121 E3 TAKE-HOME / Writing Project #3

(1) $4x^5$ in control

(2) Descartes:

$f(x) = \underbrace{4x^5}_{1} - 12x^4 - \underbrace{x^3}_{2} + \underbrace{44x^2}_{3} - \underbrace{57x}_{4} + 22$

4, 2, or 0 pos. true roots

$f(-x) = -4x^5 - 12x^4 + x^3 + 44x^2 + 57x + 22$

$\underbrace{\quad}_{1}$

Exactly one negative root.
Look for it, first!

(3) $\frac{p}{q} : \frac{22}{4} \longrightarrow$ $\pm 1, \pm 2, \pm 11, \pm 22,$

$\pm \frac{1}{2}, \cancel{\pm \frac{2}{2}}, \pm \frac{11}{2}, \cancel{\pm \frac{22}{2}},$

$\pm \frac{1}{4}, \cancel{\pm \frac{2}{4}}, \pm \frac{11}{4}, \cancel{\pm \frac{22}{4}}$

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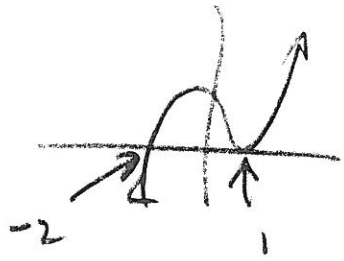
(3) Grapher shows



So, using #2, go after the one negative root, first:

-2	4	-12	-1	44	-57	22
		-8	40	-78	+68	-22
	4	-20	39	-34	11	0
1		4	-16	23	-11	
	4	-16	23	-11	0	
1		4	-12	11		
	4	-12	11	0		

Check if $4x^2 - 12x + 11$ has real zeros:

$a=4, b=-12, c=11$ ⟹

$b^2 - 4ac = (-12)^2 - 4(4)(11) = 144 - 176 = -32$

$\sqrt{-32} = i\sqrt{32} = 4i\sqrt{2}$

2 | 32
2 | 16
2 | 8
2 | 4
2

No real zeros, plus I

ANS (4) started on #6, already.

$\boxed{x=1, m=2;\ x=-2, m=1}$

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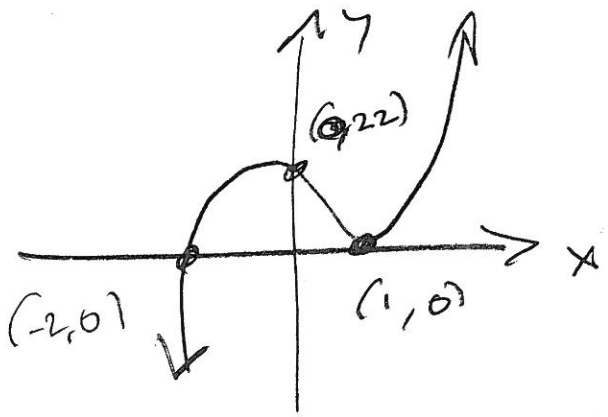
(5) $f(x) = (x-1)^2(x+2)(4x^2-12x+11)$

(6) $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{12 \pm 4i\sqrt{2}}{2(4)} = \dfrac{3 \pm i\sqrt{2}}{2}$

So $f(x) = 4(x-1)^2(x+2)\left(x - \dfrac{3+i\sqrt{2}}{2}\right)\left(x - \dfrac{3-i\sqrt{2}}{2}\right)$

(7)



(8)

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(8) $R(x) = \dfrac{2x^2 - x - 3}{x^2 + 2x - 15} = \dfrac{(2x-3)(x+1)}{(x+5)(x-3)}$

$\mathcal{D} = \mathbb{R} \setminus \{-5, 3\}$ No holes

$x = -5,\ x = 3$ V.A.

$2x - 3 = 0 \qquad x + 1 = 0$

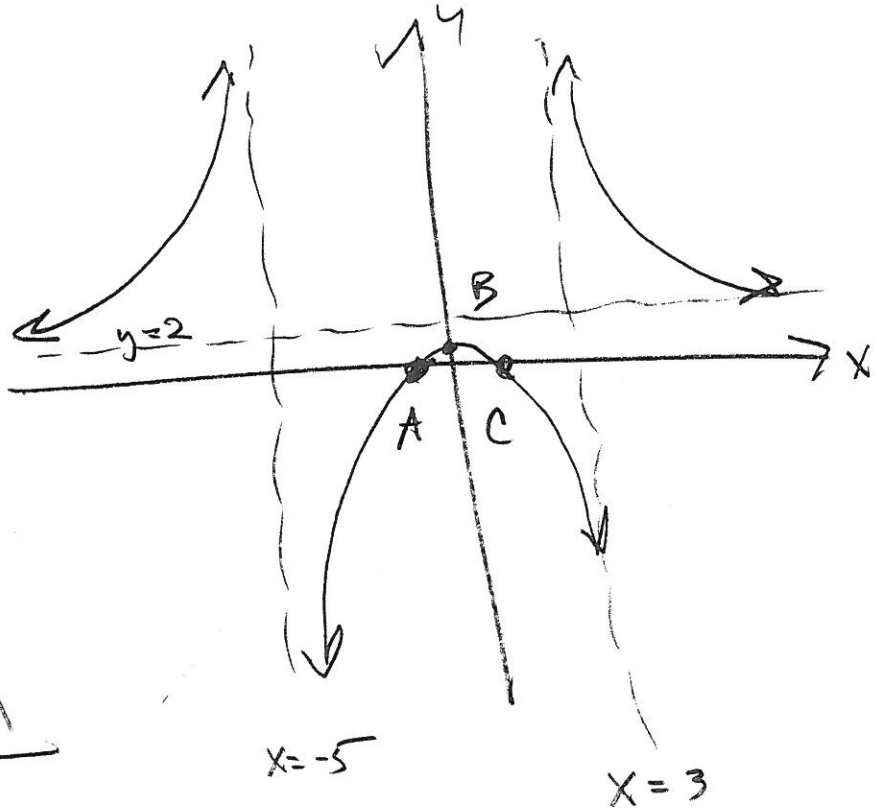
$x = \frac{3}{2} \qquad x = -1$

$\left(\frac{3}{2}, 0\right), \quad (-1, 0)$

x-int

$R(0) = \dfrac{-3}{-15} = \dfrac{1}{5}$

y-int: $(0, \frac{1}{5})$



$R(x) \xrightarrow{x \to \pm\infty} \dfrac{2x^2}{x^2} =$

$y = 2$ H.A.

$A = (-1, 0)$

$B = (0, \frac{1}{5})$

$C = (\frac{3}{2}, 0)$

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(9) $g(x) = \dfrac{2x^3-9x^2+x+12}{x^3-2x^2-23x+60}$ ⇒ R(x) with a hole.

We peel away the R(x) inside by breaking down numerator / denominator

We KNOW this

$$g(x) = \frac{(2x-3)(x+1)(x-?)}{(x+5)(x-3)(x-?)}$$

$$\frac{(2(-4)-3)(-4+1)}{(-4+5)(-4-3)} = \frac{(-8-3)(-3)}{(1)(-7)} = \frac{33}{-7}$$

M1

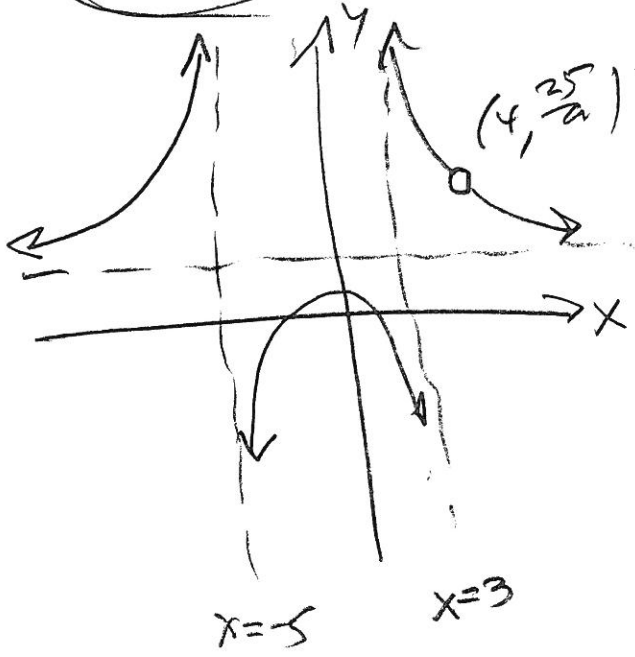
$$\begin{array}{r|rrr} -5 & 1 & -2 & -23 & 60 \\ & & -5 & 35 & -60 \\ \hline & 1 & -7 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrr} 3 & 1 & -7 & 12 \\ & & 3 & -12 \\ \hline & 1 & -4 & \to 0 \end{array}$$

$x-? = x-4$!

$$R(4) = \frac{(2(4)-3)(4+1)}{(4+5)(4-3)} = \frac{25}{9}$$

$(4, \frac{25}{9})$ = HOLE IN $g(x)$



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(10) $h(x) = -\dfrac{x^2-6x+3}{x-4} = \dfrac{-x^2+6x-3}{x-4} =$

$D = \mathbb{R} \setminus \{4\}$ Check for holes:

V..

$x^2-6x+3=0$

$x^2-6x = -3$

$x^2-6x+3^2 = -3+9$

$(x-3)^2 = 6$

$x-3 = \pm\sqrt{6}$ → 5.4495

$x = 3 \pm \sqrt{6}$ → .5505

No cancellation with $x-4$.

x-int: $(3-\sqrt{6}, 0)$

$(3+\sqrt{6}, 0)$

→ V.A.: $x=4$

Oblique (slant) asymptote:

4 | -1 6 -3

-4 8

-1 2 5

$y = -x+2$ is O.A.

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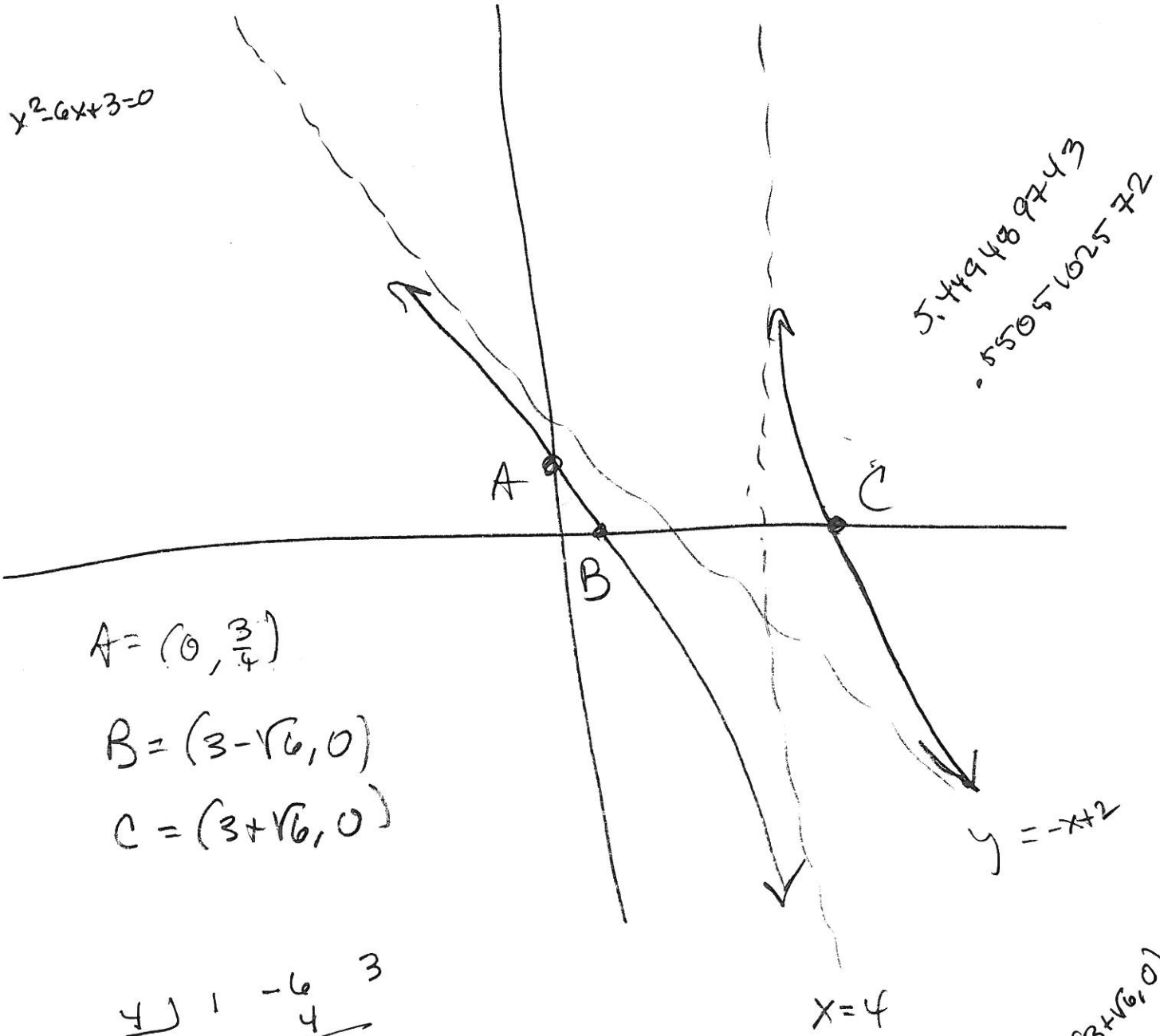
$x^2-6x+3=0$



5.449489743
.5505102572

$A=(0,\frac{3}{4})$

$B=(3-\sqrt{6},0)$

$C=(3+\sqrt{6},0)$

$y=-x+2$

$x=4$

$\frac{x^2-6x+3}{x-4}$

1 -6 3
4
1 -2

$y=x-2$