

An Independent System with a unique solution.

$$\begin{aligned} x + 3y - z &= 4 \\ -3x - 8y + z &= -16 \\ 4x + 14y - 7z &= 11 \end{aligned} \quad \text{(Gaussian) Elimination}$$

$$\begin{array}{l} \cdot 3R1 \quad 3x + 9y - 3z = 12 \\ R2 \quad -3x - 8y + z = -16 \\ \hline -3R1 + R2 \quad y - 2z = -4 \end{array} \quad \begin{array}{l} -4R1 \quad -4x - 12y + 4z = -16 \\ R3 \quad 4x + 14y - 7z = 11 \\ \hline \quad \quad \quad 2y - 3z = -5 \end{array}$$

New System

$$\begin{aligned} x + 3y - z &= 4 \\ y - 2z &= -4 \\ 2y - 3z &= -5 \end{aligned}$$

$$\begin{array}{l} -2R2 \quad -2y + 4z = 8 \\ R3 \quad 2y - 3z = -5 \\ \hline -2R2 + R3 \quad z = 3 \end{array}$$

New (last) System

$$\begin{aligned} x + 3y - z &= 4 \\ y - 2z &= -4 \\ z &= 3 \end{aligned}$$

$$\begin{aligned} y - 2z &= -4 \\ y - 2(3) &= -4 \\ y - 6 &= -4 \\ y &= 2 \end{aligned}$$

$$\begin{aligned} x + 3y - z &= 4 \\ x + 3(2) - 3 &= 4 \\ x + 6 - 3 &= 4 \\ x + 3 &= 4 \\ x &= 1 \end{aligned}$$

$$(x, y, z) = (1, 2, 3)$$

A dependent system with infinitely-many solutions.

$$x - y + 2z = 1$$

$$5x - 4y + 12z = 7$$

$$3x - 2y + 8z = 5$$

$$\begin{array}{l} -5R1 \\ R2 \end{array} \quad \begin{array}{l} -5x + 5y - 10z = -5 \\ 5x - 4y + 12z = 7 \end{array}$$

$$\begin{array}{l} -3R1 \\ R3 \end{array} \quad \begin{array}{l} -3x + 3y - 6z = -3 \\ 3x - 2y + 8z = 5 \end{array}$$

$$\begin{array}{r} -5R1+R2 \\ R2 \end{array} \quad \begin{array}{l} y + 2z = 2 \\ 5x - 4y + 12z = 7 \end{array}$$

$$\begin{array}{r} -3R1+R3 \\ R3 \end{array} \quad \begin{array}{l} y + 2z = 2 \\ 3x - 2y + 8z = 5 \end{array}$$

New System:

$$x - y + 2z = 1$$

$$y + 2z = 2$$

$$y + 2z = 2$$

$$\begin{array}{l} -R2 \\ R3 \end{array} \quad \begin{array}{l} -y - 2z = -2 \\ y + 2z = 2 \end{array}$$

$$\begin{array}{l} R3 \\ R3 \end{array} \quad \begin{array}{l} y + 2z = 2 \\ y + 2z = 2 \end{array}$$

$$\begin{array}{l} -R2+R3 \\ R3 \end{array} \quad \begin{array}{l} 0 = 0 \\ y + 2z = 2 \end{array}$$

New System:

$$x - y + 2z = 1$$

$$y + 2z = 2$$

$$0 = 0$$

$$\Rightarrow \boxed{y = -2z + 2}$$

$$x - y + 2z = 1$$

$$x - (-2z + 2) + 2z = 1$$

$$x + 2z - 2 + 2z = 1$$

$$x + 4z - 2 = 1$$

$$\boxed{x = -4z + 3}$$

z is free!

Solution set:

$$\boxed{(x, y, z) \in \{(-4z + 3, -2z + 2, z) \mid z \in \mathbb{R}\}}$$

General Sol'n

Particular Sol'n's:

$$z = 1 \Rightarrow (x, y, z) = (-4(1) + 3, -2(1) + 2, 1)$$

$$= (-1, 0, 1) \quad (z=1)$$

A dependent system that has no solution.

$$x - y + 2z = 1$$

$$5x - 4y + 12z = 7$$

$$3x - 2y + 8z = 8$$

$$\begin{array}{r} -5R1 \quad -5x + 5y - 10z = -5 \\ R2 \quad 5x - 4y + 12z = 7 \\ \hline -5R1 + R2 \quad y + 2z = 2 \end{array} \quad \begin{array}{r} -3R1 \quad -3x + 3y - 6z = -3 \\ R3 \quad 3x - 2y + 8z = 8 \\ \hline -3R1 + R3 \quad y + 2z = 5 \end{array}$$

New System:

$$x - y + 2z = 1$$

$$y + 2z = 2$$

$$y + 2z = 5$$

$$\begin{array}{r} -R2 \quad -y - 2z = -2 \\ R3 \quad y + 2z = 5 \\ \hline -R2 + R3 \quad 0 = 3 \end{array}$$

$0 = 3$?! You Kiddin'?!

New System:

$$x - y + 2z = 1$$

$$y + 2z = 2$$

$$0 = 3 \quad ?!$$

$0 = 3$ is absurd. Assuming our mechanics are all good, this absurdity tells us we're reasoning from a faulty premise, namely, that this system HAS a solution.

Therefore, there is no solution.

