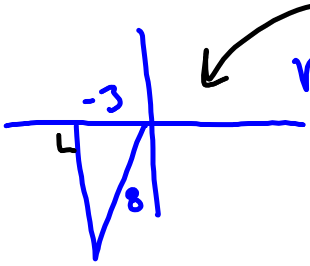
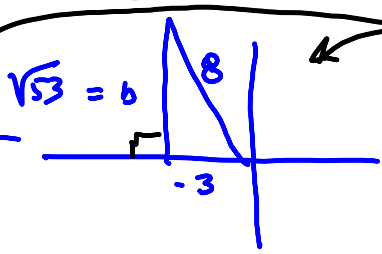
It's impossible to get all the work done in the time we have in class.

2. (10 pts) Find the values of all six trigonometric functions, given $\cos(u) = -\frac{3}{8}$ and $\sin(u) < 0$.

2 pics for $\cos u = -\frac{3}{8}$



$\sin u < 0$



$\sin u > 0$ is what we want

$(-3)^2 + b^2 = 8^2$

$8^2 - 3^2 = b^2$

$64 - 9 = b^2$

$53 = b^2$

$\pm\sqrt{53} = b$

$\sin u > 0$, we want $b = +\sqrt{53}$.

$\sin u = \frac{\sqrt{53}}{8}$ $\qquad$ $\csc u = \frac{8}{\sqrt{53}}$

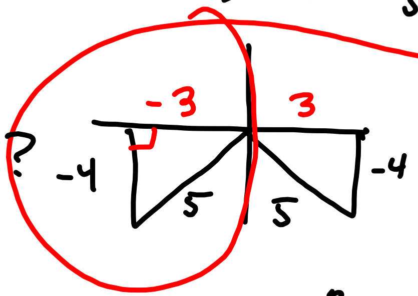
$\cos u = -\frac{3}{8}$ $\qquad$ $\sec u = -\frac{8}{3}$

$\tan u = -\frac{\sqrt{53}}{8}$ $\qquad$ $\cot u = -\frac{8}{\sqrt{53}}$

3. (10 pts) Find $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$, and $\tan\left(\frac{u}{2}\right)$, given that $\sin(u) = -\frac{4}{5}$ and $\cos(u) < 0$. Give final answers in radical form. You do not need to rationalize denominators I do not want any use of calculators on this problem.

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

$\sin u = -\frac{4}{5}$ & $\cos u < 0$



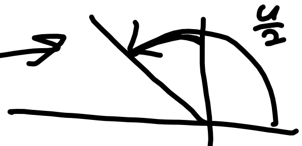
Is $\sin\frac{u}{2}$ pos or neg?

So $\sin\frac{u}{2} = \sqrt{\frac{1-\cos u}{2}}$

Look $180^\circ < u < 270^\circ$

$90^\circ < \frac{u}{2} < \frac{270^\circ}{2} = 135^\circ$

$$= \sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}}$$



$\frac{u}{2}$ is here, somewhere.

$$= \sqrt{\frac{\frac{8}{5}}{2}} = \sqrt{\frac{8}{5}\cdot\frac{1}{2}}$$

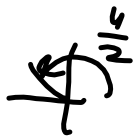
So $\sin e > 0$

$$= \sqrt{\frac{4}{5}} = \boxed{\frac{2}{\sqrt{5}}}$$

$\sin\frac{u}{2}$

is fine. $\frac{2\sqrt{5}}{5}$ is simplified radical form

$$\cos\frac{u}{2} = -\sqrt{\frac{1+\cos u}{2}}$$, because $\frac{u}{2} \in Q\,\text{II}$!



$$= -\sqrt{\frac{1+\left(-\frac{3}{5}\right)}{2}} = -\sqrt{\frac{\frac{2}{5}}{2}} = -\sqrt{\frac{1}{5}} = \boxed{-\frac{1}{\sqrt{5}}}$$

$\cos\frac{u}{2}$

$$\tan\frac{u}{2} = \frac{\frac{2}{\sqrt{5}}}{-\frac{1}{\sqrt{5}}} = -\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{1} = \boxed{-2 = \tan\frac{u}{2}}$$

Use $\sin\frac{u}{2}$ & $\cos\frac{u}{2}$ to make $\tan\frac{u}{2}$.

Consider the equation $3\sec^4(x) - 16\sec^2(x) + 16 = 0$.

a. (10 pts) Find all solutions x, in radians, to the equation, above, in the interval $[0, 2\pi)$. Give *exact* answers, here. (Hint: Factor by grouping.)

b. (10 pts) Find *all* real solutions x, in radians.

Let $u = \sec(x)$. Then $3u^4 - 16u^2 + 16 = 0$

Let $v = u^2$. Then $3v^2 - 16v + 16 = 0$

$a = 3,\ b = -16,\ c = 16$

$b^2 - 4ac = (-16)^2 - 4(3)(16) = 256 - 192$

$= 64$ (whew!)

$$\begin{array}{r} 12 \\ 256 \\ -64 \\ \hline 192 \end{array} \qquad \begin{array}{r} 16 \\ 12 \\ \hline 32 \\ 16\,6 \\ \hline 192 \end{array} \checkmark$$

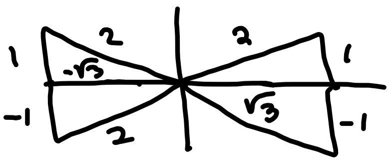
$v = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{16 \pm \sqrt{64}}{2(3)}$

$= \dfrac{16 \pm 8}{6}$ → $\dfrac{24}{6} = 4$, $\dfrac{8}{6} = \dfrac{4}{3}$

$v = \frac{4}{3} = u^2 \Rightarrow u = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}} = \sec x$!

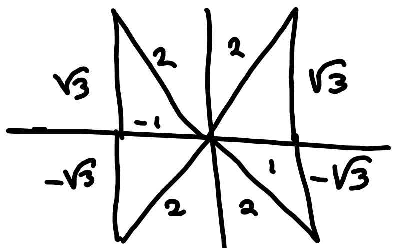
$v = 4 = u^2 \Rightarrow u = \pm 2 = \sec x$!

$\sec x = \pm \frac{2}{\sqrt{3}}$



These all have 30° reference angles.

$\sec x = \pm 2$



All w/ 60° reference angles

(b) So, 30°, 150°, 210°, 330° | 60°, 120°, 240°, 300°

(a) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ | $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

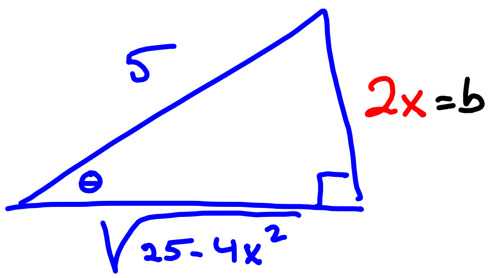
This prob. & previous will be at least 20pts EACH.

4. (10 pts) Re-write $\sin\left(\operatorname{arcsec}\left(\frac{5}{\sqrt{25-4x^2}}\right)\right)$ as an algebraic expression.

$= \sin\theta$.

Test 1 question, here

$\theta = \operatorname{arcsec}\left(\frac{5}{\sqrt{25-4x^2}}\right)$



$5^2 - \sqrt{25-4x^2}^{\,2} = b^2$

$25 - (25-4x^2) = b^2$

$4x^2 = b^2$

$\pm 2x = b$

Take the positive for these.

So, $\boxed{\sin\theta = \frac{2x}{5}}$!

5. (10 pts) Solve $\csc^2(x) - 4\csc(x) = -4$. Find all solutions in $[0, 2\pi)$. You may certainly use degrees to "see" things, better, but I expect (require) an exact answer, in radians, as your final answer.

This is an easier repeat of the more difficult (and un-numbered) equation, between #3 and #4.

Idiot instructor...

$\csc^2 x - 4\csc x = -4$

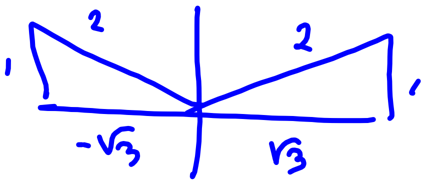
$u^2 - 4u + 4 = 0$

$(u-2)^2 = 0$

$u - 2 = \pm 0$ (Heh.)

$u = 2$

$\csc x = 2$



30°, $\frac{\pi}{6}$

150°, $\frac{5\pi}{6}$

$x \in \left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\}$

6. Evaluate $\sin\left(\frac{13\pi}{12}\right)$ in two ways: (Give *exact* answers, in simplified radical form.)

a. (10 pts) Use a Sum identity.

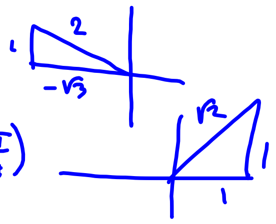
b. (10 pts) Use a Half-Angle identity.

(a) Find two angles that add up to $\frac{13\pi}{12}$ that we "know"

$$\frac{13\pi}{12} = \frac{12\pi + 1\pi}{12} = \frac{11\pi + 2\pi}{12} = \frac{10\pi + 3\pi}{12}$$

Ahh. $\frac{10\pi}{12} = \frac{5\pi}{6}$, $\frac{3\pi}{12} = \frac{\pi}{4}$

$= 150°$ $\quad = 45°$



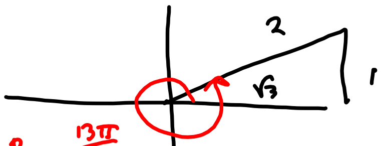
So $\sin\frac{13\pi}{12} = \sin\left(\frac{5\pi}{6} + \frac{\pi}{4}\right)$

$= \sin\frac{5\pi}{6}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{5\pi}{6}$

$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{\sqrt{3}}{2}\right)$

$= \boxed{\frac{1-\sqrt{3}}{2\sqrt{2}}}$

(b) $\sin\left(\frac{13\pi}{12}\right) = \sin\left(\frac{1}{2}\left(\frac{13\pi}{6}\right)\right)$



$390° = \frac{13\pi}{6}$

Reference Angle $30°$

$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}}$

$\frac{13\pi}{12} = 195°$. Sine is NEGATIVE.

$\sin\left(\frac{13\pi}{12}\right) = -\sqrt{\frac{1-\cos\left(\frac{13\pi}{6}\right)}{2}}$

$= -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$= -\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}}$

$= -\sqrt{\frac{2-\sqrt{3}}{4}} = -\frac{\sqrt{2-\sqrt{3}}}{2}$

So $-\frac{\sqrt{2-\sqrt{3}}}{2} = \frac{1-\sqrt{3}}{2\sqrt{2}}$!

Weird!