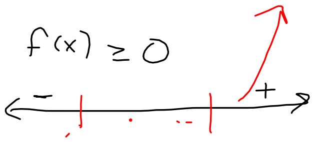
Let $f(x) = x^5 - x^4 - 5x^3 + 9x^2 - 16x - 60$. We'll say everything we can about this polynomial that's worth saying.

1. (3 pts) Describe the end behavior of the graph of f with a simple graphic.

x^5

$f(x) \geq 0$



$-3x^6 +$ lower . . .

$3x^8 +$ lower . . .

$-11x^{17}$

2. (3 pts) Use Descartes' Rule of Signs to determine the *possible* number of positive and negative zeros of f.

$$f(x) = \underbrace{x^5 - x^4}_{1} \underbrace{- 5x^3 + 9x^2}_{2} \underbrace{- 16x}_{3} - 60.$$

$$f(-X) = -x^5 \underbrace{- x^4 + \overset{d}{5x^3}}_{1} + 9x^2 \underbrace{+ 16x - 60}_{2}$$

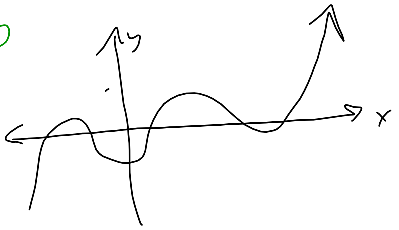
3, 1 possible positive zero

Always go for the odd #

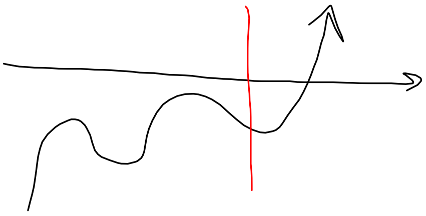
2 or 0 negative zeros.

	①	②	③	④	⑤
Pos	3	3	1	1	1
Neg	2	0	2	0	4
Unreal	0	2	2	4	0

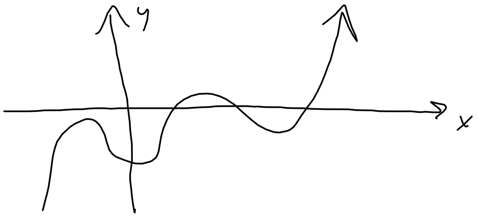
①



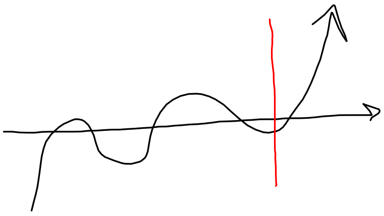
④



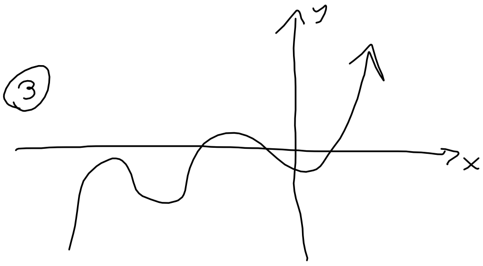
②

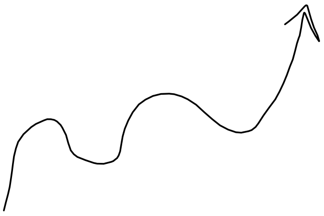


⑤



③





3. (3 pts) Use the Rational Zero~~e~~s Theorem to determine the possible rational zero~~e~~s of f.

$$f(x) = \underset{\substack{\uparrow \\ q}}{x^5} - x^4 - 5x^3 + 9x^2 - 16x - \underset{\substack{\uparrow \\ p}}{60}.$$

$$\begin{array}{r|l} 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ & 5 \end{array}$$

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 5, \pm 4, \pm 6, \pm 10, \pm 15, \pm 12, \pm 20, \pm 60$$

$2x^5$ + lower + 60

± 30

$$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 5, \pm 4, \pm 6, \pm 10, \pm 15, \pm 12, \pm 20, \pm 60,$$

$$\pm\frac{1}{2}, \cancel{\pm\frac{2}{2}}, \pm\frac{3}{2}, \pm\frac{5}{2}, \cancel{\pm\frac{4}{2}}, \cancel{\pm\frac{6}{2}}, \cancel{\pm\frac{10}{2}}, \pm\frac{15}{2}, \cancel{\pm\frac{12}{2}}, \cancel{\pm\frac{20}{2}}, \cancel{\pm\frac{60}{2}}, \cancel{\pm\frac{30}{2}}$$

4. (3 pts) Informed by your work, above, *and maybe a graphing utility of some sort*, use synthetic division to find the zeros. Each time you find a zero, it *should* reduce (depress) the question by one degree. Each time you find a zero, you should thereafter be working with a *depressed polynomial* that is of lesser degree.

$$f(x) = x^5 - x^4 - 5x^3 + 9x^2 - 16x - 60.$$

This is take-home & graphers are O.K.

On sit-down test, I'll make $\underset{\text{one of}}{\uparrow}$ ±1, ±2, ±3 work

Graph suggests

$x = -2$, $m = 2$ (or $m = 4$)

$x = 3$

$x=-2$:

$$\begin{array}{r|rrrrrrl} -2 & 1 & -1 & -5 & 9 & -16 & -60 & \\ & & -2 & 6 & -2 & -14 & 60 & \\ \hline -2 & 1 & -3 & 1 & 7 & -30 & 0 & \text{sweet} \\ & & -2 & 10 & -22 & 30 & & \\ \hline -2 & 1 & -5 & 11 & -15 & 0 & & \text{Sweet} \\ & & -2 & 14 & -50 & & & \\ \hline & 1 & -7 & 25 & \text{Nope} & & & \end{array}$$

$(x+2)\boxed{(x^4 - 3x^3 + 1x^2 + 7x - 30)}$

$(x+2)^2\boxed{(x^3 - 5x^2 + 11x - 15)}$

Zeros: $-2, 3, 1 \pm 2i$

$x = -2$ has $m = 2$

$x = 3$

$$\begin{array}{r|rrrrl} 3 & 1 & -5 & 11 & -15 & \\ & & 3 & -6 & 15 & \\ \hline & 1 & -2 & 5 & & \text{sweet!} \end{array}$$

$(x+2)^2(x-3)(x^2 - 2x + 5)$

$a = 1,\ b = -2,\ c = +5$

$b^2 - 4ac = (-2)^2 - 4(1)(+5)$

$= 4 - 20 = -16$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{-16}}{2(1)} = \frac{2 \pm i4}{2} = \frac{2(1 \pm 2i)}{2} = 1 \pm 2i$$

$(x+2)^2(x-3)(x - (1+2i))(x - (1-2i))$

$= (x+2)^2(x-3)\ (x - 1 - 2i)(x - 1 + 2i)$

5. (3 pts) From you work, above, factor f over the real numbers. This will involve an irreducible quadratic factor.

$$(x+2)^2(x-3)(x^2-2x+5)$$

6. (3 pts) From your work above, factor f over the complex numbers. This should split f into linear factors.

$$(x+2)^2(x-3)(x-(1+2i))(x-(1-2i))$$

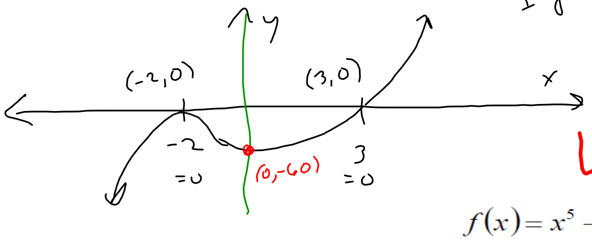
7. (3 pts) Give a rough sketch of f that shows all intercepts.

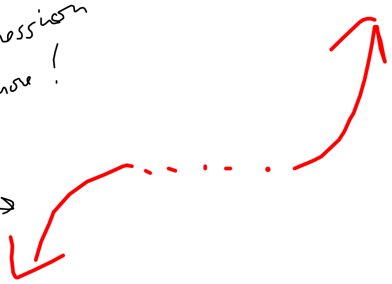
Use the info given from zeros to graph. Do this one like you DON'T have a grapher.

$x=-2 \quad m=2$

$x=3 \quad m=1$

$\boxed{x = 1 \pm i}$ → Nonreal zeros have NO expression in the graph Ignore!





$$f(x) = x^5 - x^4 - 5x^3 + 9x^2 - 16x - 60.$$

$(x+2)^2 (x-3)(x^2-2x+5)$

$(x+2)^2 (x-3)(x^2-2x+5)$

8. (3 pts) Sketch the graph of $R(x)=\dfrac{x^2-5x-6}{x^2-5x+6}$. Show all asymptotes, intercepts and any holes.

$R(x)=\dfrac{(x-6)(x+1)}{(x-3)(x-2)}$

$\mathcal{D}=\mathbb{R}\setminus\{2,3\}=\{x \mid x\neq 2 \text{ and } x\neq 3\}$

V.A. : $\boxed{x=2\ ,\ x=3}$

No holes

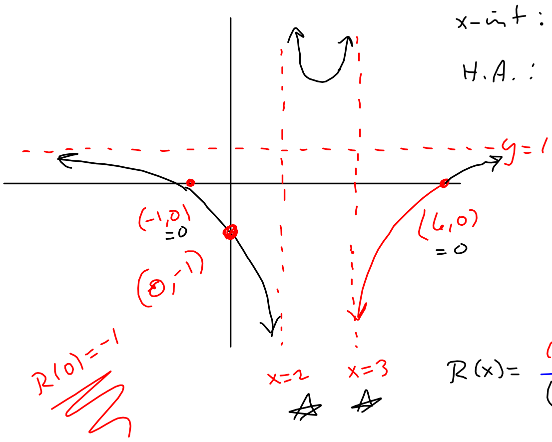
x-int : $x=-1,6 \rightsquigarrow \boxed{(-1,0),\ (6,0)}$

H.A. : deg num = deg denom

$\Rightarrow$ just look @ biggest stuff:

$\dfrac{x^2+\text{smaller}}{x^2+\text{smaller}} \xrightarrow{x\to\pm\infty} \dfrac{x^2}{x^2}=\boxed{1=y}$ H.A.

Gives us our "end behavior."



y=1 is positive

$R(x)=\dfrac{(x-6)\,(x+1)}{(x-3)\,(x-2)}$

9. (3 pts) The graph of $g(x)=\dfrac{x^3-9x^2+14x+24}{x^3-9x^2+26x-24}$ differs from the graph of *f*, in #8, in only one small detail.

Sketch the graph of g, showing all asymptotes, intercepts and holes.

This is the one where they stick a hole, somewhere ...

$R(x)$ was $\dfrac{(x+1)(x-6)}{(x-2)(x-3)}$

Bet on it, it's living inside this follow-up! Let's reveal the changes:

$$\begin{array}{r|rrrr} -1 & 1 & -9 \cdot & 14 & 24 \\ & & -1 & 10 & -24 \\ \hline 6 & 1 & -10 & 24 & 0 \\ & & 6 & -24 & \\ \hline & 1 & -4 & 0 & \end{array}$$

$(x+1)(x-6)(x-4)$

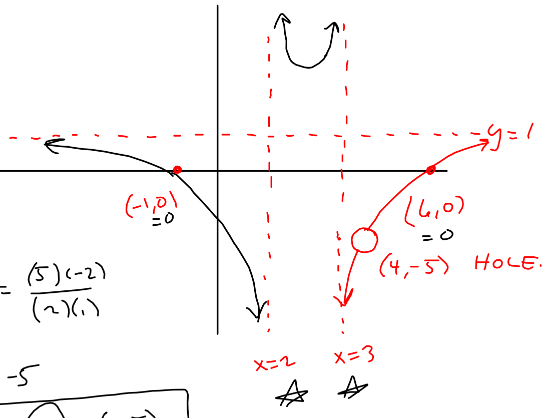
So, $g(x)= \dfrac{(x+1)(x-6)\cancel{(x-4)}}{(x-2)(x-3)\cancel{(x-4)}} = \dfrac{(x+1)(x-6)}{(x-2)(x-3)}$, but

$x \neq 4$

$x=4$ is forbidden!

$$\begin{array}{r|rrrr} 2 & 1 & -9 & 26 & -24 \\ & & 2 & -14 & 24 \\ \hline 3 & 1 & -7 & 12 & 0 \\ & & 3 & -12 & \\ \hline & 1 & -4 & 0 & \end{array}$$

$(x-2)(x-3)(x-4)$



Handle the hole @ $x=4$:

$R(4)= \dfrac{(4+1)(4-6)}{(4-2)(4-3)} = \dfrac{(5)(-2)}{(2)(1)}$

$= \dfrac{-10}{2} = -5$

$\boxed{\text{HOLE @ } (4,-5)}$

10. (3 pts) Sketch the graph of $R(x)=\dfrac{x^3-4x^2-7x+10}{x^2-x-6}$, showing all asymptotes, intercepts and holes.

degree num. > degree denom :

Oblique (Slant) asymptote. DIVIDE !

$\mathcal{D} = \mathbb{R} \setminus \{-2, 3\}$

$(x-3)(x+2) = x^2-x-6$

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -7 & 10 \\ & & -2 & 12 & -10 \\ \hline & 1 & -6 & 5 & 0 \end{array}$$ Sweet!

$(x+2)(x-5)(x-1) = x^3-4x^2-7x+10$

So this $R(x) = \dfrac{\boxed{(x+2)}(x-5)(x-1)}{(x-3)\boxed{(x+2)}}$ Lowest Terms $= \dfrac{(x-5)(x-1)}{(x-3)}$, $x \neq -2$

V.A. : $x=3$

HOLE : $x=-2 \rightsquigarrow \boxed{\left(-2, -\frac{21}{5}\right) = \text{HOLE}}$

$\dfrac{(-2-5)(-2-1)}{(-2-3)} = \dfrac{(-7)(-3)}{-5} = -\dfrac{21}{5}$

x-int : $x = 1, 5, \boxed{-2}$ → NOT Defined @ $x=-2$

$\boxed{(1,0),\ (5,0)}$

y-int :

$\boxed{\left(0, -\frac{5}{3}\right)}$

H.A. : None. Has Slant Asymptote

S.A. : $y = x-3$

Slant Asymptote

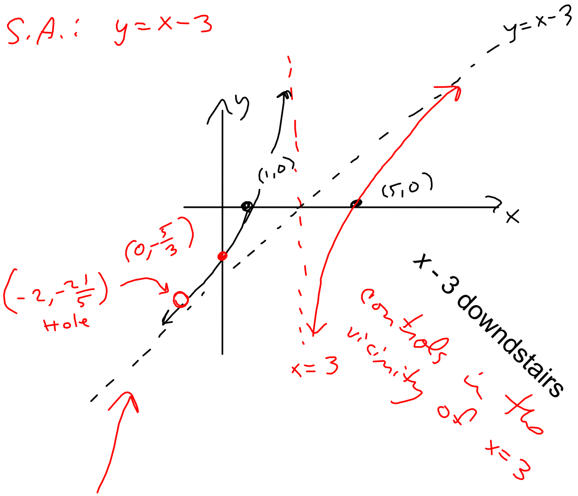
$y=x-3$ is slant asymp.

$$\begin{array}{r|l} & \boxed{x-3} \qquad r\ -4x-8 \\ \hline x^2-x-6 & x^3-4x^2-7x+10 \\ & -(x^3-x^2-6x) \\ \hline & \quad -3x^2-x+10 \\ & \quad -(-3x^2+7x+18) \\ \hline & \qquad -4x-8 \end{array}$$

This gives :

$$R(x) = x-3 + \frac{-4x-8}{x^2-x-6}$$

$$= x-3 - \frac{4(x+2)}{(x-3)(x+2)} = x-3 + \frac{4}{x-3}$$



Far away from x=3, this function is close to y=x-3