Sum and Difference Formulas

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$\tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$

Double-Angle Formulas

$\sin 2u = 2 \sin u \cos u$

$\tan 2u = \dfrac{2 \tan u}{1 - \tan^2 u}$

$\cos 2u = \cos^2 u - \sin^2 u$
$= 2\cos^2 u - 1$
$= 1 - 2\sin^2 u$

Power-Reducing Formulas

$$\boxed{\sin^2 u = \frac{1-\cos 2u}{2} \qquad \cos^2 u = \frac{1+\cos 2u}{2}}$$

$\tan^2 u = \dfrac{1-\cos 2u}{1+\cos 2u}$

Half-Angle Formulas

$\sin \dfrac{u}{2} = \pm\sqrt{\dfrac{1-\cos u}{2}}$

$\cos \dfrac{u}{2} = \pm\sqrt{\dfrac{1+\cos u}{2}}$

$\tan \dfrac{u}{2} = \dfrac{1-\cos u}{\sin u} = \dfrac{\sin u}{1+\cos u}$

The signs of $\sin \frac{u}{2}$ and $\cos \frac{u}{2}$ depend on the quadrant in which $\frac{u}{2}$ lies.

Product-to-Sum Formulas

$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$

$\cos u \cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$

$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$

Sum-to-Product Formulas

$\sin u + \sin v = 2\sin\left(\dfrac{u+v}{2}\right)\cos\left(\dfrac{u-v}{2}\right)$

$\cos u + \cos v = 2\cos\left(\dfrac{u+v}{2}\right)\cos\left(\dfrac{u-v}{2}\right)$

$\sin(u-v)$
$= \sin(u+(-v))$
$= \sin u \cos(-v) + \sin(-v)\cos u$
$= \sin u \cos v - \sin v \cos u$

$s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$

$\sin^2 x = \dfrac{1-\cos(2x)}{2}$

$\Rightarrow \sin x = \pm\sqrt{\dfrac{1-\cos(2x)}{2}}$

$\sin\left(\dfrac{u}{2}\right) = \pm\sqrt{\dfrac{1-\cos(u)}{2}}$

15. –/2 points LarTrig9 2.5.061.

Find all solutions of the equation in the interval $[0, 2\pi)$. (Enter your answers as a comma-separated list.)

$\sin 6x + \sin 2x = 0$

$$= 2\sin(3x)\cos(3x) + \sin(2x) = 0$$

$$= 2[\sin(2x+x)][\cos(2x+x)] + \sin(2x) = 0$$

$$= 2[\sin(2x)\cos(x) + \sin(x)\cos(2x)] \cdot$$
$$[\cos(2x)\cos(x) - \sin(2x)\sin(x)] + \sin(2x) = 0$$

$$2[(2\sin x\cos x)(\cos x) + (\sin x)(\cos^2 x - \sin^2 x)] \cdot$$
$$[[\cos^2 x - \sin^2 x](\cos x) - 2\sin x\cos x \cdot \sin x] + 2\sin x\cos x$$

$$= 2[\underline{2\sin x\cos^2 x + \sin x\cos^2 x} - \sin^3 x] \cdot$$
$$[\cos^3 x - \underline{\sin^2 x\cos x - 2\sin^2 x\cos x}] + 2\sin x\cos x = 0$$

$$= 2[3\sin x\cos^2 x - \sin^3 x][\cos^3 x - 3\sin^2 x\cos x]$$
$$+ 2\sin x\cos x$$

$$= 2[3\sin x\cos^2 x - \sin x[1 - \cos^2 x]][\cos x(1 - \sin^2 x) - 3\sin^2 x\cos x]$$
$$+ 2\sin x\cos x$$

$$= 2\left[3\sin x\cos^2 x - \sin x + \sin x\cos^2 x\right]\cdot$$
$$\left[\cos x - \cos x\sin^2 x - 3\sin^2 x\cos x\right] + 2\sin x\cos x$$

$$= 2\left[4\sin x\cos^2 x - \sin x\right]\left[-4\sin^2 x\cos x + \cos x\right] + 2\sin x\cos x$$

$$= 2\left[4\sin x\left[\cos^2 x - 1\right]\right]\left[-4\cos x\left[-\sin^2 x + 1\right]\right] + 2\sin x\cos x$$

$$= 2\left[(4\sin x)(-\sin^2 x)\right]\left[-4\cos x\left[\cos^2 x\right]\right] + 2\sin x\cos x$$

$$= 2\left[-4\sin^3 x\right]\left[-4\cos^3 x\right] + 2\sin x\cos x$$

$$= 32\sin^3 x\cos^3 x + 2\sin x\cos x$$

$$= (\sin x\cos x)\left[32(\sin^2 x)(\cos^2 x) + 1\right] =$$

$$= (\sin x\cos x)\left[32\sin^2 x(1 - \sin^2 x) + 1\right]$$

$$= (\sin x\cos x)\left[32\sin^2 x - 32\sin^4 x + 1\right]$$

$$= (\sin x\cos x)(-32\sin^4 x + 32\sin^2 x + 1) = 0$$

$\Rightarrow$ $\sin x = 0$ $\qquad$ $\cos x = 0$ $\qquad$ $32\sin^4 x - 32\sin^2 x - 1 = 0$

$0, \pi, 2\pi$ $\qquad$ $\frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ $32u^4 - 32u^2 - 1 = 0$

$32v^2 - 32v - 1 = 0$

$a = 32,\ b = -32,\ c = -1$

$b^2 - 4ac = (-32)^2 - 4(32)(-1)$

$= 1152$

$$v = \frac{32 \pm 24\sqrt{2}}{64}$$

$$= \frac{8(4 \pm 3\sqrt{2})}{64}$$

$$= \frac{4 \pm 3\sqrt{2}}{8}$$

$$= \frac{1}{2} \pm \frac{3\sqrt{2}}{8} = v$$

Ouch!

Double-Angle Formulas

$\sin 2u = 2 \sin u \cos u$ $\qquad$ $\cos 2u = \cos^2 u - \sin^2 u$

$\sin(u + v) = \sin u \cos v + \cos u \sin v$

$\cos(u + v) = \cos u \cos v - \sin u \sin v$ $\qquad$ $\tan 2u = \dfrac{2 \tan u}{1 - \tan^2 u}$