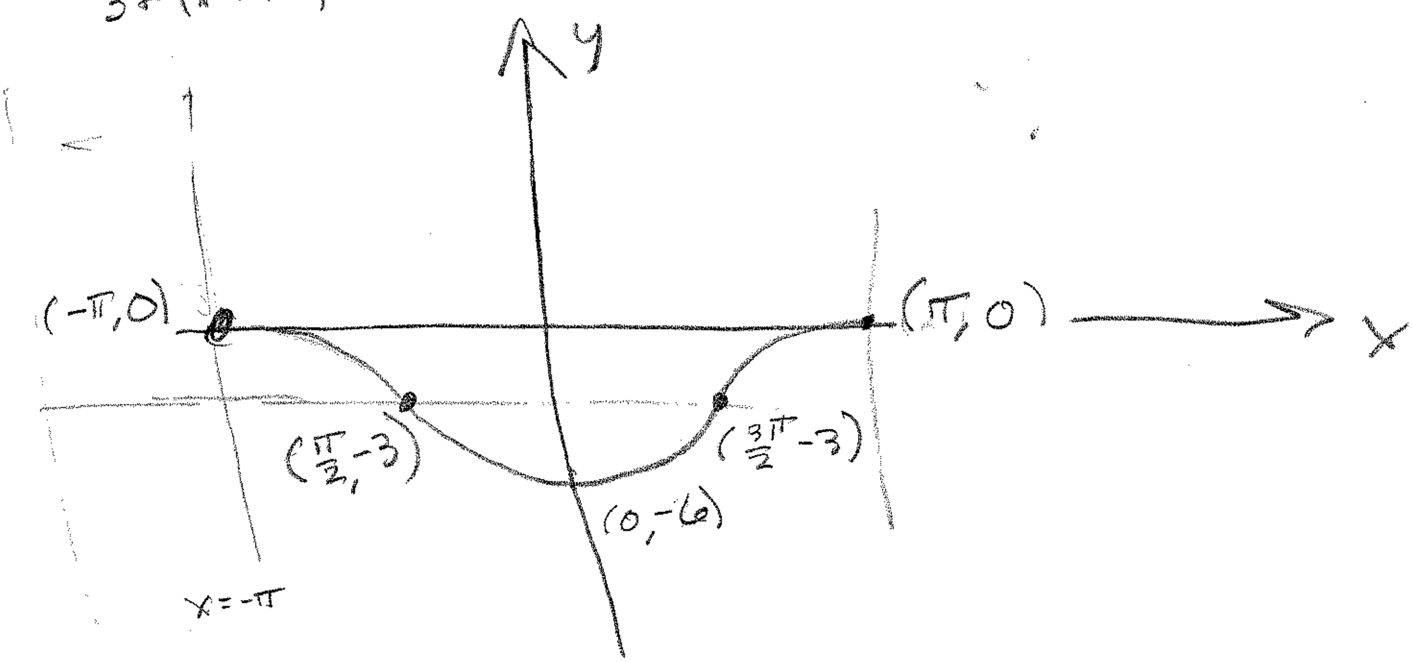
122 §1.5 #s 57-61, 65, 67, 83, 87

57 $y = 3\cos(x+\pi) - 3 = g(x)$. $f(x) = \cos(x)$

$3f(x) = 3\cos x$ left π Down 3



$3f(x+\pi) - 3$

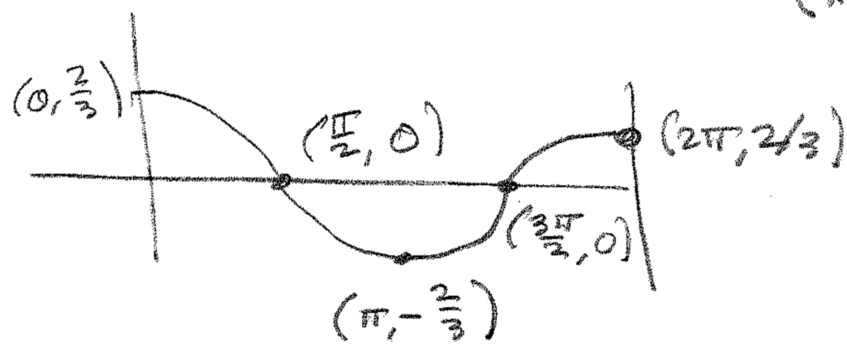


122 §1.5 #s 59, 61, 65, 83, 85, 87

59 $y = \frac{2}{3}\cos\left(\frac{x}{2} - \frac{\pi}{4}\right) = \frac{2}{3}\cos\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right)$

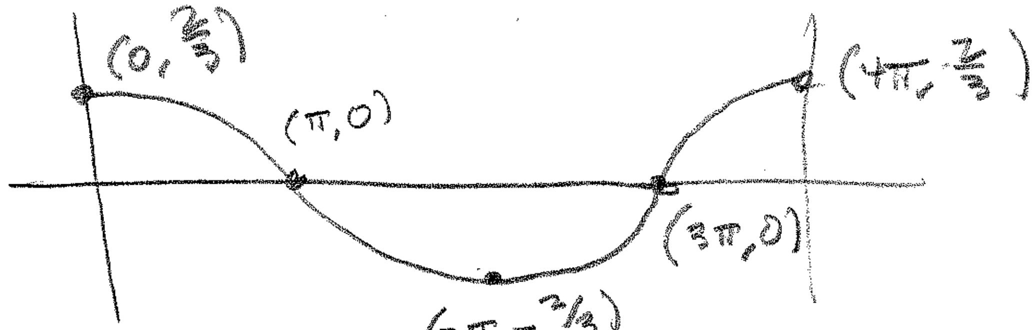
$\frac{2}{3}\cos(x) = \frac{2}{3}f(x)$

$(x, y) \longmapsto (\frac{2}{3}x, y)$



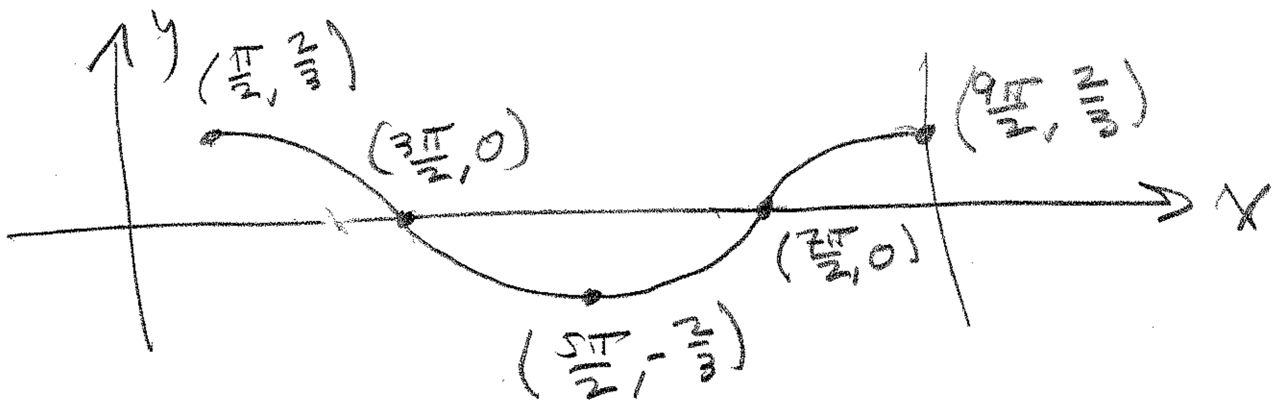
$\frac{2}{3}\cos\left(\frac{1}{2}x\right) = \frac{2}{3}f\left(\frac{1}{2}x\right)$

$(x, y) \longmapsto (2x, y)$



$\frac{2}{3}\cos\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right) = \frac{2}{3}\cos\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right)$

$(x, y) \longmapsto (x + \frac{\pi}{2}, 0)$



122 §1.5 #s 61, 65, 83, 85, 87

#s 61–66 g is related to basic (parent) function $f(x) = \sin(x)$ or $f(x) = \cos(x)$

(a) Describe sequence of transformations from f to g

(b) Sketch the graph of g

(c) Use function notation to write g in terms of f.

Here's #62, in response to question on #61

62 $g(x) = \sin(2x + \pi)$

(a) $g(x) = \sin\left(2\left(x + \frac{\pi}{2}\right)\right)$

$f(x) = \sin x \longrightarrow f(2x) = \sin(2x)$

compress by factor of $\frac{1}{2}$:

$(x, y) \longmapsto \left(\frac{1}{2}x, y\right)$

Answer to part (c)

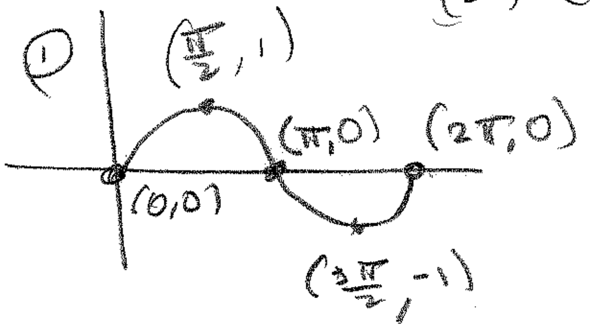
$\boxed{f\left(2\left(x + \frac{\pi}{2}\right)\right)} = \sin\left(2\left(x + \frac{\pi}{2}\right)\right)$

Phase shift left by $\frac{\pi}{2}$ units $\quad (x, y) \longmapsto \left(x - \frac{\pi}{2}, y\right)$

62 cntd

(b) Graph

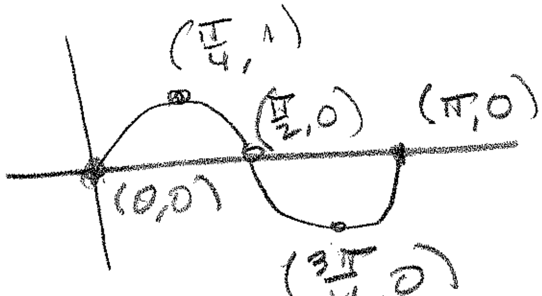
1



$f(x)=\sin x$

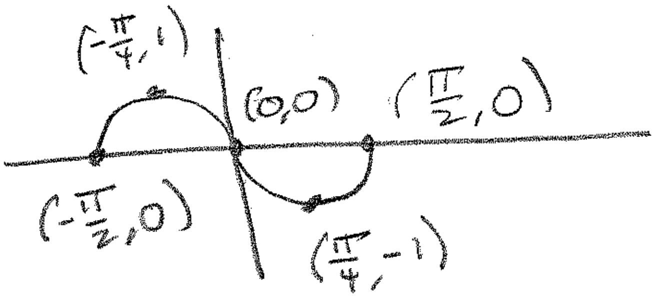
2 $f(2x)=\sin(2x)$

<u>Compress</u>



<u>Left $\frac{\pi}{2}$</u>

3 $f(2(x+\frac{\pi}{2}))$



(c) $g(x)=\sin(2x+\pi)=\sin(2(x+\frac{\pi}{2}))$

$= f(2(x+\frac{\pi}{2}))$

.122 §1.5 #s 61, 65, 83, 85, 87

(61) ~~Des~~ #s 61–66 Describe the moves from

f(x) to g(x)

(61) $g(x) = \sin(4x - \pi) = \sin\left(4\left(x - \frac{\pi}{4}\right)\right)$

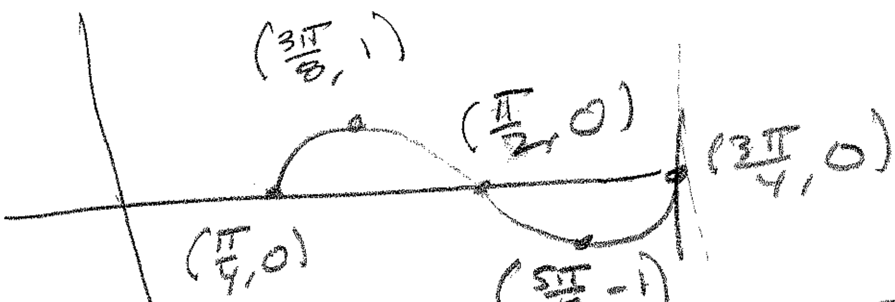
Moves

$f(x) \longmapsto f(4x) \longmapsto f\left(4\left(x - \frac{\pi}{4}\right)\right)$

$\sin x \longmapsto \sin(4x) \longmapsto \sin\left(4\left(x - \frac{\pi}{4}\right)\right)$

$(x, y) \longmapsto \left(\frac{1}{4}x, y\right)$ Horizontal shrink by factor of $\frac{1}{4}$.

$(x, y) \longmapsto \left(x + \frac{\pi}{4}, y\right)$ Right shift (delay) by $\frac{\pi}{4}$.



$\left(\frac{\pi}{2}, 1\right) \longmapsto \left(\frac{\pi}{8}, 1\right) \longmapsto \left(\frac{\pi}{8} + \frac{\pi}{4}, 1\right) = \left(\frac{3\pi}{8}, 1\right)$

$\left(\frac{3\pi}{2}, -1\right) \longmapsto \left(\frac{3\pi}{8}, -1\right) \longmapsto \left(\frac{3\pi}{8} + \frac{\pi}{4}, -1\right) = \left(\frac{5\pi}{8}, -1\right)$

$(\pi, 0) \longmapsto \left(\frac{\pi}{4}, 0\right) \longmapsto \left(\frac{\pi}{4} + \frac{\pi}{4}, 0\right) = \left(\frac{\pi}{2}, 0\right)$

122 §1.5 #s 65, 83, 85, 87 $\qquad 4x-\pi = 4\left(x-\frac{\pi}{4}\right)$

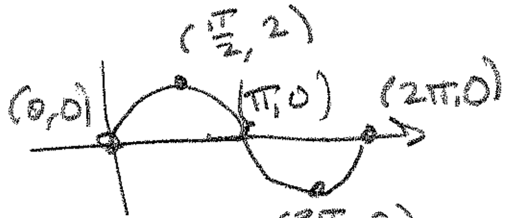
65 $g(x) = 2\sin(4x-\pi) - 3$

$f(x) = \sin x \implies g(x) = 2f\left(4\left(x-\frac{\pi}{4}\right)\right) - 3$

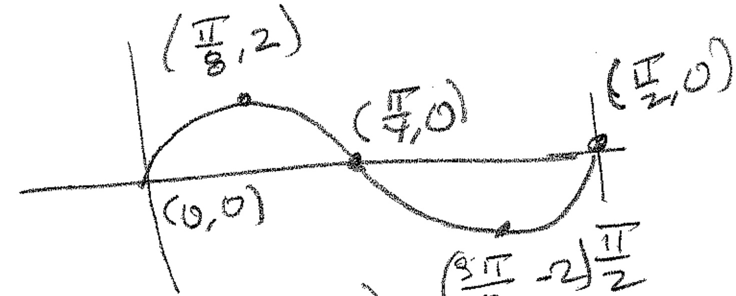
$2\sin x \longrightarrow 2\sin(4x) \longrightarrow 2\sin\left(4\left(x-\frac{\pi}{4}\right)\right)$

$y \to 2y \qquad x \to \frac{1}{4}x \qquad x \to x+\frac{\pi}{4}$

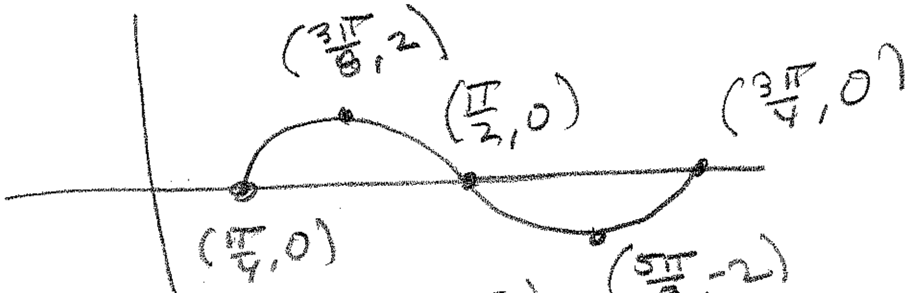
$\longrightarrow 2\sin\left(4\left(x-\frac{\pi}{4}\right)\right) - 3$



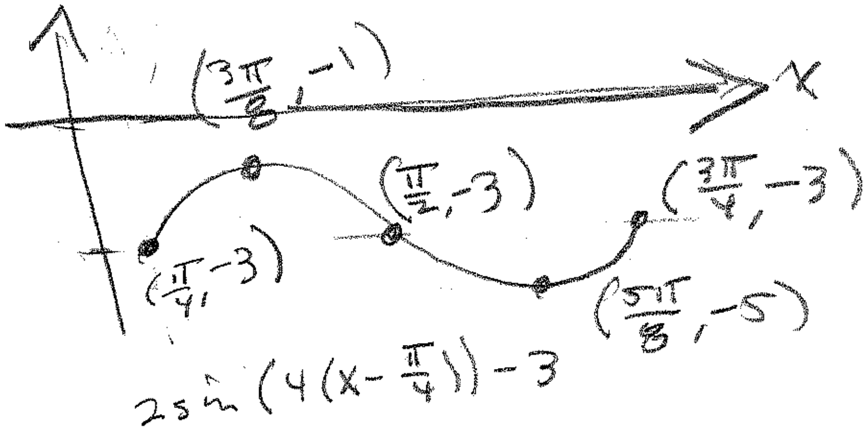
$2\sin x$



$2\sin(4x)$



$2\sin\left(4\left(x-\frac{\pi}{4}\right)\right)$



$2\sin\left(4\left(x-\frac{\pi}{4}\right)\right) - 3$

122 §1.5 #583, 585, 587

#583–586 Write the function described.

83 sine curve $\quad \sin(x)$

period is π : $\quad b\pi = 2\pi \Longrightarrow b = 2$

$\sin(2x)$

Right $\frac{\pi}{2}$ $\quad \sin\left(2\left(x-\frac{\pi}{2}\right)\right)$

up 1 $\quad \boxed{\sin(2x-\pi)+2}$

85 Cosine curve $\quad \cos(x)$

Period is π $\quad \cos(2x)$

Amp is 1 $\quad \cos(2x)$

left π $\quad \cos(2(x-\pi))$

Down $\frac{3}{2}$ $\quad \boxed{\cos(2x-2\pi)-\frac{3}{2}}$

87 Velocity of air flow after exercise is

$v = 1.75\sin\left(\frac{\pi}{2}t\right)$, where t = time in S.

"In" = "positive"

(a) One full cycle : $\frac{\pi}{2}t \overset{\text{set}}{=} 2\pi \Longrightarrow$

$t = \frac{2\pi}{\frac{\pi}{2}} = \boxed{4\ \text{s}}$

(b) $\frac{4\ \text{s}}{1\ \text{cycle}} \Longrightarrow \frac{\frac{1}{4}\ \text{cycle}}{\text{s}} \Longrightarrow$

$\left(\frac{1}{4}\frac{\text{cycle}}{\text{s}} \cdot \frac{60\ \text{s}}{1\ \text{min}}\right) = \boxed{15\ \text{cycles}/1\ \text{min}}$

(c)