An Independent System with a unique solution.

$$x+3y-z=4$$
$$-3x-8y+z=-16$$
$$4x+14y-7z=11$$

(Gaussian) Elimination

$3R1 \quad 3x+9y-3z=12$
$R2 \quad -3x-8y+z=-16$
$-3R1+R2 \quad y-2z=-4$

$-4R1 \quad -4x-12y+4z=-16$
$R3 \quad 4x+14y-7z=11$
$2y-3z=-5$

New System

$$x+3y-z=4$$
$$y-2z=-4$$
$$2y-3z=-5$$

$-2R2 \quad -2y+4z=8$
$R3 \quad 2y-3z=-5$
$-2R2+R3 \quad z=3$

New (last) System

$$x+3y-z=4$$
$$y-2z=-4$$
$$z=3$$

$$y-2z=-4$$
$$y-2(3)=-4$$
$$y-6=-4$$
$$y=2$$

$$x+3y-z=4$$
$$x+3(2)-3=4$$
$$x+6-3=4$$
$$x+3=4$$
$$x=1$$

$$\boxed{(x,y,z)=(1,2,3)}$$

Apr 25-2:03 PM

A dependent system with infinitely-many solutions.

$$x - y + 2z = 1$$
$$5x - 4y + 12z = 7$$
$$3x - 2y + 8z = 5$$

$$\begin{array}{ll} -5R1 & -5x + 5y - 10z = -5 \\ R2 & 5x - 4y + 12z = 7 \\ \hline -5R1+R2 & y + 2z = 2 \end{array} \qquad \begin{array}{ll} -3R1 & -3x + 3y - 6z = -3 \\ R3 & 3x - 2y + 8z = 5 \\ \hline -3R1+R3 & y + 2z = 2 \end{array}$$

New System:

$$x - y + 2z = 1$$
$$y + 2z = 2$$
$$y + 2z = 2$$

$$\begin{array}{ll} -R2 & -y - 2z = -2 \\ R3 & y + 2z = 2 \\ \hline -R2+R3 & 0 = 0 \end{array}$$

New System:

$$x - y + 2z = 1$$
$$y + 2z = 2$$
$$0 = 0$$

$$y + 2z = 2$$
$$\Longrightarrow \boxed{y = -2z + 2}$$
$$x - y + 2z = 1$$
$$x - (-2z + 2) + 2z = 1$$
$$x + 2z - 2 + 2z = 1$$
$$x + 4z - 2 = 1$$
$$\boxed{x = -4z + 3}$$

z is free!

Solution set:

$$\boxed{(x,y,z) \in \{(-4z+3, -2z+2, z) \mid z \in \mathbb{R}\}}$$

General Sol'n

Particular Sol'ns:

$$z = 1 \Longrightarrow (x,y,z) = (-4(1)+3, -2(1)+2, 1)$$
$$= (-1, 0, 1) \quad (z = 1)$$

A dependent system that has no solution.

$$x - y + 2z = 1$$
$$5x - 4y + 12z = 7$$
$$3x - 2y + 8z = 8$$

$$\begin{array}{ll} -5R1 & -5x + 5y - 10z = -5 \\ R2 & 5x - 4y + 12z = 7 \\ \hline -5R1+R2 & y + 2z = 2 \end{array} \qquad \begin{array}{ll} -3R1 & -3x + 3y - 6z = -3 \\ R3 & 3x - 2y + 8z = 8 \\ \hline -3R1+R3 & y + 2z = 5 \end{array}$$

New System:

$$x - y + 2z = 1$$
$$y + 2z = 2$$
$$y + 2z = 5$$

$$\begin{array}{ll} -R2 & -y - 2z = -2 \\ R3 & y + 2z = 5 \\ \hline -R2+R3 & 0 = 3 ?! \end{array}$$

You Kiddin'?!

New System:

$$x - y + 2z = 1$$
$$y + 2z = 2$$
$$0 = 3 \quad ?!$$

0 = 3 is absurd. Assuming our mechanics are all good, this absurdity tells us we're reasoning from a faulty premise, namely, that this system HAS a solution.

Therefore, there is no solution.

Apr 25-2:16 PM