MAT 121
30 Points

Take-Home Test 3 – Fall, 2020
WRITING PROJECT #3

Name KEY
GRAPHING UTILITY RECOMMENDED

Remember to use separate paper for everything except your name. Leave a margin in the top left corner. Spread out your work. Work in one column. Just because there's room on the right doesn't mean you have to fill all of it. Submit problems in order.

A grapher (calculator or online or computer algebra system) will give you some quick, instant information about where zeros and asymptotes are located, but will not give you the essense of the shape. For that, you need to have good understanding about smoothness of these functions and the ideas I present in videos.

Get me an early PDF version electronically in e-mail OR mail it by USPS (Snail Mail) for the Early Bird Bonus by Monday, November 2nd. Snail-mail projects may or may not be graded and returned before the test. But the bonus will still stand. (My USPS address: Harry Mills, 2358 50th Ave, Greeley, CO, 80634)

BEGIN TEST:

⊕

We will be working with $f(x)=4x^5+16x^4\overset{\downarrow}{-}21x^3\overset{\downarrow}{-}13x^2-38x+12$

for most of this test. We'll say everything about this polynomial that's worth saying.

1. (2 pts) Describe the end behavior of f with a simple graphic.

2. (2 pts) Use Descartes' Rule of Signs to determine the *possible* number of positive and negative zeros.

3. (2 pts) Use the Rational Zeros Theorem to determine the *possible* rational zeros (roots) of f.

4. (2 pts) Using the information, above, find all real zeros of f. Finding all zeros includes finding the multiplicity of each. This means performing multiple synthetic divisions. Always check for multiplicity greater than 1 with another synthetic division, just in case.

5. (2 pts) From your work, above, factor f over the real numbers. This will involve an irreducible quadratic factor that your grapher has no way of helping you to see. without the synthetic divisions in #4, bringing you closer and closer, step by step, to the irreducible quadratic.

6. (2 pts) Give a rough sketch of f from all of the above information. This is an *art* whose essence is really only found in my videos. If you're too tied to your grapher's output, you'll not capture the real essence of what's going on, or the key features I'm always looking for. Your picture will be more "vertical" than it should be.

7. (2 pts) Now we've covered everything *real* about f. Let's use that work to find *all* the roots of f and *split f into linear factors*. 5 roots are *guaranteed by the Fundamental Theorem of Algebra,* and we have found the 3 real ones. The other 2 are nonreal, hiding inside the irreducible quadratic polynomial that remains as the last, very very depressed piece that's not broken all the way down in #5. Now do your quadratic equation thing to *find* the 2 nonreal roots. *Finally*, apply the Factor Theorem to *all* the above work, and represent f as a product of linear factors, $f(x)=a(x-r_1)^{m_1}(x-r_2)^{m_2}\cdots(x-r_w)^{m_w}$. Don't forget the leading coefficient, a.

This wrings (almost) every useful drop of the Theorems on Polynomials out of f, so now on to Rational Functions, which are *quotients* of polynomials!

8. (5 pts) Sketch the graph of $R(x)=\dfrac{6x^2+x-2}{x^2+x-12}$, showing all intercepts, asymptotes, and capturing the *essential features* of the shape of the graph. If you're a slave to your grapher, and oblivious to the features I'm looking for, it'll jump off the page at me (and be bad).

Note: There *is* a subtle feature to this graph that I downplay on tests, but you should pick up on with a take-home, namely, the horizontal asymptote *does* intersect the graph of the function.

I'm willing to part with **5 bonus points** if you can find the point of intersection of $R(x)$ with its horizontal asymptote and label it with an ordered-pair label. I'm also looking for its effect on the graph. There's a little wiggle to this graph in the 1st quadrant.

9. (2 pts) Sketch the graph of $Q(x)=\dfrac{6x^3-41x^2-9x+14}{x^3-6x^2-19x+84}$. All the work you did for #8 applies to this one, *except* for the *hole* in the graph of Q, which I expect you to find and clearly label in your graph.

10. (5 pts) Sketch the graph of $T(x)=\dfrac{6x^3-29x^2-7x+10}{x^2+x-12}$, showing all intercepts and asymptotes. This was also built off #8, so use the zeros you found for the numerator in #8 to help you find the 3rd zero of this new numerator.

Now for a pair of questions many struggle with on the sit-down test, but which are actually *very simple* if you can synthesize your skills and *apply* them to these sorts of questions. Often the downfall of people on the sit-down, but designed to be easy points for people who are putting things together.

For HELP on these problems, you want to look at Test Prep Videos, in particular the Test-Prep Videos for the SIT-DOWN Test 3, because the old Take-Home 3/Writing Project #3 didn't have these type-questions.

11. (2 pts) What is the domain of $W(x)=\sqrt{(x+4)^2(x-2)^3(x+5)^4(x-8)^5}$?

12. (2 pts) What is the domain of $\mathrm{K}(x)=\sqrt{\dfrac{(x-2)^3(x+5)^4}{(x+4)^2(x-8)^5}}$?

121 WP #3 TAKE-HOME TEST 3 ①

$f(x) = 4x^5 + 16x^4 + 21x^3 \underbrace{+13x^2}_{1} \underbrace{-38x + 12}_{2}$

①

② 2 sign changes in f(x) ⟹ $\boxed{\text{2 or 0 pos. zeros}}$

$f(-x) = \underbrace{-4x^5 + 16x^4}_{1} \underbrace{- 21x^3}_{2} \underbrace{+ 13x^2}_{3} + 38x + 12$

ODD # → at least one!

3 sign changes in f(−x) ⟹ 3 or 1 negative zeros
(CHECK neg's first.)

③ $a_0 = 12 \rightarrow p$'s, $a_5 = 4 \rightarrow q$'s

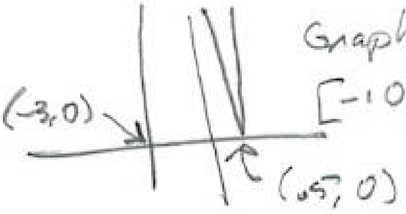
$\frac{p}{q}$'s : ±1, ±2, ±3, ±4, ±6, ±12

$\pm\frac{1}{2}$, $\cancel{\pm\frac{2}{2}}$, $\pm\frac{3}{2}$, $\cancel{\pm\frac{4}{2}}$, $\cancel{\pm\frac{6}{2}}$, $\cancel{\pm\frac{12}{2}}$

$\pm\frac{1}{4}$, $\cancel{\pm\frac{2}{4}}$, $\pm\frac{3}{4}$, $\cancel{\pm\frac{4}{4}}$, $\cancel{\pm\frac{6}{4}}$, $\cancel{\pm\frac{12}{4}}$

So, $\boxed{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}}$

④ Cheat w/ grapher ⟹ $x = -3$, $x = \frac{1}{2}$



Grapher image
[-10,10] by [-10,10]

−3	4	16	21	13	−38	12
		−12	−12	−27	42	−12
	4	4	9	−14	4	0
−3		−12	24	−99	339	
	4	−8	33	−113	NO	

sweet!

Always check to see if it works twice, if it works once!

$$\begin{array}{r|rrrrrr}
\frac{1}{2} & 4 & 4 & 9 & -14 & 4 & 0 \\
 & & 2 & 3 & 6 & -4 & \\
\hline
 & 4 & 6 & 12 & -8 & 0 & \text{sweet!}
\end{array}$$

$$\begin{array}{r|rrrr}
\frac{1}{2} & 4 & 6 & 12 & -8 \\
 & & 2 & 4 & 8 \\
\hline
 & 4 & 8 & 16 & 0
\end{array} \quad \text{Sweet!}$$

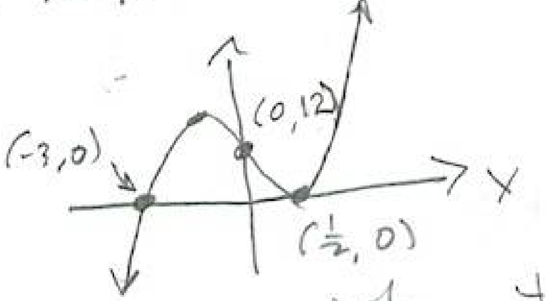
By #2, that's all the positives

NOTE I used the depressed polynomial $4x^4+4x^3+9x^2-14x+4$, after I found $x=3$ was a zero. NEVER throw away the work you did!

⑤ By #4, $\boxed{f(x) = (x+3)(x-\frac{1}{2})^2\underbrace{(4x^2+8x+16)}}$

Bottom row from previous step.

⑥ Keepin' it real:



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⑦ Imaginary bits. $4x^2+8x+16=0 \longrightarrow$

$4(x^2+2x+4)=0$

$x^2+2x+4=0 \longrightarrow$

$x^2+2x+1^2=-4+1=-3 \longrightarrow (x+1)^2=-3 \Longrightarrow \boxed{x=-1\pm i\sqrt{3}}$

$\Longrightarrow \boxed{f(x)= 4(x-3)(x+\frac{1}{2})^2(x-(-1+i\sqrt{3}))(x-(-1-i\sqrt{3}))}$

⑧ $R(x) = \dfrac{6x^2+x-2}{x^2+x-12}$

$12 \to 4, 3$

$6x^2+4x-3x-2$

$2x(3x+2)-1(3x+2)$

$=(3x+2)(2x-1)$

$=0 \Rightarrow$

$x \in \{-\frac{2}{3}, \frac{1}{2}\}$

$\mathcal{D}$: $x^2+x-12 = x^2+4x-3x-12$

$= x(x+4)-3(x+4)$

$= (x+4)(x-3) \overset{SET}{=} 0 \Rightarrow$

$x \in \{-4, 3\}$, the set on which $R(x) \nexists$.

$\mathcal{D} = \mathbb{R} \setminus \{3, -4\} = \{x \mid x \neq 3 \;\&\; x \neq -4\}$

$\boxed{\text{V.A.: } x=3,\ x=-4}$

H.A.: $\dfrac{6x^2}{x^2} = 6 \longrightarrow$ $\boxed{y=6 \text{ is H.A.}}$

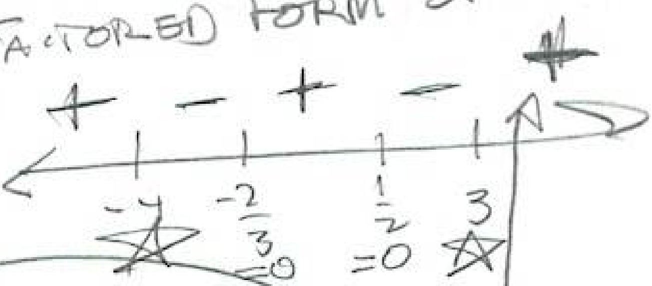
y-int: $\dfrac{-2}{-12} = \dfrac{1}{6} \rightsquigarrow$ $\boxed{(0, \tfrac{1}{6}) \text{ is y-int}}$

x-int: See top right corner, b/c I just did it by reflex, there, for some reason

$\boxed{\text{x-int: } (-\tfrac{2}{3}, 0), (\tfrac{1}{2}, 0)}$

Sign pattern: FACTORED FORM OF R(x) is

$R(x) = \dfrac{(3x+2)(2x-1)}{(x+4)(x-3)}$



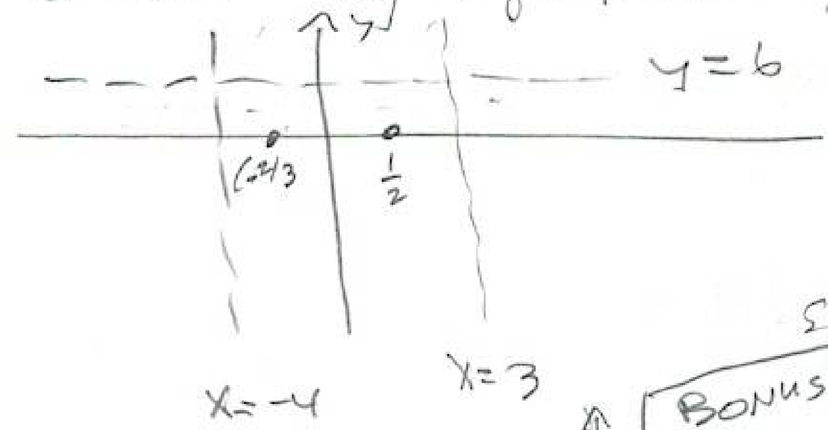
$-4, 3, -\frac{2}{3}, \frac{1}{2}$

$-4, -\frac{2}{3}, \frac{1}{2}, 3$

y=6 is H.A.

121 WP#3 – T3T-H ④

#8 cnt'd By sign pattern, we have



Bonus

$R(x) = 6$

$6x^2+x-2=6(x^2+x-12)$

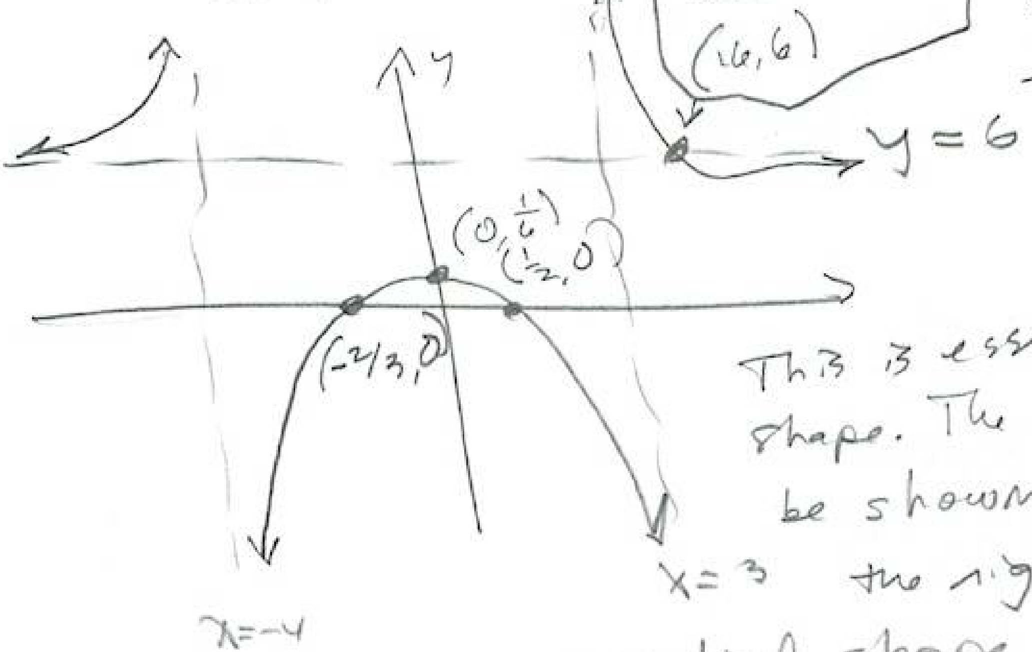
$x-2=6x-72$

$-5x=-70$

$x=\frac{70}{5}=16$

So, BONUS 5pts

(16, 6)



This is essence of the shape. The (16, 6) should be shown farther to the right, but the essential shape is

show approach to y=6 off to right

121 WP #3 T3T-H FALL '20

⑨ is #8 w/ a hole. $Q(x) = \dfrac{6x^3-41x^2-9x+14}{x^3-6x^2-19x+84}$ ⑤

we just have to find the hole.

x^2+x-12 is easiest, so tackle the denominator, knowing $x=3,-4$ work.

$x^3-6x^2-19x+84$

$$\begin{array}{r|rrrr} 3 & 1 & -6 & -19 & 84 \\ & & 3 & -9 & -84 \\ \hline -4 & 1 & -3 & -28 & 0 \\ & & -4 & 28 & \\ \hline & 1 & -7 & & \end{array}$$

$x-7=0$

$x=7$.

$Q(x) = \dfrac{(3x+2)(2x-1)(x-7)}{(x+4)(x-3)(x-7)}$

by leaning on #8.

Leaning on #8, again,

find $R(7)=$

$R(7) = \dfrac{6(7)^2+7-2}{7^2+7-12} = \dfrac{6(49)+5}{49-5} = \dfrac{294+5}{44}$

$= \dfrac{299}{144} = 6.79\overline{54}$

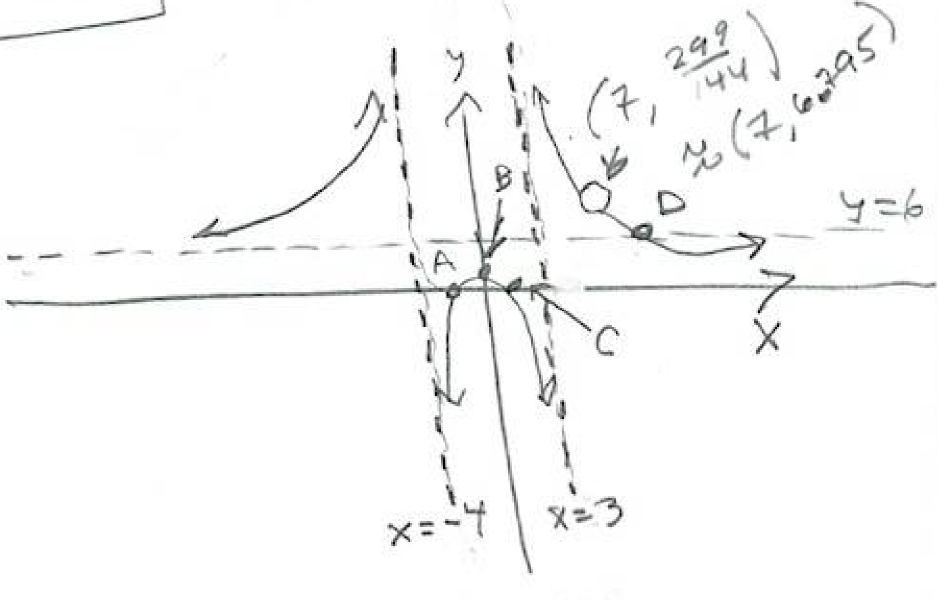
HOLE (7 $\boxed{(7, \frac{77}{36})}$

$A = (-\frac{2}{3}, 0)$

$B = (0, \frac{L}{6})$

$C = (\frac{1}{2}, 0)$

$D = (16, 6)$



121 WP #3 T3T-H ⑥

⑩ $T(x) = \frac{6x^3-29x^2-7x+10}{x^2+x-12}$

Using $6x^2+x-2$ from #8 "inside,"

$$\begin{array}{r|rrrr} -\frac{2}{3} & 6 & -29 & -7 & 10 \\ & & -4 & 22 & -10 \\ \hline \frac{1}{2} & 6 & -33 & 15 & \\ & & 3 & -15 & \\ \hline & 6 & -30 & & \end{array}$$

$6x-30=0$

$6x=30$

$x=5$ gives other x-int., $(5,0)$, rather.

$\boxed{\text{x-int: } (-\tfrac{2}{3},0), (\tfrac{1}{2},0), (5,0)}$

$\boxed{\text{V.A.: } x=3, x=-4}$ by #8

$\boxed{\text{y-int: } -\frac{10}{12}=-\frac{5}{6} \quad (0,-\frac{5}{6})}$

SLANT ASYMPTOTE:

$$\begin{array}{r|l} & 6x-35 \\ \hline x^3+x-12 & 6x^3-29x^2-7x+10 \\ & -(6x^3+6x^2-72x\quad) \\ \hline & -35x^2+65x+10 \end{array}$$

BONUS: Does T(x) ever equal 6x-35?

$6x^3-29x^2-7x+10 = (6x-35)(x^2+x-12)$

$= 6x^3+6x^2-72x-35x^2-35x+420 = 6x^3-29x^2-107x+420$

$= 6x^3+6x^2-72x-35x^2-35x+420$

$6x^3-29x^2-107x+420$

$\Rightarrow 6x^3-29x^2-7x+10 = 6x^3-29x^2-107x+420 \Rightarrow 100x=410 \Rightarrow$ $\boxed{x=\frac{41}{10}}$, $\boxed{y=-\frac{52}{5}}$

$-7x+10=-107x+420$

121 wp#3

#10 Cnt'd

slant asymptote $y=6x-35$ intersects $T(x)$

at $\left(\frac{41}{10}, -\frac{52}{5}\right) = (4.1, -10.4)$

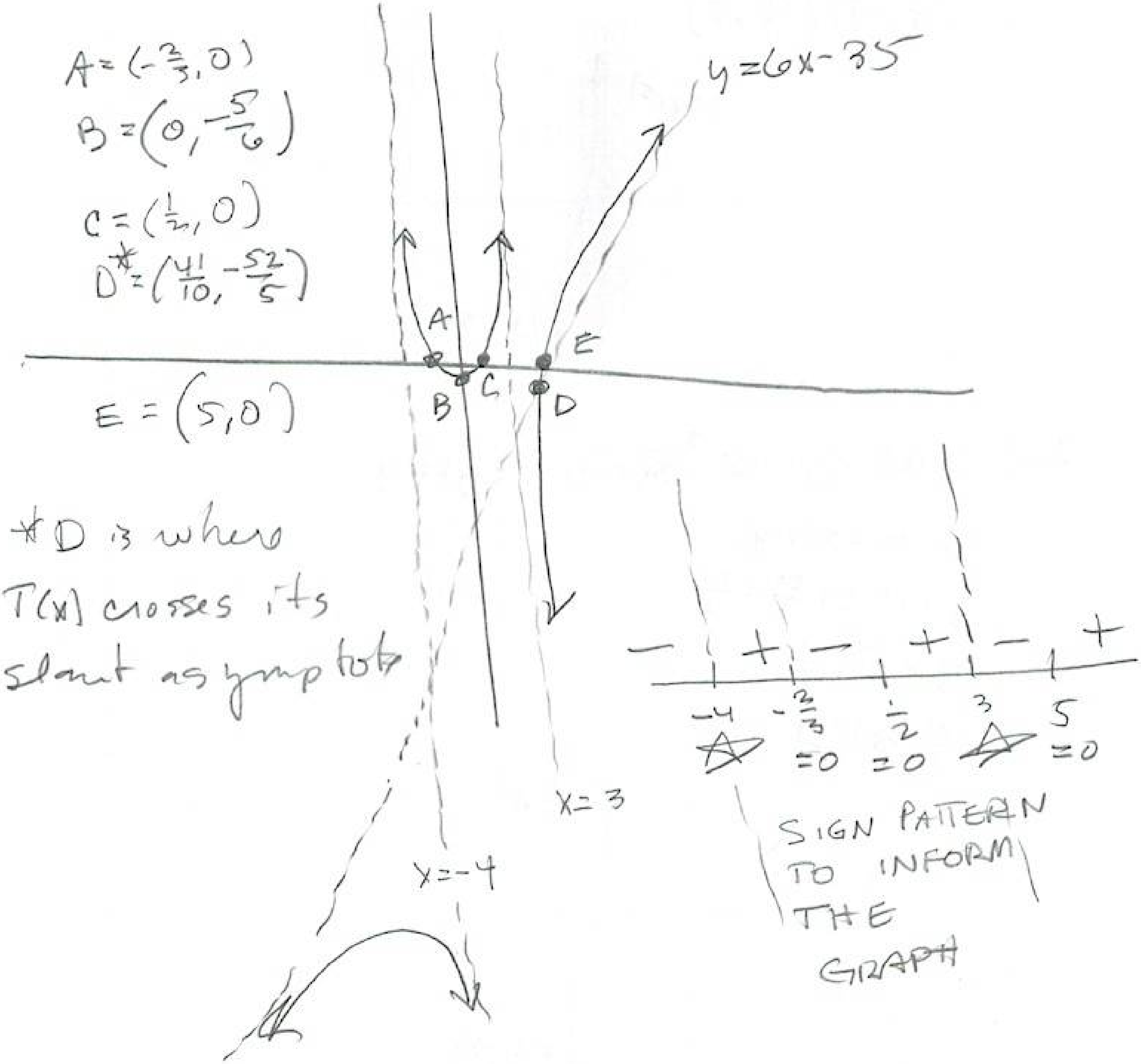
$A=\left(-\frac{2}{3}, 0\right)$

$B=\left(0, -\frac{5}{6}\right)$

$C=\left(\frac{1}{2}, 0\right)$

$D^{*}=\left(\frac{41}{10}, -\frac{52}{5}\right)$

$E=(5,0)$



*D is where T(x) crosses its slant asymptote

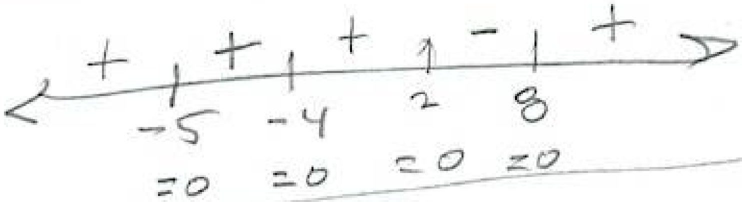
SIGN PATTERN TO INFORM THE GRAPH

121 wp#3 T3T-H

⑨

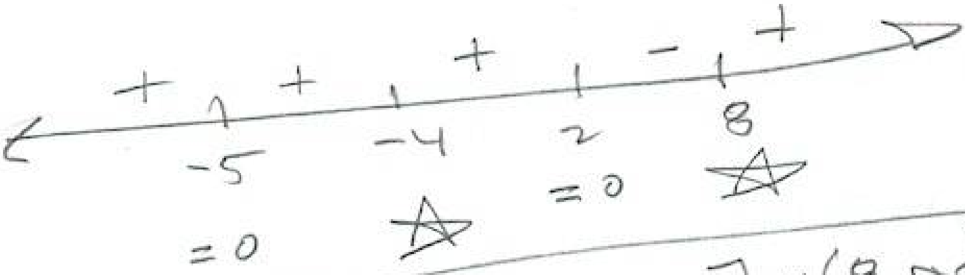
⑪ $w(x) = \sqrt{(x+4)^2(x-2)^3(x+5)^4(x-8)^5} = \sqrt{f(x)}$

Need $f(x) \geq 0$



$\boxed{D(w) = (-\infty, 2] \cup [8, \infty)}$

⑫ $K(x) = \sqrt{\dfrac{(x-2)^3(x+5)^4}{(x+4)^2(x-8)^5}}$



$\boxed{D(K) = (-\infty, -4) \cup (-4, 2] \cup (8, \infty)}$