§4.2 #80

Conjugate pairs theorem.

$f(x) = (x-3+i\sqrt{2})(x-3-i\sqrt{2})(x+2)$

$f(-2)=0$ $(-2,0) \Rightarrow -2$ is a root.

Given $3+i\sqrt{2}$ is a root $\Rightarrow$
$3-i\sqrt{2}$ " " "

$(x-3+i\sqrt{2})(x-3-i\sqrt{2})$ ~~FOIL~~ Distributive Law.

$= x^2-3x-i\sqrt{2}\,x-3x+9+3i\sqrt{2}+i\sqrt{2}\,x-3i\sqrt{2}-i^2\sqrt{2}\sqrt{2}$

$= x^2-6x+9+2 = x^2-6x+11$

$(x+2)(x^2-6x+11) = x^3-6x^2+11x+2x^2-12x+22$

$= x^3-4x^2-x+22$

a. (5 pts) Express z in standard form. $z = 1\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)$

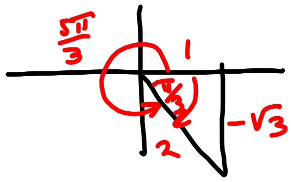
b. (5 pts) Find the principal 5th root of z, i.e., find $\sqrt[5]{z}$, in trigonometric form.

c. (5 pts) Now, find *all* the 5th roots of z, in trigonometric form.

d. (5 pts) Find the trigonometric form of z^2

e. (5 pts) Finally, let $w = 5\left(\cos\left(\frac{4\pi}{7}\right) + i\sin\left(\frac{4\pi}{7}\right)\right)$, and find the trigonometric form of the product $z \cdot w$.

7 a) $\frac{5\pi}{3}$



$\cos\frac{5\pi}{3} = \frac{1}{2}$, $\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$,

so $z = 1\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right)$

$= \boxed{\frac{1}{2} - \frac{i\sqrt{3}}{2}}$

b) $\sqrt[5]{z} = r^{\frac{1}{5}}\left(\cos\left(\frac{5\pi}{3}\cdot\frac{1}{5}\right) + i\sin\left(\frac{5\pi}{3}\cdot\frac{1}{5}\right)\right)$ 5th root of magnitude "r"

1 $= \cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)$ and $\frac{1}{5}$ the argument.

c) to find the rest, add increments of $\frac{2\pi}{5}$:

$\left(\frac{2\pi}{5}\cdot\frac{3}{3} = \frac{6\pi}{15}\right.$ is the increment

2 $\frac{\pi}{3} + \frac{2\pi}{5} = \frac{5\pi + 6\pi}{15} = \frac{11\pi}{15}$

3 $\frac{11\pi}{15} + \frac{2\pi}{5} = \frac{11\pi + 6\pi}{15} = \frac{17\pi}{15}$ → See pattern?

4 $\frac{17\pi}{15} + \frac{6\pi}{15} = \frac{23\pi}{15}$

5 $\frac{23\pi}{15} + \frac{6\pi}{15} = \frac{29\pi}{15}$

NOTE: One more & we're coterminal with $\frac{\pi}{3}$:
(Full circle)

$\frac{35\pi}{15} = \frac{30\pi}{15} + \frac{5\pi}{15} = 2\pi + \frac{\pi}{3}$. See?

So
$\cos\left(\frac{11\pi}{15}\right) + i\sin\left(\frac{11\pi}{15}\right)$
$\cos\left(\frac{17\pi}{15}\right) + i\sin\left(\frac{17\pi}{15}\right)$
$\cos\left(\frac{23\pi}{15}\right) + i\sin\left(\frac{23\pi}{15}\right)$
$\cos\left(\frac{29\pi}{15}\right) + i\sin\left(\frac{23\pi}{15}\right)$

I did about twice as much writing as necessary

Once you get that $\frac{6\pi}{15}$ for convenient adding, you can write this

$$z = 1\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)$$

d. (5 pts) Find the trigonometric form of z^3

e. (5 pts) Finally, let $w = 5\left(\cos\left(\frac{4\pi}{7}\right) + i\sin\left(\frac{4\pi}{7}\right)\right)$, and find the trigonometric form of the product $z \cdot w$.

(d) $z^3 = 1^3\left(\cos\left(3\cdot\frac{5\pi}{3}\right) + i\sin\left(3\cdot\frac{5\pi}{3}\right)\right)$

$= \boxed{\cos(5\pi) + i\sin(5\pi)}$

$= -1$!

(e) $zw = \left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)\left(5\cos\frac{4\pi}{7} + i\sin\frac{4\pi}{7}\right)$

$5(1) = 5$, $\frac{5\pi}{3} + \frac{4\pi}{7} = \frac{35\pi + 12\pi}{21} = \frac{47\pi}{21}$, so,

$\boxed{zw = 5\left(\cos\frac{47\pi}{21} + i\sin\frac{47\pi}{21}\right)}$