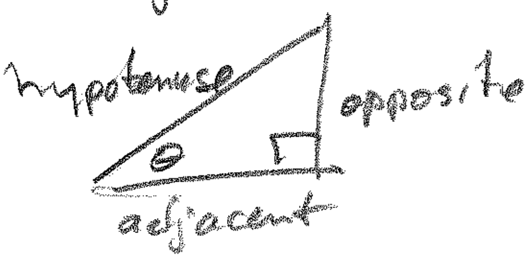
122 §1.3 #5   1-5 All, 7-37, 41-51, 57-65, 67?, 73, 75

(1) sine = $\frac{\text{opposite}}{\text{hypotenuse}}$, cosine = $\frac{\text{adjacent}}{\text{hypotenuse}}$

tangent = $\frac{\text{opposite}}{\text{adjacent}}$, cosecant = $\frac{\text{hypotenuse}}{\text{opposite}}$

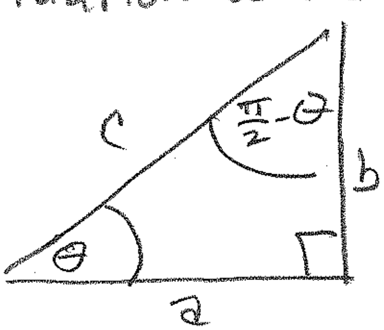
secant = $\frac{\text{hypotenuse}}{\text{adjacent}}$, cotangent = $\frac{\text{adjacent}}{\text{hypotenuse}}$



(2) Relative to the acute angle θ, the three sides of a right triangle are the adjacent, opposite, and hypotenuse.

(3) Cofunctions of complementary angles are equal. This is something we didn't discuss in lecture. Basically
$$\sin(\theta) = \cos(\tfrac{\pi}{2} - \theta)$$
$$\sin(\theta) = \cos(90^\circ - \theta)$$

I always end up using logic to handle any situation where this idea might be handy.



$\cos(\frac{\pi}{2} - \theta) = \frac{b}{c} = \sin\theta$ is the idea. Well, duh! One picture & you don't have to clutter your brain with this.

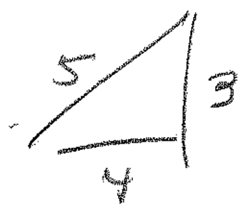
122 §1.3 #s 4, 5-37, 41-51, 57-65, 67?, 73, 75

(4) From the horizontal up to an object is angle of <u>elevation</u>; whereas, angle of <u>depression</u> is angle from horizontal <u>down</u> to an object

#5-8 Find exact value of the 6 trigs

(5)

a version of the 3-4-5!



Just scaled-up by factor of 2!

$\sin\theta = \frac{3}{5}$ $\csc\theta = \frac{5}{3}$

$\cos\theta = \frac{4}{5}$ $\sec\theta = \frac{5}{4}$

$\tan\theta = \frac{3}{4}$ $\cot\theta = \frac{4}{3}$

(7)

$\sqrt{41^2 - 9^2} = \sqrt{1600} = \sqrt{16\cdot100}$

$= 4\cdot10 = 40$

$\sqrt{1600} = \sqrt{2^6\cdot5^2} = (2^6\cdot5^2)^{\frac{1}{2}}$

$= 2^{\frac{6}{2}}\cdot5^{\frac{2}{2}} = 2^3\cdot5 = 40$

$$\begin{array}{r|l} 2 & 1600 \\ 2 & 800 \\ 2 & 400 \\ 2 & 200 \\ 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ & 5 \end{array}$$

$\sin\theta = \frac{9}{41}$ $\csc\theta = \frac{41}{9}$

$\cos\theta = \frac{40}{41}$ $\sec\theta = \frac{41}{40}$

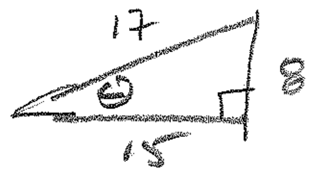
$\tan\theta = \frac{9}{40}$ $\cot\theta = \frac{40}{9}$

122 § 1.3 #s 9-37, 41-51, 57-65, 67?, 73, 75

#s 9-12 Find exact values of the six trigs.
Explain why they're the same for both triangles.

They're the same, because each pair of triangles has the same interior angles and the same ratios between sides, as a result.

(9)

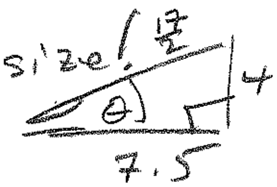


$\sin\theta = \frac{8}{17}$ $\csc\theta = \frac{17}{8}$

$\cos\theta = \frac{15}{17}$ $\sec\theta = \frac{17}{15}$

$\tan\theta = \frac{8}{15}$ $\tan\theta = \frac{15}{8}$

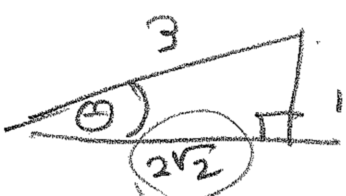
$\frac{1}{2}$-size!



$\sqrt{8^2+15^2} = \sqrt{64+225}$

$= \sqrt{289} = 17$

(11)



$\sin\theta = \frac{1}{3}$ $\csc\theta = 3$

$\cos\theta = \frac{2\sqrt{2}}{3}$ $\sec\theta = \frac{3}{2\sqrt{2}}$ (or $\frac{3\sqrt{2}}{4}$)

$\tan\theta = \frac{1}{2\sqrt{2}}$ $\left(= \frac{\sqrt{2}}{4}\right)$ $\cot\theta = 2\sqrt{2}$

Double-size!

$\sqrt{3^2-1^2} = \sqrt{8} = 2\sqrt{2}$

$2\,|\,8$
$2\,|\,4$
2

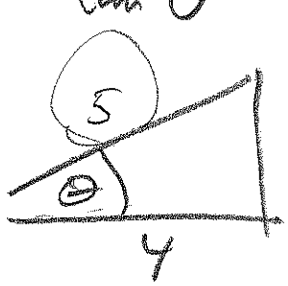
$2^3 = 2^2 \cdot 2^1$

$\sqrt{2^3} = \sqrt{2^2 \cdot 2^1} = \sqrt{2^2}\,\sqrt{2} = 2\sqrt{2}$

122 §1.3 #s 13–37, 41–51, 57–65, 67?, 73, 75

#s 13–20 Sketch a right triangle corresponding to the trig value for the **acute** angle.

(13) $\tan\theta = \frac{3}{4}$

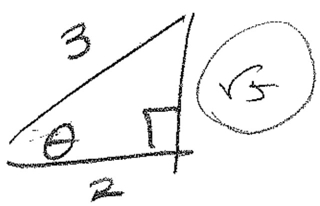


$\sin\theta = \frac{3}{5}$ $\quad \csc\theta = \frac{5}{3}$

$\cos\theta = \frac{4}{5}$ $\quad \sec\theta = \frac{5}{4}$

$\cot\theta = \frac{4}{3}$

(15) $\sec\theta = \frac{3}{2}$



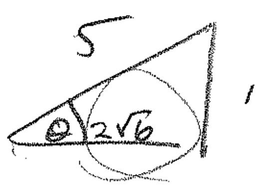
$\sqrt{3^2 - 2^2} = \sqrt{5}$

$\sin\theta = \frac{\sqrt{5}}{3}$ $\quad \csc\theta = \frac{3}{\sqrt{5}}$

$\cos\theta = \frac{2}{3}$

$\tan\theta = \frac{\sqrt{5}}{2}$ $\quad \cot\theta = \frac{2}{\sqrt{5}}$ OR $\frac{2\sqrt{5}}{5}$

(17) $\sin\theta = \frac{1}{5}$



$\sqrt{5^2 - 1^2} = \sqrt{24} = 2\sqrt{6}$

$\csc\theta = \frac{5}{1}$

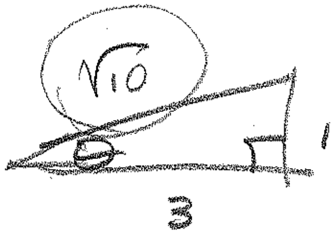
$\cos\theta = \frac{2\sqrt{6}}{5}$ $\quad \sec\theta = \frac{5}{2\sqrt{6}}$ OR $\frac{5\sqrt{6}}{12}$

$\tan\theta = \frac{1}{2\sqrt{6}}$ OR $\frac{\sqrt{6}}{2 \cdot 6} = \frac{\sqrt{6}}{12}$

$\cot\theta = 2\sqrt{6}$

122 §1.3 #s 19-37, 41-51, 57-65, 67?, 73, 75

(19) $\cot\theta = 3$



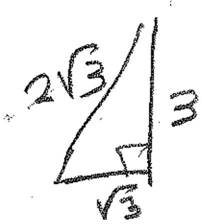
$\sin\theta = \frac{1}{\sqrt{10}}$ OR $\frac{\sqrt{10}}{10}$ $\qquad \csc\theta = \sqrt{10}$

$\cos\theta = \frac{3}{\sqrt{10}}$ OR $\frac{3\sqrt{10}}{10}$ $\qquad \sec\theta = \frac{\sqrt{10}}{3}$

$\tan\theta = \frac{1}{3}$ $\qquad \cot\theta$ ✓

#S 21-30 Construct an appropriate triangle to deduce the rest of the table (θ is acute)

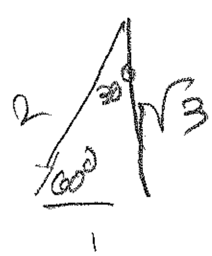
	Func	θ (deg)	θ (rad)	Func val
(21)	sin	30^0	$\frac{\pi}{6}$	$\frac{1}{2}$
(23)	sec	45^0	$\frac{\pi}{4}$	$\sqrt{2}$
(25)	cot	60^0	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{3}$



$\frac{2\sqrt{3}}{2\sqrt{3}} = 1$

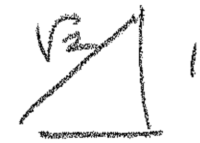
$\frac{3}{2\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

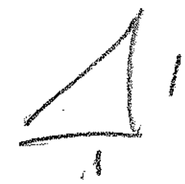
$\frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$ Oh! 1-2-$\sqrt{3}$!



122 §1.3 #s 27-37, 41-51, 57-65, 67?, 73, 75

	f	θ°	θ rad	$f(\theta)$
27	csc	45°	$\frac{\pi}{4}$	$\sqrt{2}$
29	cot	45°	$\frac{\pi}{4}$	1





#s 31-40 Evaluate to 4 decimal-place accuracy.

31 (a) $\sin 10^\circ \approx \boxed{.1736}$ (b) $\cos 80^\circ \approx \boxed{.1736}$ (cofunc.)

33 (a) $\sin 16.35^\circ \approx \boxed{.2815}$ (b) $\csc 16.35^\circ \approx \boxed{3.5523}$

35 (a) $\cos(4^\circ 50' 15'')$

$= \cos\left(4^\circ + \left(\frac{50}{60}\right)^\circ + \left(\frac{15}{3600}\right)^\circ\right)$

$= \cos(4.8375^\circ) \approx \boxed{.9964}$

(b) $\sec(4^\circ 50' 15'')$

$= \sec(4.8375^\circ)$

$\approx \boxed{1.0036}$

37 (a) $\cot(11^\circ 15')$

$= \cot\left(11^\circ + \left(\frac{15}{60}\right)^\circ\right)$

$= \cot(11.25^\circ)$

$\approx \boxed{5.0273}$

(b) $\tan(11^\circ 15')$

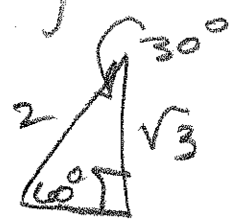
$= \tan(11.25^\circ)$

$\approx \boxed{.1989}$

122 §1.3 #s 41-51, 57-65, 67?, 73, 75

#s 41-46 Use given func values and trig identities to find the indicated trig values.

(41) $\sin 60^\circ = \frac{\sqrt{3}}{2}$ $\quad \cos 60^\circ = \frac{1}{2}$



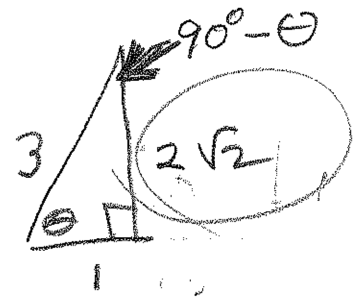
(a) $\sin 30^\circ = \frac{1}{2}$

(b) $\cos 30^\circ = \frac{\sqrt{3}}{2}$

(c) $\tan 60^\circ = \sqrt{3}$

(d) $\cot 60^\circ = \frac{1}{\sqrt{3}}$

I just draw the triangle. Cofunction identities aren't really big/important to me.



(43) $\cos \theta = \frac{1}{3}$.

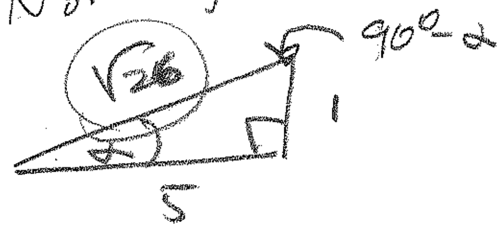
(a) $\sin \theta = \frac{2\sqrt{2}}{3}$

(b) $\tan \theta = 2\sqrt{2}$

(c) $\sec \theta = 3$

(d) $\csc(90^\circ - \theta) = 3$

} cofunc ident. Not huge deal 2 me.



(45) $\cot \alpha = 5$

(a) $\tan \alpha = \frac{1}{5}$

(b) $\csc \alpha = \sqrt{26}$

(c) $\cot(90^\circ - \alpha) = \frac{1}{5}$

(d) $\cos \alpha = \frac{5}{\sqrt{26}}$ OR $\frac{5\sqrt{26}}{26}$

122 § 1.3 #s 47–51, 57–65, 67?, 73, 75

#s 47–56 Use trig id's to "prove" the given identity.

(47) $\tan\theta \cot\theta = \tan\theta \cdot \frac{1}{\tan\theta} = 1$

(49) $\tan\alpha \cos\alpha = \frac{\sin\alpha}{\cos\alpha} \cos\alpha = \sin\alpha$

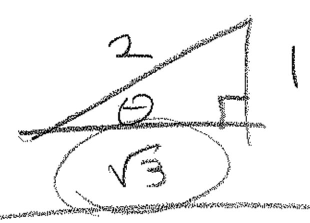
(51) $\cot\alpha \sin\alpha = \frac{\cos\alpha}{\sin\alpha} \cdot \sin\alpha = \cos\alpha$

#s 57–62 Find θ in degrees and radians. Assume θ is acute.

(57) (a) $\sin\alpha = \frac{1}{2}$

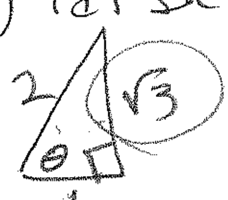
$\boxed{\alpha = 30^\circ = \frac{\pi}{6}}$

(b) $\csc\theta = 2$



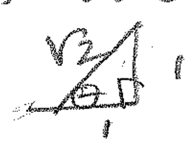
$\boxed{\theta = 30^\circ = \frac{\pi}{6}}$

(59) (a) $\sec\theta = 2$



$\boxed{\alpha = 60^\circ = \frac{\pi}{3}}$

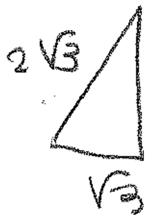
(b) $\cot\theta = 1$



$\boxed{\theta = 45^\circ = \frac{\pi}{4}}$

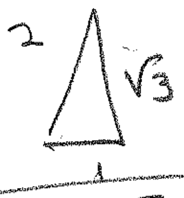
122 §1.3 #s 61–65, 67?, 73, 78

61 (a) $\csc\theta = \frac{2\sqrt{3}}{3}$



$3 \div \sqrt{3} \longrightarrow$

$\rightarrow$



$\boxed{\theta = 60^\circ = \frac{\pi}{3}}$

(b) $\sin\theta = \frac{\sqrt{2}}{2}$

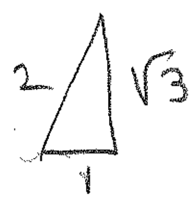
$\sqrt{1^2 - \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{1 - \frac{2}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

oh. $\boxed{45^\circ = \frac{\pi}{4} = \theta}$

#s 63–66 Find exact values of given variables

63 Find x & y



$\frac{y}{18} = \sin 60^\circ$

$y = 18 \sin 60^\circ$

$y = \frac{18\sqrt{3}}{2} = \boxed{9\sqrt{3} = y}$

$x = 18 \cos 60^\circ$

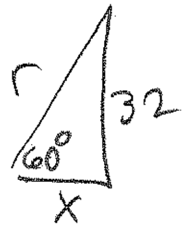
$x = 18 \cdot \frac{1}{2} = \boxed{9 = x}$

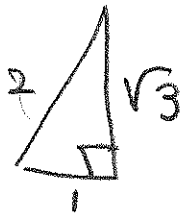
$\boxed{x = r\cos\theta, \; y = r\sin\theta}$

These are both EASILY derived from def'n of sine & cosine. You may or may not want to memorize.

122 §1.3 #'s 65, 67 ?, 73, 75

65 Find x & r





$\frac{32}{r} = \sin 60^\circ$

$\frac{32}{\sin 60^\circ} = r = 32\csc 60^\circ$

$= 32\left(\frac{2}{\sqrt{3}}\right) = \boxed{\frac{64}{\sqrt{3}} = r}$

OR $\frac{64\sqrt{3}}{3}$

$\frac{x}{32} = \cot 60^\circ \longrightarrow$

$\boxed{x = 32\left(\frac{1}{\sqrt{3}}\right) = \frac{32}{\sqrt{3}}}$ OR $\frac{32\sqrt{3}}{3}$

67 $x = 45$ m from building

$\theta = 82^\circ$ = angle of elevation to top of 86th floor.

Total height of building = 123 m above 86th floor. (Top of floor, we must assume... ?)

Let y = height from ground to top of 86th floor of building. Find y.

Then $y + 123$ = height of building

$\frac{y}{45} = \tan 82^\circ$

$y = 45 \tan 82^\circ$ m

≈ 320.1916375 m

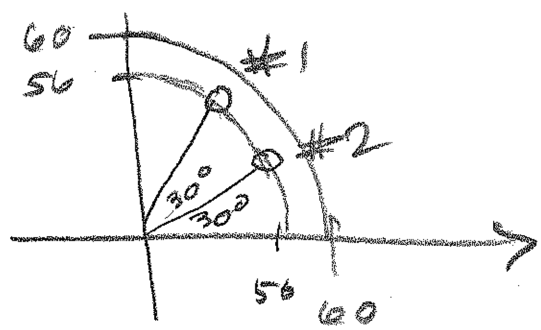
≈ 320 m $\approx y$

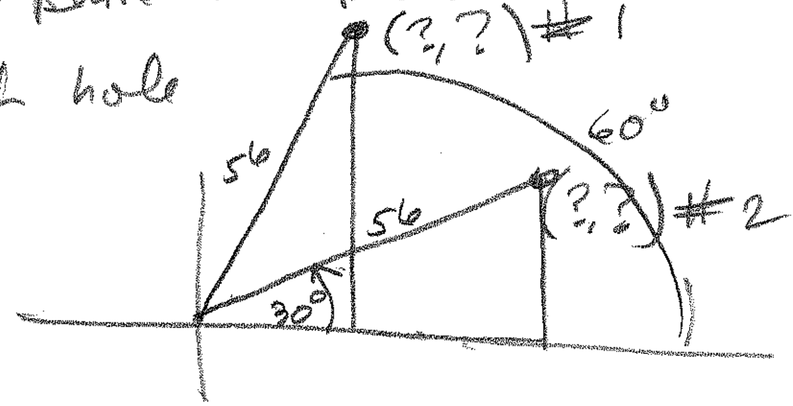
$\therefore$ Height of building is approximately $320 + 123\,\text{m} = \boxed{443\text{m}}$

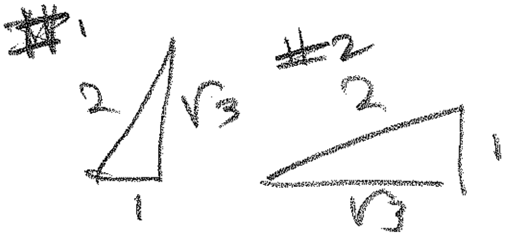
122 §1.3 #s 73, 75

(73) A steel plate is a $\frac{1}{4}$-circle w/ radius $r = 60$ cm. Two 2-cm holes are to be drilled in the plate (see pic.)

Find center of each hole







#1 hole:

$x_1 = r\cos\theta = 56\cos 60°$

$= 56 \cdot \frac{1}{2} = 28$ cm

$y_1 = r\sin\theta = 56\sin 60°$

$= 56 \cdot \frac{\sqrt{3}}{2}$ cm

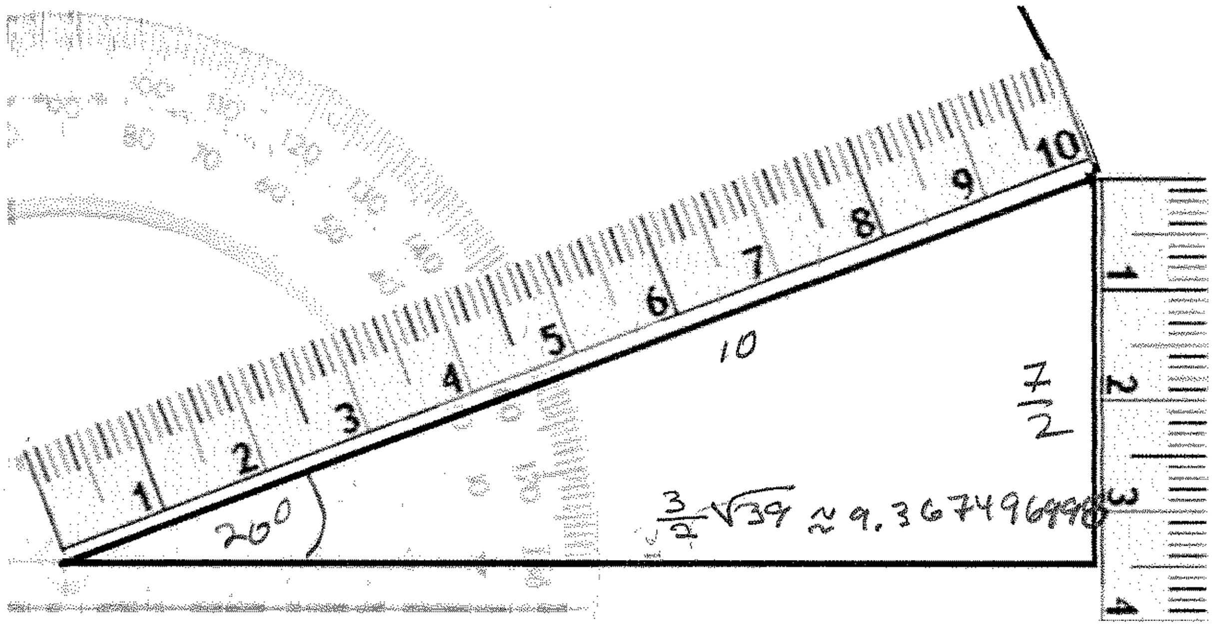
$= 28\sqrt{3}$ cm

$$\boxed{\begin{aligned}(x_1, y_1) &= (28, 28\sqrt{3})\\ (x_2, y_2) &= (28\sqrt{3}, 28)\end{aligned}}$$

by logic / co func. / brute force

(75)

122 §1.3 #75



$r = 10$, $y \approx 3.5 = \frac{35}{10} = \frac{7}{2}$

$\Longrightarrow x = \sqrt{r^2 - y^2} \approx \sqrt{10^2 - \left(\frac{7}{2}\right)^2} = \sqrt{100 - \frac{49}{4}}$

$3 \,\underline{|351}$
$3 \,\underline{|117}$
$3 \,\underline{|39}$
13

$= \sqrt{\frac{400-49}{4}} = \sqrt{\frac{351}{4}} = \frac{3\sqrt{39}}{2} \approx 9.367496998$

CHECK

$\sin 20^0 \approx \frac{\frac{7}{2}}{10} = \frac{7}{20} = .35 \longleftrightarrow .3420201433$

$\cos 20^0 \approx \frac{3\sqrt{39}}{20} \approx .9367496998 \longleftrightarrow .9396926208$

$\tan 20^0 \approx .3736323589 \longleftrightarrow .3639702343$

$\csc 20^0 \approx \frac{20}{7} \approx 2.857142857 \longleftrightarrow 2.9238044$

$\sec 20^0 \approx 1.067521025 \longleftrightarrow 1.064177772$

$\cot 20^0 \approx 2.676427714 \longleftrightarrow 2.747477419$