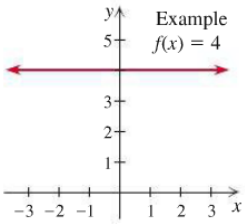
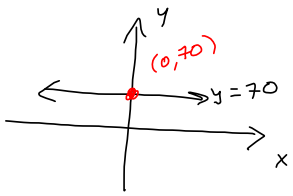
Graphing by Transforming Basic Functions

Basic Functions you should know:

Constant Function $f(x) = k$



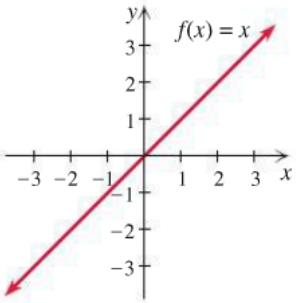
Domain $(-\infty, \infty)$
Range $\{4\}$
Constant on $(-\infty, \infty)$
Symmetric about y-axis



$y = 70$

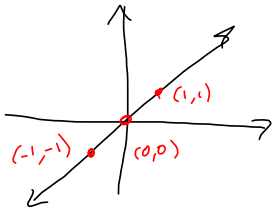
$(x = \text{anything!})$

Identity Function



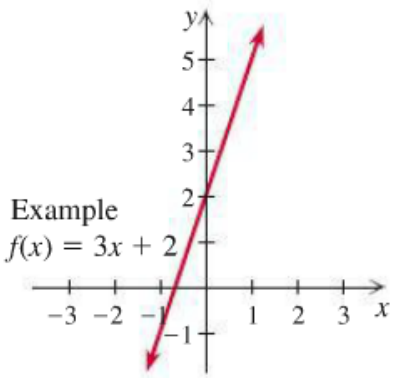
Domain $(-\infty, \infty)$
Range $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
Symmetric about origin

$y = x = f(x)$



$y = 5(x+7) - 11$
$= 5x + 35 - 11$
$= 5x + 24$ Bleah!

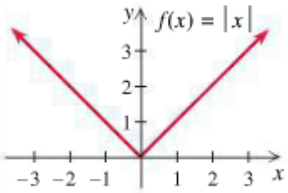
Linear Function $f(x) = mx + b$



Ugh!
You've heard enough on this guy, before MAT 121.
(Almost TOO much!)

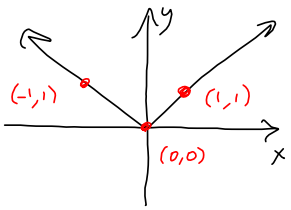
Domain $(-\infty, \infty)$
Range $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$

Absolute Value Function

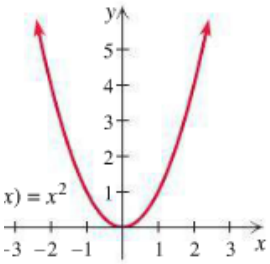


Domain $(-\infty, \infty)$
Range $[0, \infty)$
Increasing on $[0, \infty)$
Decreasing on $(-\infty, 0]$
Symmetric about y-axis

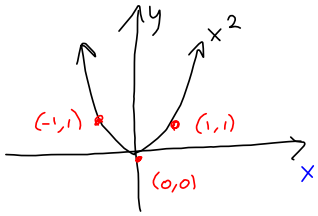
$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



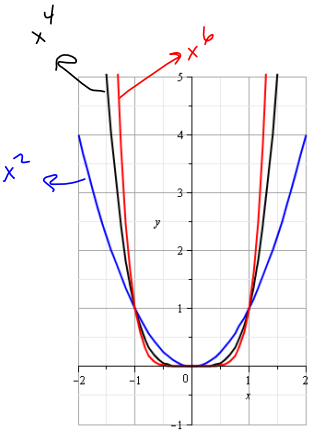
Square Function



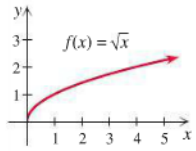
Domain $(-\infty, \infty)$
Range $[0, \infty)$
Increasing on $[0, \infty)$
Decreasing on $(-\infty, 0]$
Symmetric about *y*-axis



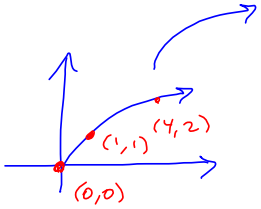
$x^2, x^4, x^6, \ldots, x^{2n}$



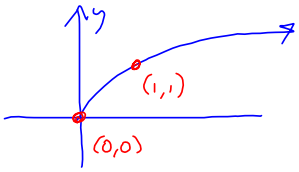
Square-Root Function



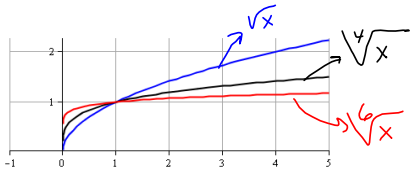
Domain $[0, \infty)$
Range $[0, \infty)$
Increasing on $[0, \infty)$

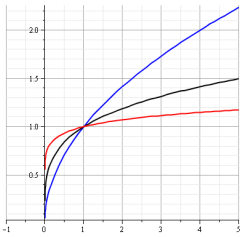


$\sqrt{x} = \sqrt[2]{x}$



$\sqrt{x}, \sqrt[4]{x}, \sqrt[6]{x}, \sqrt[8]{x}, \ldots, \sqrt[2n]{x}$





$\frac{1}{16} = x$

$\sqrt{\frac{1}{16}} = \frac{\sqrt{1}}{\sqrt{16}}$

$= \frac{1}{4} = \sqrt{x}$

$\sqrt[4]{\frac{1}{16}} = \frac{1}{\sqrt[4]{16}} = \frac{1}{2}$

$2^4 = 16$

$\sqrt[4]{2^4} = \sqrt[4]{16} = 2$

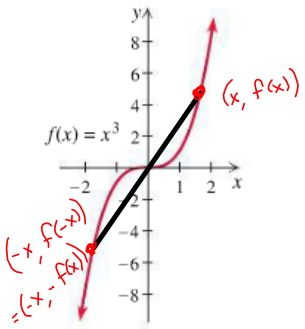
Numerical justification for $\sqrt[4]{x} > \sqrt{x}$ when $0 < x < 1$

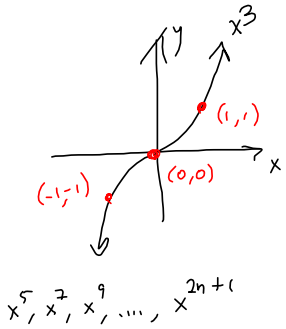
$16 = x \quad \sqrt[4]{x} = \sqrt[4]{16} = 2$

$\sqrt{x} = \sqrt{16} = 4$

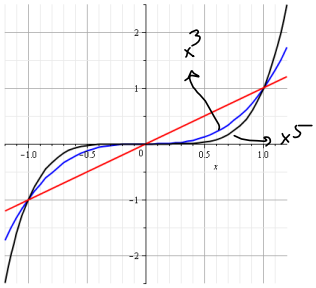
$\sqrt{x} > \sqrt[4]{x}$ when $x > 1$

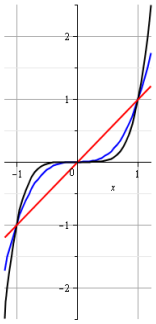
Cube Function





$x^5, x^7, x^9, \ldots, x^{2n+1}$





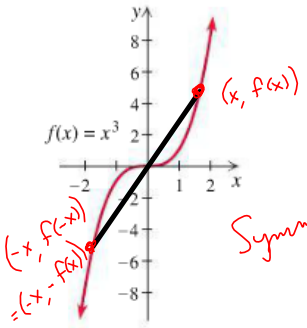
Domain $(-\infty, \infty)$
Range $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
Symmetric about origin

x	$x^3 = y$
0	0
1	1
2	8
-2	-8
-1	-1

ODD

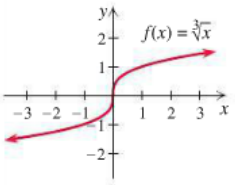
$f(-x) = -f(x)$

Cube Function

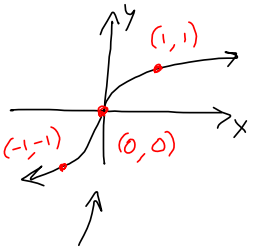


Domain $(-\infty, \infty)$
Range $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
Symmetric about origin

Cube-Root Function

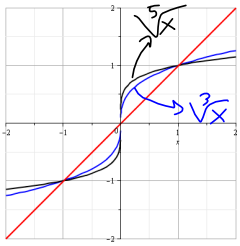


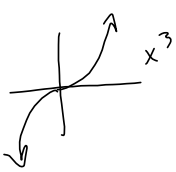
Domain $(-\infty, \infty)$
Range $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
Symmetric about origin

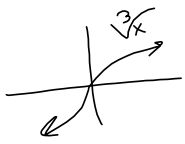


$\sqrt[5]{x}$, $\sqrt[7]{x}$, ..., $\sqrt[2n+1]{x}$

x	$\sqrt[3]{x}$
0	0
1	1
8	2
27	3
−1	−1
−8	−2
−27	−3



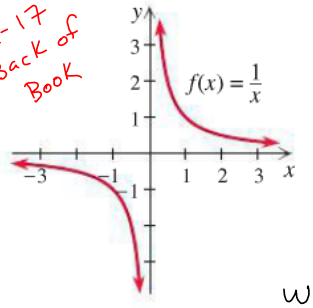


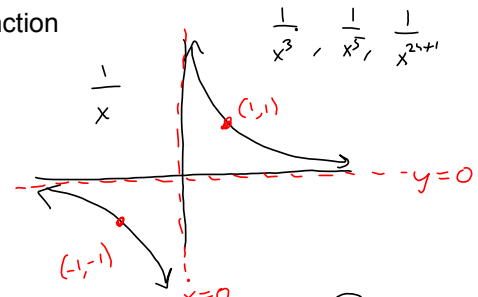


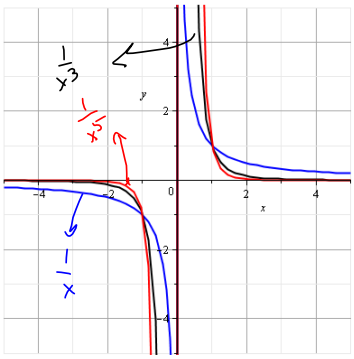
Similar, but very different.

Recipcrocal Function

I-17 Back of Book







What's happening at $x=0$?

As x approaches 0 from the right, $\frac{1}{x}$ approaches ∞.

x	y
0	∄
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$
-1	-1
-2	$-\frac{1}{2}$
-3	$-\frac{1}{3}$

Mathematically $\frac{1}{x} \to \infty$ means "Pick any BIG #. I can make $\frac{1}{x}$ bigger than that #, by letting x get sufficiently close to $x=0$ (without touching)."

$y \to \infty$.

PICK $y = 1000$. I can make $\frac{1}{x} > 1000$.

$$x = \frac{1}{1001} \Rightarrow y = \frac{1}{\left(\frac{1}{1001}\right)} = 1 \cdot \left(\frac{1001}{1}\right) = 1001$$

$x=0$ is vertical Asymptote

$y=0$ is horizontal Asymptote

Asymptote means a line the graph comes closer & closer to, without touching

$y = 1000000$

$$x = \frac{1}{1000001} \Rightarrow y = 1000001 > 1000000$$

Shorthand: $\lim_{x \to 0^+} \frac{1}{x} = \infty$

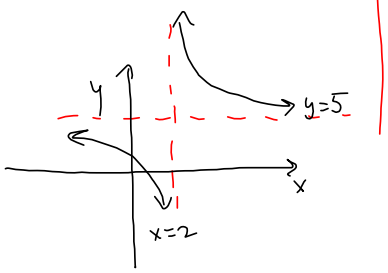
$\frac{1}{x} \xrightarrow{x \to 0^+} \infty$

FROM the LEFT

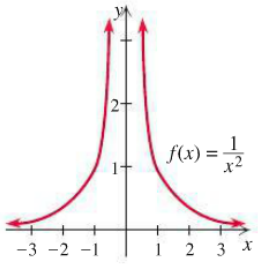
$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

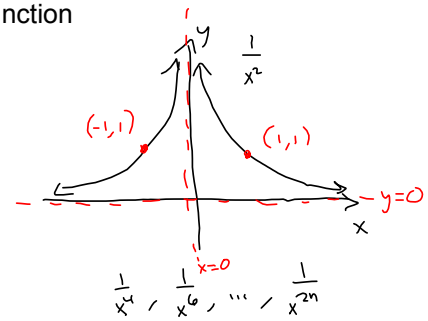
$\frac{1}{x} \xrightarrow{x \to 0^-} -\infty$

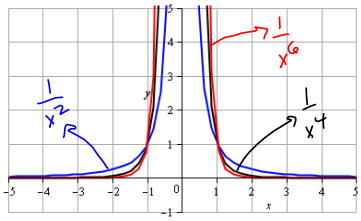
$\frac{1}{x-2} + 5 = g(x)$



Reciprocal-squared Function







Inc. $(-\infty, 0)$

Dec. $(0, \infty)$

$(0, -\infty)$ is junk

$\lim_{x \to 0} \frac{1}{x^2} = \infty$

$\frac{1}{x^2}$ $x \to 0$ $\longrightarrow \infty$

$-\infty \longleftrightarrow \infty$

$(-\infty, 7)$

$(-\infty, 25]$

$(-\infty, -10002359]$