Test 3 – Fall, 2016

I think you know the drill on margins and legibility. I can't give points for what I can't read. Take a minute, at the end, to make sure your work is organized and submitted in proper order.

Only use one side of each blank sheet of paper you use. Don't write on the test EXCEPT FOR YOUR NAME.

1. Consider the triangle in the figure. Assume lengths are in centimeters.
   a. (5 pts) Use the Law of Cosines to find the length of side a.
   b. (5 pts) Use the Law of Sines to find angles B and C.

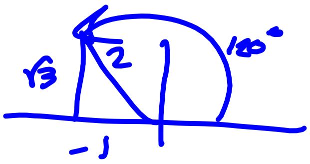
I'll specify ~~as~~ precision.

$a^2 = b^2 + c^2 - 2bc\cos A$

$= 8^2 + 7^2 - 2(56)\cos(120°)$

$= 64+49-112\left(-\frac{1}{2}\right)$

$= 113 + 56 = 169$



$\Rightarrow a = \sqrt{169} = \boxed{13 = a}$

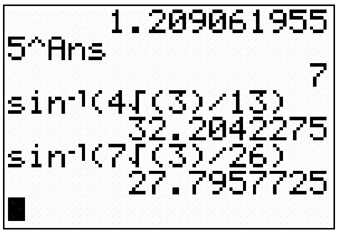
Math Olivia

$\frac{\sin B}{b} = \frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin B}{8} = \frac{\frac{\sqrt{3}}{2}}{13} = \frac{8\sqrt{3}}{26} = \frac{4\sqrt{3}}{13}$

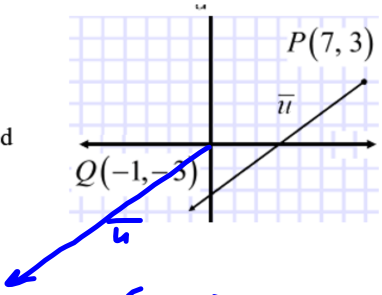
$32.204° \approx B = \sin^{-1}\left(\frac{4\sqrt{3}}{13}\right)$

$27.796° \approx C = \sin^{-1}\left(\frac{7\sqrt{3}}{26}\right)$



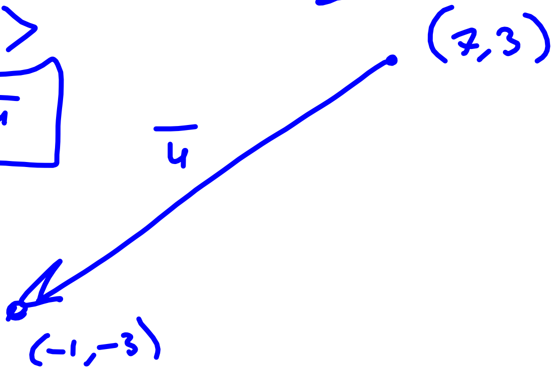
```
     1.209061955
5^Ans
               7
sin⁻¹(4√(3)/13)
      32.2042275
sin⁻¹(7√(3)/26)
      27.7957725
```

2. Consider the directed line segment $P\dot{Q}$ in the figure on the right. I want you to provide some basic facts about the vector $\bar{u}$:
   a. (5 pts) Express the vector $\bar{u} = \overrightarrow{PQ}$ in component form.
   b. (5 pts) Compute the magnitude of $\bar{u}$. Leave your answer in simplified radical form.
   c. (5 pts) Find the direction angle of $\bar{u}$. Use degrees, rounded to 4 places.



ⓐ $\bar{u} = \langle -1-7, -3-3 \rangle$

$= \boxed{\langle -8, -6 \rangle = \bar{u}}$

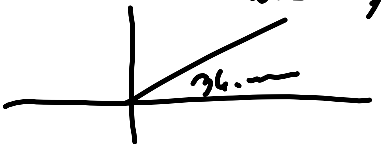


ⓑ $\|\bar{u}\| = \sqrt{8^2+6^2}$

$= \sqrt{64+36} = \sqrt{100} = \boxed{10 = \|\bar{u}\|}$

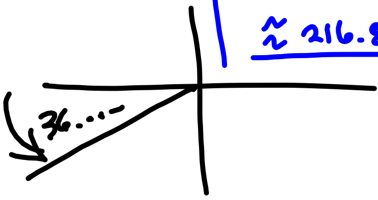
ⓒ $\arctan\left(\frac{6}{8}\right) \approx 36.86989765^\circ$

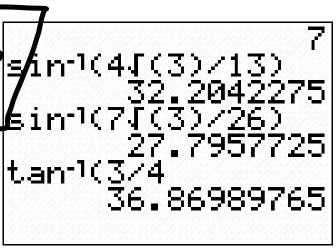
arctangent



$\Rightarrow \boxed{\theta \approx 216.86989765^\circ}$

$\boxed{\approx 216.8699^\circ}$





```
                    7
sin⁻¹(4√(3)/13)
          32.2042275
sin⁻¹(7√(3)/26)
          27.7957725
tan⁻¹(3/4
         36.86989765
```

3. Let $\bar{u} = \langle 4, 5\rangle$.

   a. (5 pts) Express $\bar{u}$ as a linear combination of the canonical (standard) unit vectors $\bar{i}$ and $\bar{j}$.

   b. (5 pts) What's another word for the sum of 2 vectors?

ⓐ $\bar{u} = 4\bar{i} + 5\bar{j}$

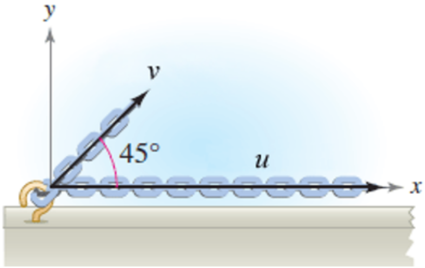
ⓑ Resultant.

4. Forces with magnitudes $\|\bar{u}\| = 90$ **N** and $\|\bar{v}\| = 25\sqrt{2}$ **N** are acting on a hook, as shown in the figure.

   a. (5 pts) Express $\bar{u}$ and $\bar{v}$ in component form.

   b. (5 pts) Express the resultant force, in component form.

   c. (5 pts) Find the direction angle of the resultant force, in degrees, rounded to 4 decimal places.



ⓐ $\bar{u} = 90\langle 1, 0\rangle = \langle 90, 0\rangle$

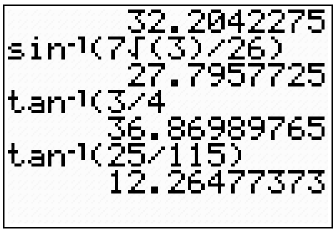
$\bar{v} = 25\sqrt{2}\langle \cos 45°, \sin 45°\rangle = \langle 25, 25\rangle$

Length · unit vector in proper direction.

ⓑ $\rightarrow \boxed{\bar{u} + \bar{v} = \langle 115, 25\rangle}$

ⓒ $\arctan\left(\frac{25}{115}\right) \approx 12.26477373°$

$\approx \boxed{12.2648°}$



```
                32.2042275
sin⁻¹(7√(3)/26)
                27.7957725
tan⁻¹(3/4
               36.86989765
tan⁻¹(25/115)
               12.26477373
```

5. Let $f(x)=3x^3-8x^2+10x-4$.
   a. (5 pts) Use synthetic division to find $f(2)$.
   b. (5 pts) Use synthetic division to show that $x=1+i$ is a solution of the equation $f(x)=0$.
   c. (5 pts) Find the linear factorization of f that is promised to us in the Fundamental Theorem of Algebra.

5a

$$\begin{array}{r|rrrr} 2 & 3 & -8 & 10 & -4 \\ & & 6 & -4 & 12 \\ \hline & 3 & -2 & 6 & \boxed{8 = P(2)} \end{array}$$

Dividing by x-2.

$(a+bi)(a-bi)$

$= a^2+b^2$

$a^2-(b^2i^2)$

$x-(1+i)$

b

$$\begin{array}{r|rrrr} 1+i & 3 & -8 & 10 & -4 \\ & & 3+3i & -8-2i & 4 \\ \hline 1-i & 3 & -5+3i & 2-2i & 0 \\ & & 3-3i & -2+2i & \\ \hline & 3 & -2 & 0 & \\ & x' & c & r & \end{array}$$

$\Rightarrow f(x) = (x-(1+i))(x-(1-i))(3x-2)$

$2(1+i)(1-i) = 2(1^2+1^2) = 4$

$(-5+3i)(1+i) = -5 -5i +3i \boxed{+3i^2} \to = -3$

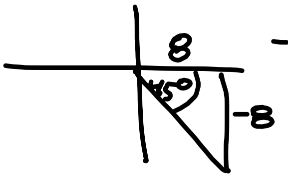
$= -8-2i$

6. Let $z=8-8i$
   a. (5 pts) Find $z+\bar{z}$ and $z\bar{z}$, where $\bar{z}$ is the complex conjugate of z.
   b. (5 pts) Express z in trigonometric form.

a $z+\bar{z} = 8-8i+(8+8i) = 16 \quad (= 2\,\mathrm{Re}(z))$

$z\bar{z} = 8^2+8^2 = \underline{128} = ||\bar{z}||^2$

b



$-\frac{\pi}{4}$

$\bar{z} = \sqrt{128}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

$= 8\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

2 | 128
2 | 64
2 | 32
2 | 16
2 | 8
2 | 4
2

7. Let $z=16\left(\cos\left(\frac{5\pi}{3}\right)+i\sin\left(\frac{5\pi}{3}\right)\right)$. $= 16\left(\frac{1}{2}+i\left(-\frac{\sqrt{3}}{2}\right)\right)$

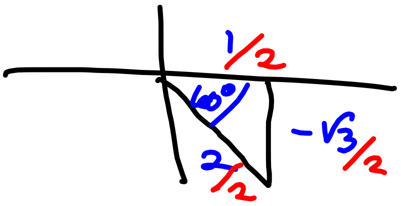
a. (5 pts) Express z in standard form.

b. (5 pts) Find the principal 4th root of z, i.e., find $\sqrt[4]{z}$. Leave z in trigonometric form for this.

c. (5 pts) Now, find *all* the 4th roots of z, in trigonometric form. $=$

d. (5 pts) Find the trigonometric form of z^2.

e. (5 pts) Finally, let $w=3\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)$, and find the trigonometric form of the product $z \cdot w$.



$\frac{5\pi}{3}\cdot\frac{\cancel{180}^{60}}{\cancel{\pi}} = 300°$

z ⓐ $z = 16\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) = \boxed{8-8\sqrt{3}i} = \bar{z}$

ⓑ $\sqrt[4]{z} = 16^{\frac{1}{4}}\left(\cos\left(\frac{\frac{5\pi}{3}}{4}\right)+i\sin\left(\frac{5\pi}{12}\right)\right)$

$= \boxed{2\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)=\sqrt[4]{z}}$

ⓒ $\frac{2\pi}{4} = \frac{6\pi}{12}$ ADD $\frac{6\pi}{12}$ to argument 'til done.

$2\left(\cos\left(\frac{11\pi}{12}\right)+i\sin\left(\frac{11\pi}{12}\right)\right)$

$2\left(\cos\left(\frac{17\pi}{12}\right)+i\sin\left(\frac{17\pi}{12}\right)\right)$

$2\left(\cos\left(\frac{23\pi}{12}\right)+i\sin\left(\frac{23\pi}{12}\right)\right)$

$2\left(\cos\left(\frac{29\pi}{12}\right)+i\sin\left(\frac{29\pi}{12}\right)\right) \longleftrightarrow 2\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

is coterminal with $\sqrt[4]{z}$

ⓓ $z^2 = 16^2\left(\cos\left(\frac{10\pi}{3}\right) + i\sin\left(\frac{10\pi}{3}\right)\right)$

$= r^2\left(\cos(2\theta) + i\sin(2\theta)\right)$

Also $= 256\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$.

ⓔ $w = 3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$zw = 48\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right)$

$\frac{5\pi}{3} + \frac{\pi}{4}$

$= \frac{20\pi + 3\pi}{12}$

B1 (5 pts) Find the area of the triangle in the 1st problem.

Heron's. Arithmetic. Tedious. Simple.

B2 A gun with a muzzle velocity of 370 meters per second is fired, with an angle of 15^0 from the horizontal.

a. (5 pts) Find the horizontal and vertical components of the bullet, as it leaves the muzzle, accurate to 4 decimal places.

b. (5 pts) Use a half-angle formula to find the *exact* value for the answer to the previous.

c. (5 pts) Using $-9.8\ \frac{m}{s^2}$ for the acceleration due to gravity, and neglecting air friction, predict where and when the bullet will hit the ground, in the gun question.

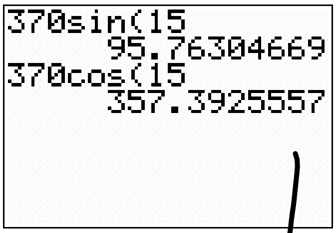
ⓐ $\boxed{370\langle\cos(15^\circ), \sin(15^\circ)\rangle} \approx \langle 357.3926, 95.3\ldots\rangle$

$370\cos(15^\circ)$ = Horizontal Component

$\approx \boxed{357.3926\ m/s}$

$370\sin(15^\circ)$ = Vertical Component.

$\approx \boxed{95.7630\ m/s}$



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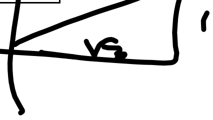
370sin(15
         95.76304669
370cos(15
         357.3925557
```

$\frac{u}{2} = 15^0 \longrightarrow$

$u = 30^\circ$ $\boxed{QI}$



$$\cos(15^\circ) = +\sqrt{\frac{1+\cos(30^\circ)}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

$$\text{HOR: } \frac{370\sqrt{2+\sqrt{3}}}{2} = \boxed{185\sqrt{2+\sqrt{3}} = \text{HOR}}$$

$$\sin 15^\circ = \sqrt{\frac{1-\cos(30^\circ)}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

$$\text{VER: } \frac{370\sqrt{2-\sqrt{3}}}{2} = \boxed{185\sqrt{2-\sqrt{3}} = \text{VER}}$$

$$s = -\tfrac{1}{2}gt^2 + v_0t + s_0$$

$$\approx -4.9t^2 + 95.7630t \overset{\text{SET}}{=} 0$$

$$-4.9t\left(t - \frac{95.7630}{4.9}\right) = 0$$

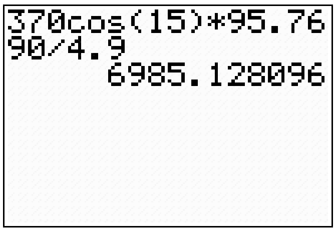
$$t = 0 \quad \text{OR} \quad t \approx \frac{95.7690}{4.9}$$

$$at^2 + bt = at\left(t + \frac{b}{a}\right)$$

How far down field?

$$x\left(\frac{95.76\ldots}{4.9}\right) \approx$$

$$\left(357.3926 \text{ m/s}\right)\left(\frac{95.76\ldots}{4.9}\right) \approx \boxed{6985.1 \text{ meters}}$$



```
370cos(15)*95.76
90/4.9
        6985.128096
```

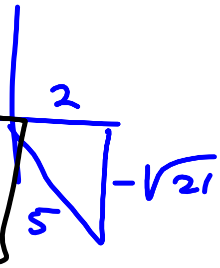
B3 (5 pts) Find $\sin(2u)$, $\cos(2u)$ and $\tan(2u)$, given that $\cos(u) = \frac{2}{5}$ and $\sin(u) < 0$.

Use the 1st two answers to *build* the 3rd. It's *silly* to go back to your cheat sheet and deal with the mess.

I'd like to put a half-angle in there, also.

$$\sin(2u) = 2\sin u \cos u$$

$$= 2\left(-\frac{\sqrt{21}}{5}\right)\left(\frac{2}{5}\right) = \boxed{\frac{-4\sqrt{21}}{25} = \sin(2u)}$$



$$\cos(2u) = 2\cos^2(u) - 1$$

$$= 2\left(\frac{4}{25}\right) - 1 = \frac{8-25}{25} = \boxed{-\frac{17}{25} = \cos(2u)}$$

$$\tan(2u) = -\frac{4\sqrt{21}}{25} \cdot \left(-\frac{25}{17}\right) = \frac{\text{sine}}{\text{cosine}}$$

$$= \boxed{\frac{4\sqrt{21}}{17} = \tan(2u)}$$

B4 (5 pts) Build a sine function that achieves its maximum height of $y = 62$ meters at time $x = 5$ seconds and its minimum height of $y = -8$ meters at $x = 13$ seconds.

$(5, 62)$ +

$$y = 35\cos\left(\frac{\pi}{8}(x-5)\right) + 27$$

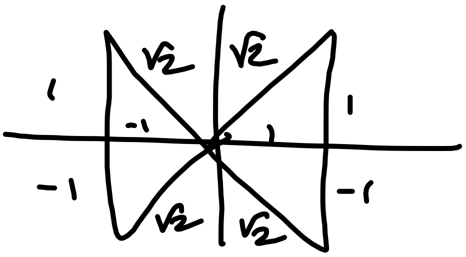
$(13, -8)$

B5 (5 pts) Find all solutions of the equation $2\sin^2(3x)-1=0$ in the interval $[0,2\pi)$.

$2\sin^2(3x)-1=0$

$\sin^2(3x)=\frac{1}{2}$

$\sin(3x)=\pm\frac{1}{\sqrt{2}}$



$3x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4},\frac{9\pi}{4},\frac{11\pi}{4},$

$\frac{13\pi}{4},\frac{15\pi}{4},\frac{17\pi}{4},\frac{19\pi}{4},\frac{21\pi}{4},\frac{23\pi}{4},$ ~~$\frac{25\pi}{4}>6\pi$~~

$x\in[0,2\pi)$

$3x\in[0,6\pi)$

$x\in\left\{\frac{\pi}{12},\frac{\pi}{4},\frac{5\pi}{12},\frac{7\pi}{12},\frac{3\pi}{4},\frac{11\pi}{12},\frac{13\pi}{12},\frac{5\pi}{3},\frac{17\pi}{12},\frac{19\pi}{12},\frac{23\pi}{12}\right\}$

B6 (5 pts) Sketch the graph of $4\sin\left(\frac{2\pi}{7}x-\frac{26\pi}{7}\right)-11$.

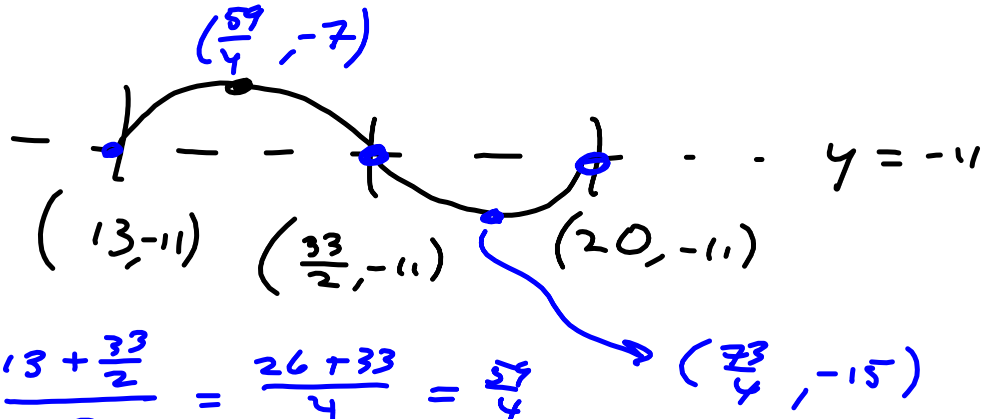
$\frac{26\pi}{7}\cdot\frac{7}{2\pi}=13$

$\pi\cdot\frac{2}{7}x=2\pi$

$x=\frac{2\pi\cdot 7}{2\pi}=7$

Period is 7

$=4\sin\left(\frac{2\pi}{7}(x-13)\right)-4$



$\frac{13+\frac{33}{2}}{2}=\frac{26+33}{4}=\frac{59}{4}$

$\frac{\frac{33}{2}+20}{2}=\frac{33+40}{4}=\frac{73}{4}$

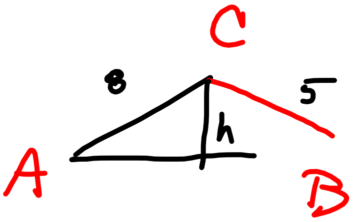
B7 The triangle described has 2 possible solutions:

Angle $A = 30^0$, side $b = 8$ and side $a = 5$.

a. (5 pts) Prove there are 2 possible triangles from this ambiguous information.

b. (5 pts) Find both triangles.

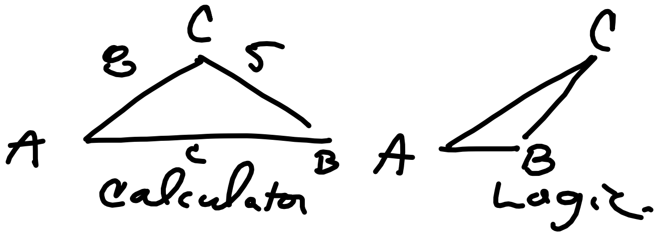
c. (5 pts) Use your work to find the area of both triangles.



$$\boxed{h < 5 < 8 \Longrightarrow \text{2 solns}}$$

$$\frac{h}{8} = \sin 30^\circ = \frac{1}{2} \longrightarrow h = 8\cdot\frac{1}{2} = 4$$

ⓑ



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\sin B = \frac{b \sin A}{a} = \frac{8\left(\frac{1}{2}\right)}{5} = \frac{4}{5}$$

$$B = \sin^{-1}\left(\frac{4}{5}\right) \approx 53.1301^\circ$$

$$C = 180 - 30^\circ - B \approx 150^\circ - 53.1301$$

$$\approx \boxed{96.8699^\circ \approx C}$$