122 §1.1 #s 1-6, 7-57 odds, 60, 63, 69

(1) Two angles with the same initial and terminal sides are coterminal. ($270°$ & $-90°$)

(2) One radian is the measure of a central angle that intercepts an arc equal to the radius of the circle.

(3) Two POSITIVE angles whose sum is $\frac{\pi}{2}$ are complementary angles. ($\alpha, \beta > 0$, $\alpha + \beta = \frac{\pi}{2}$)

Two POSITIVE angles whose sum is $\pi$ are supplementary angles.

(4) The angle measure that is equivalent to a rotation of $\frac{1}{360}$ of a complete revolution about an angle's vertex is one degree.

(5) The (average) linear speed of a particle is the ratio of the arc length to the time elapsed ($\frac{\Delta s}{\Delta t}$), and the (average) angular speed is the ratio of the central angle to the elapsed time. ($\frac{\Delta \theta}{\Delta t}$).

(6) The area $A$ of a sector of a circle with radius, $r$, and central angle, $\theta$, is given by the formula $A = \frac{1}{2} r^2 \theta$ (if $\theta$ is in radians, only)

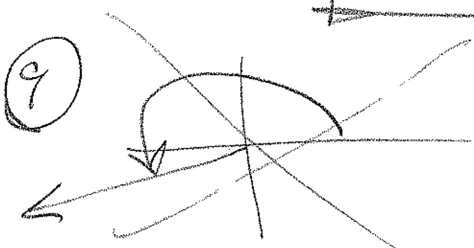
122 §1.1 #s 7-57 odds, 60, 63, 69

#s 7-10 Estimate the angle to the nearest $\frac{1}{2}$ radian. ( $\frac{57^\circ}{2} \approx 28.5^\circ$, so about $30^\circ$ is our tolerance for error.

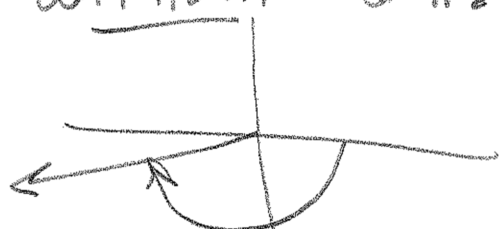
(7) I guess I was fuzzy on this problem. Last time I covered this book I left these guys out...

My instinct is to say $\frac{\pi}{4}$, but I see why the book says $\boxed{1 \text{ radian}}$ They're trying to get you to think in radians WITHOUT the $\pi$.

(9)



oops!



This looks close to $\boxed{-3 \text{ radians}}$, since

1 radian $\approx 57^\circ$, so

3 radians $\approx (3)(57) < (3)(60) = 180^\circ$

#s 11, 12 Determine the quadrant in which the (terminal side of the) angle lies.

(11) (a) $\frac{\pi}{4}$ $\boxed{QI}$ (b) $\frac{5\pi}{4} = \frac{4\pi}{4} + \frac{1\pi}{4} = \pi + \frac{\pi}{4}$

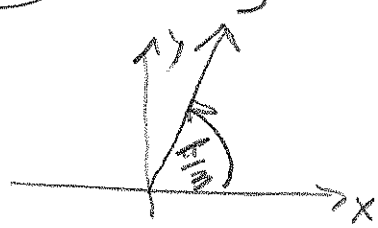
$\boxed{QIII}$

122 §1.1 #s 13–57 odds, 60, 63, 69

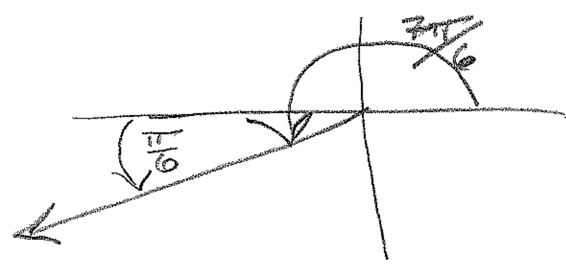
I do the header all the time, to keep track and give me that margin at the top that the teacher requires! Don't staple thru your work. Always leave room @ top left corner!

#s 13, 14 Sketch each angle in standard position.

13 (a) $\frac{\pi}{3}$




(b) $\frac{7\pi}{6} = \frac{6\pi}{6} + \frac{1\pi}{6} = \pi + \frac{\pi}{6}$




#s 15, 16 Find positive and negative angles coterminal with the given angle.

15 (a) $\frac{\pi}{6}$:

$$\frac{\pi}{6} + 2\pi = \frac{\pi + 12\pi}{6} = \boxed{\frac{13\pi}{6}}$$

$$\frac{\pi}{6} - 2\pi = \frac{\pi}{6} - \frac{2\pi}{1}\cdot\frac{6}{6} = \frac{\pi - 12\pi}{6} = \boxed{\frac{-11\pi}{6}}$$

(b) $\frac{7\pi}{6}$:

$$\frac{7\pi}{6} + 2\pi = \frac{7\pi + 12\pi}{6} = \boxed{\frac{19\pi}{6}}$$

$$\frac{7\pi}{6} - 2\pi = \frac{(7-12)\pi}{6} = \boxed{-\frac{5\pi}{6}}$$

122 §1.1 #s 17-57 odds, 60, 63, 69

#s 17-20. Find (if possible), supplementary (Sup) & complementary (comp) angles.

17 (a) $\frac{\pi}{3}$:

$\frac{\pi}{2} - \frac{\pi}{3} = \frac{3\pi - 2\pi}{6} = \boxed{\frac{\pi}{6} \text{ comp}}$

$\pi - \frac{\pi}{3} = \boxed{\frac{2\pi}{3} \text{ sup}}$

"S'up!"

(b) $\frac{\pi}{4}$:

$\frac{\pi}{2} - \frac{\pi}{4} = \boxed{\frac{\pi}{4} \text{ comp}}$

$\pi - \frac{\pi}{4} = \boxed{\frac{3\pi}{4} \text{ Sup}}$

19 (a) 1:

$\boxed{\frac{\pi}{2} - 1}$ comp

$\boxed{\pi - 1 \text{ Sup}}$

(b) 2:

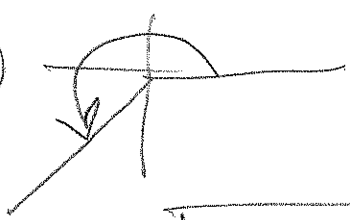
$\frac{\pi}{2} - 2 \approx -.4292036732.$

$\Longrightarrow 2 > \frac{\pi}{2} \Longrightarrow$ No Comp!

$\boxed{\pi - 2 \text{ Sup}}$

#s 21-24 Estimate the angle measure, in degrees. Your teacher is biased towards 30°, 45°, 60°!

21



$180^\circ + 45^\circ = \boxed{225^\circ}$

Answers may vary!

23

$\boxed{-60^\circ}$

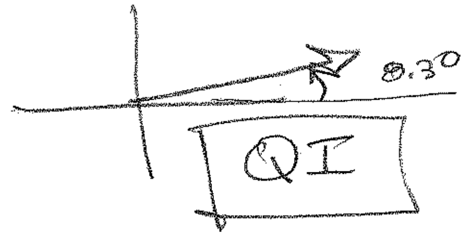
122 § 1.1 #s 25-57 odds, 60, 65, 69

25–26 Determine the quadrant in which the (terminal side of the) angle lies.

25 (a) $130^\circ$

Since $90^\circ < 130^\circ < 180^\circ$, it's in $\boxed{QII}$

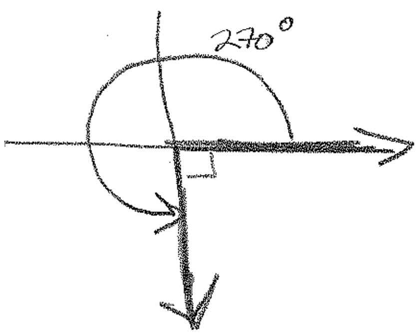

(b) $8.3^\circ$




$\boxed{QI}$

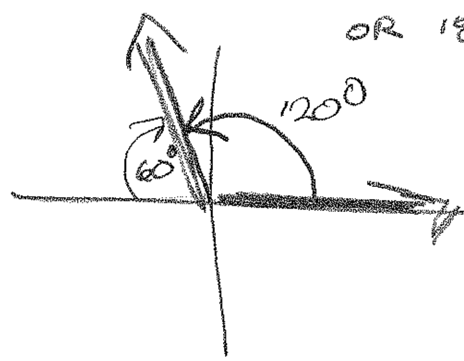
#s 27-8 Sketch the angle in standard position

27 (a) $270^\circ$




(b) $120^\circ = 90^\circ + 30^\circ$

OR $180^\circ - 60^\circ$




#s 29, 30 Determine a positive & negative coterminal angle.

29 (a) $45^\circ$

$45^\circ + 360^\circ = \boxed{405^\circ}$

$45^\circ - 360^\circ = \boxed{-315^\circ}$

(b) $-36^\circ$:

$-36^\circ + 360^\circ = \boxed{324^\circ}$

$-36^\circ - 360^\circ = \boxed{-396^\circ}$

122 §4.1 #s 31-57 odds, 60, 63, 69

#s 31-34 Find complementary and supplementary angles, if possible.

31 (a) $18^\circ$:

$90^\circ - 18^\circ = \boxed{72^\circ \text{ comp.}}$

$180^\circ - 18^\circ = \boxed{162^\circ \text{ sup}}$

(b) $85^\circ$:

$90^\circ - 85^\circ = \boxed{5^\circ \text{ comp}}$

$180^\circ - 85^\circ = \boxed{95^\circ \text{ sup}}$

33 (a) $150^\circ$

$\boxed{\text{No comp.}}$

$180^\circ - 150^\circ = \boxed{30^\circ \text{ Sup}}$

(b) $79^\circ$;

$90^\circ - 79^\circ = \boxed{11^\circ \text{ comp.}}$

$180^\circ - 79^\circ = \boxed{101^\circ \text{ sup.}}$

#s 35-6 Rewrite the angle in radians:

35 (a) $120^\circ = (120^\circ)\left(\frac{\pi \text{ radians}}{180^\circ}\right) = \frac{12\pi}{18} = \boxed{\frac{2\pi}{3}}$

(b) $-20^\circ = (-20^\circ)\left(\frac{\pi}{180^\circ}\right) = -\frac{2\pi}{18} = \boxed{-\frac{\pi}{9}}$

#s 37-8 Rewrite the angle measure in degrees

37 (a) $\frac{3\pi}{2} = \left(\frac{3\pi}{2}\right)\left(\frac{180^\circ}{\pi}\right) = (3)(90)^\circ = \boxed{270^\circ}$

(b) $\frac{7\pi}{6} = \left(\frac{7\pi}{6}\right)\left(\frac{180^\circ}{\pi}\right) = (7)(30)^\circ = \boxed{210^\circ}$

122 §1.1 #s 39–57 odds, 60, 63, 69

~~39~~ #s 39–42 Convert to radians, Round to "3 decimal places."

CONVENTION:

Radians is a unitless measure.

39) $45^\circ = \frac{\pi}{4} \approx \boxed{.785}$

41) $(0.54^\circ)\left(\frac{\pi}{180^\circ}\right) \approx \boxed{.009}$

#s 43–46 Convert from radians to degrees Round to 3 digits beyond the decimal

43) $\left(\frac{5\pi}{11}\right)\left(\frac{180^\circ}{\pi}\right) \approx \boxed{81.818^\circ}$

45) $-4.2\pi = (-4.2\pi)\left(\frac{180^\circ}{\pi}\right) = -756.000^\circ$

#s 47–8 Convert to DECIMAL Degree form w/o a calculator. Then check w/ calculator.

47) (a) $54^\circ 45' = 54^\circ + .75^\circ = 54.75^\circ$

$\frac{45}{60} = 3/4 = .75$

$54^\circ + (45')\left(\frac{1^\circ}{60'}\right) = 54 + .75^\circ = 54.75^\circ$

(b) $-128^\circ 30' = -128.5^\circ$

$-128^\circ - (30')\left(\frac{1^\circ}{60'}\right) =$

Adding a negative, here.

(30 minutes is a half a degree. works like hours = degrees, minutes, seconds.

122 §1.1 #s 49-57 odds, 60, 63, 69

#s 49-50 Convert to $D^\circ M' S''$ form.

49 (a) $240.6^\circ = 240^\circ + (.6^\circ)\left(\frac{60'}{1^\circ}\right)$

$= 240^\circ + 36'$, i.e., $\boxed{240^\circ 36'}$

(b) $-145.8^\circ = -145^\circ - (.8^\circ)\left(\frac{60'}{1^\circ}\right)$

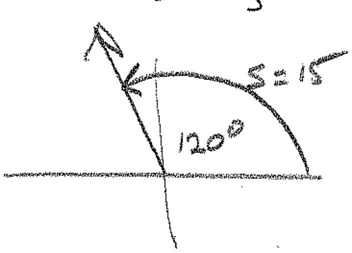
$= -145^\circ - 48'$ OR $\boxed{-145^\circ 48'}$

If there'd been a decimal in the minutes, do the same sort of thing with minutes to seconds $\left(\frac{60''}{1'}\right)$ that we did with degrees to minutes.

#s 51-2 Find length, s, of the arc on a circle of radius, r, intercepted by a central angle $\theta$. $\theta = \frac{s}{r} \longrightarrow s = r\theta$ from def'n of radian measure.

51 $r = 15$ inches, $\theta = 120^\circ = (120^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{2\pi}{3}$

$\Longrightarrow s = r\theta = (\cancel{15}^{5})\left(\frac{2\pi}{\cancel{3}_{1}}\right) = \boxed{10\pi \text{ in}}$ EXACT!




$\approx \boxed{31.41592654 \text{ in}}$ Approx

122 § 11.1 #s 53-57 odds, 60, 63, 69

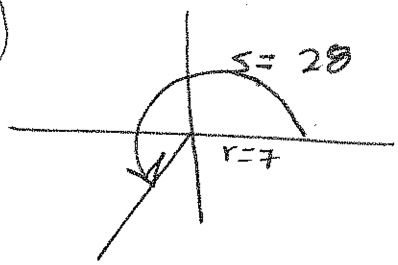
#s 53-4 Find the radian measure of the central angle, given radius, r, and arc length, s.

(53) $r = 80$ km, $s = 150$ km $\Longrightarrow$

$\theta = \frac{150}{80} = \boxed{\frac{15}{8} \text{ (rad)}}$

#s 55-6 Use s & r to find $\theta$.

(55)




$\theta = \frac{s}{r} = \frac{28}{7} = \boxed{4 \text{ (rad)}}$

(57) Find the area of the sector of a circle of radius, r, and central angle, $\theta$.

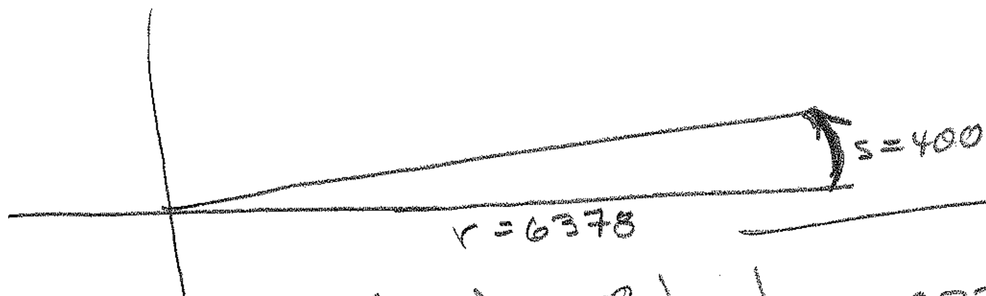
$r = 12$ mm, $\theta = \frac{\pi}{4} \Longrightarrow A = \frac{1}{2} r^2 \theta = \frac{1}{2}(12)^2\left(\frac{\pi}{4}\right)$

$= \frac{144\pi}{8} = \frac{36\pi}{2} = \boxed{18\pi \text{ mm}^2}$

$\approx 56.5486776$ mm

122 §1.1 #s 60, 63, 69

(60) Assume Earth's radius is 6378 Km. What's the difference in latitude between 2 cities, 400 Km apart, if one is due North of the other?




$$\theta = \frac{s}{r} = \frac{400}{6378} = \left(\frac{400}{6378}\right)\left(\frac{180^\circ}{\pi}\right) \approx \boxed{3.59333832^\circ}$$

$\approx .0627155848$ (rad)

$\frac{400}{6738} = \boxed{\frac{200}{3369}}$ (rad) is lowest terms & Exact!

Rounding either answer to 3 places is arbitrary and (bad) habit-forming. If you resort to calculator, take it on out as far as you can!

122 §1.1 ~~#~~s 63, 69

63 Cirular blade rotates @ 500 rpm $\left(\frac{\text{revs}}{\text{min}}\right)$

(a) Find angular speed $\frac{\Delta\theta}{\Delta t} =$

$$\left(500\ \frac{\text{revolutions}}{1\ \text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{revolution}}\right) = \boxed{1000\pi\ \frac{\text{rad}}{\text{min}}}$$

$$\approx \boxed{3141.592654\ \frac{\text{rad}}{\text{min}}}$$

(b) If blade diameter, D, is $7\frac{1}{4} = 7 + \frac{1}{4} = 7.25$

Find linear speed in $\frac{\text{ft}}{\text{min}}$.

$$r = \frac{D}{2} = \frac{7.25}{2} = 3.625\ \text{in} \longrightarrow$$

$$\frac{\Delta s}{\Delta t} = \frac{r\Delta\theta}{\Delta t} = (3.625\ \text{in})\left(1000\pi\ \frac{\text{rad}}{\text{min}}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$

$$= \left(\frac{29}{8}\right)(1000\pi)\left(\frac{1}{12}\right)$$

$s = r\theta$

$\Delta s = r\Delta\theta$

$$= \frac{29000\pi}{96} = \boxed{\frac{3625}{12}\pi\ \frac{\text{ft}}{\text{min}}}$$

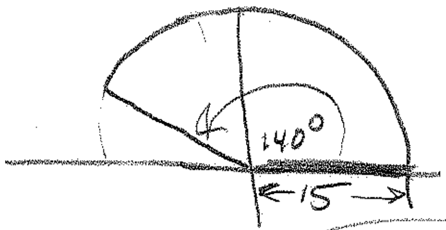
Exact, in lowest terms.

$$3.625 = 3 + \frac{625}{1000} = 3 + \frac{5}{8} = \frac{29}{8}$$

$$\approx \boxed{949.0227808\ \frac{\text{ft}}{\text{min}}}$$

122 § 1.1 #69

69 A sprinkler ~~has a~~ waters a radius of 15 m through an angle of 140°. Draw a diagram. Find the area it waters.




$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2}(15)^2 (140^\circ)\left(\frac{\pi}{180^\circ}\right)$$

$$= \frac{(15)^2(70)\pi}{180} \text{ ft}^2$$

$$(15 \text{ ft})^2 = 15^2 \text{ ft}^2$$

$$= \frac{(15)^2(7)\pi}{18} = \frac{(15)(15)(7)\pi}{18}$$

$$= \frac{(5)(15)(7)\pi}{6} = \frac{(5)(5)(7)\pi}{2} = \boxed{\frac{175\pi}{2} \text{ ft}^2}$$

$$\approx \boxed{274.8893572 \text{ ft}^2}$$