§4.1 $a+bi$

$\boxed{i^2 = -1 \quad \sqrt{-1} = i}$

Conjugate of $a+bi = z$

is $\bar{z} = a-bi$

$\sqrt{-4} = 2i$

$z\bar{z} = a^2+b^2 = (a+bi)(a-bi) = a^2-(bi)^2$

$= a^2+b^2$

$(3+2i)(5-7i) = 15 - 21i + 10i - 14i^2$

$= 15 - 11i + 14$

$= 29 - 11i$

write $\frac{1}{2+3i}$ in standard form $(a+bi)$

$\left(\frac{1}{2+3i}\right)\left(\frac{2-3i}{2-3i}\right) = \frac{2-3i}{2^2+3^2}$

$(a-b)(a+b) = a^2-b^2$

$(a-bi)(a+bi) = a^2-(bi)^2$

$= a^2+b^2$

$\boxed{\frac{2}{13} - \frac{3}{13}i} = \frac{2}{13} + \left(-\frac{3}{13}\right)i$

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§4.2

$x^2 + 2x + 5 = 0$

x^2+2x+1^2-1+5

$= (x+1)^2+4$

b^2-4ac = discriminant

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a=1, b=2, c=5$

$b^2-4ac = 2^2-4(1)(5)$

$= 4-20$

$= -16$

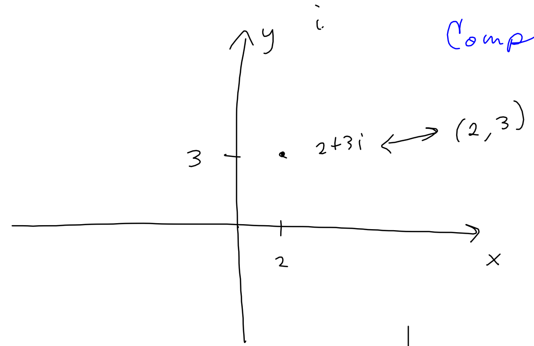
$\frac{-2 \pm \sqrt{2^2-4(1)(5)}}{2(1)}$

$\sqrt{-16} = 4i$

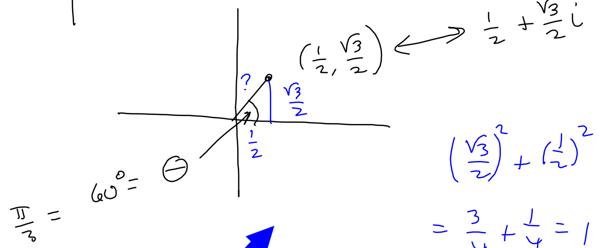
$x = \frac{-2 \pm 4i}{2}$

$\boxed{= -1 \pm 2i}$

Complex Plane.

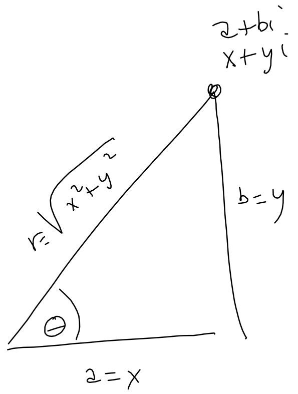


$? = r$



$$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4} = 1$$



$$\frac{1}{2} + \frac{\sqrt{3}}{2} i$$

$$= r\cos\theta + (r\sin\theta) i$$

$$= 1\cdot\cos 60^\circ + (1\cdot\sin 60^\circ) i$$

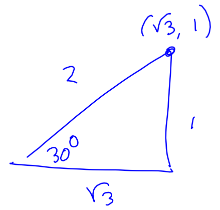
$$= 1\,(\cos 60^\circ + i\sin 60^\circ$$

$$\boxed{= r(\cos\theta + i\sin\theta)} = re^{i\theta}$$

Trigonometric form of

$$a+bi = \frac{1}{2} + \frac{\sqrt{3}}{2} i$$

$$(a+bi)^2$$



$$a+bi = \sqrt{3} + i$$

$$(\sqrt{3} + i)^2 = 3 + 2\sqrt{3}i + i^2$$

$$= 2 + 2\sqrt{3}i$$

$$= 2(1 + \sqrt{3}i)$$

$$\boxed{2(\cos 30^\circ + i\sin 30^\circ)}$$

$$2\cdot\frac{\sqrt{3}}{2} \qquad \sqrt{3} = 2\cdot\frac{\sqrt{3}}{2}$$

$$2^2(\cos 60^\circ + i\sin 60^\circ)$$

Square the 2. Double the angle.

$$4\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$$

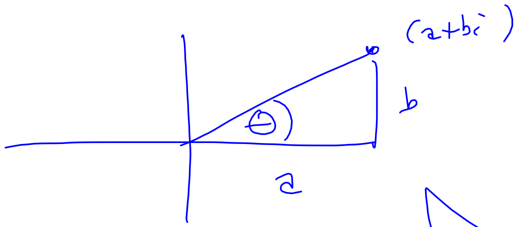
$$= 2 + 2\sqrt{3}i$$

$$\left(r(\cos\theta + i\sin\theta)\right)^5$$

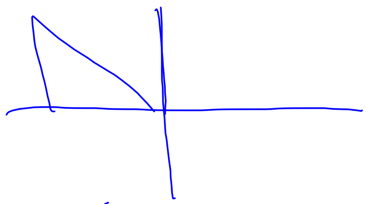
$$= r^5\left(\cos(5\theta) + i\sin(5\theta)\right)$$

$$\left(r_1(\cos\theta_1 + i\sin\theta_1)\right)\left(r_2(\cos\theta_2 + i\sin\theta_2)\right)$$

$$= r_1 r_2\left(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right)$$

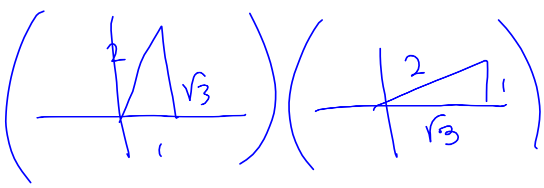


$$\theta = \arctan\left(\frac{b}{a}\right)$$



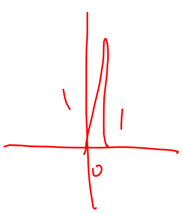
$$(1+\sqrt{3}i)(\sqrt{3}+i) = (\sqrt{3} + i + 3i - \sqrt{3})$$

$$= 4i$$



$$\left(2(\cos 60^\circ + i\sin 60^\circ)\right)\left(2(\cos 30^\circ + i\sin 30^\circ)\right)$$

$$4\left(\cos 90^\circ + i\sin 90^\circ\right)$$



$$= 4(0 + i) = 4i$$

$n^{\underline{th}}$ roots is Big

$$\left(r\left(\cos\theta + i\sin\theta\right)\right)^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\left(\frac{\theta}{n}\right) + i\sin\left(\frac{\theta}{n}\right)\right)$$

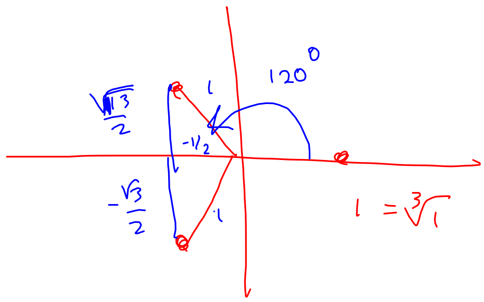
So cool!

$$\left(re^{i\theta}\right)^{\frac{1}{n}} = r^{\frac{1}{n}}e^{i\left(\frac{\theta}{n}\right)}$$

$n^{\underline{th}}$ roots : There are always n of them

$x^3 = 1$

$x = \sqrt[3]{1} = 1$ is only the PRINCIPLE cube root of 1. There are 2 others & they're evenly dispersed about the $\mathbb{C}$-plane.



3^{rd} roots of 1:

$1, \ -\frac{1}{2} + \frac{\sqrt{3}}{2}i, \ -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

$\cos 0 + i\sin 0, \ \cos 120^0 + i\sin 120^0,$

$\cos 240^0 + i\sin 240^0$!