Use the Quadratic Formula to solve the following quadratic equation:

$x^2 + x - 6 = 0$

$ax^2+bx+c = 0 \implies x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a=1, b=1, c=-6$

The b^2-4ac is the discriminant

DO IT FIRST. ALWAYS!

$b^2-4ac = 1^2-4(1)(-6) = 1+24 = 25 > 0$ 2 real zeros

$25 = 5^2$ is perfect square, tells me it factors, old-school, and the zeros are rational (No $\sqrt{7}$ or $\sqrt{3}$ in it)

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{-1 \pm \sqrt{25}}{2(1)} = \dfrac{-1 \pm 5}{2}$

$\nearrow \dfrac{-1+5}{2} = \dfrac{4}{2} = 2$

$\searrow \dfrac{-1-5}{2} = \dfrac{-6}{2} = -3$

$x \in \{-3, 2\}$

CHECK!

Suppose you're not good at factoring, but you *can* use the quadratic formula. You can use your answers to *reverse-engineer* the factored form of the quadratic expression!

Use the Quadratic Formula to solve the following quadratic equation:

$x^2 + x - 6 = 0$

$x \in \{-3, 2\}$

$x = -3 \implies (x - (-3))$ is a factor.

$x = 2 \implies (x - 2)$ is a factor.

$(x+3)(x-2)$

$x^2 + x - 6 = (x+3)(x-2) = x^2 - 2x + 3x - 6$

$= x^2 + x - 6$

See?

Sweet!

Use the Quadratic Formula to solve the following quadratic equation:

$6.2x^2 + 15.63x - 20 = 0$ $a = 6.2$, $b = 15.63$, $c = -20$

100(↑) multiply by 100 to ditch the decimals!

$620x^2 + 1563x - 2000 = 0$ $a = 620$, $b = 1563$, $c = -2000$

$b^2 - 4ac = 1563^2 - 4(620)(-2000)$

$= 7{,}402{,}969$

$\Longrightarrow$

$$x = \frac{-1563 \pm \sqrt{7402969}}{2(620)}$$

0.9337417419952291825963200250201801150690447995435566595855...

$\approx .9337$ to 4 places.

-3.4547094839307130535640619605040510828109802834145244015 2...

≈ -3.4547 $x \in \{-3.4547, .9337\}$

Use the Quadratic Formula to solve the following quadratic equation:

$ex^2 - 2wx + 27 = 0$

$a = e,\ b = -2w,\ c = 27$

$b^2 - 4ac = (-2w)^2 - 4e(27)$

$= 4w^2 - 108e$

$$\boxed{x = \frac{2w \pm \sqrt{4w^2 - 108e}}{2e}}$$

Sometimes, you'll see letters (variables) where you're expecting explicit real numbers

$(-2w)^2 = (-2w)(-2w)$

$= 4w^2$

Use Factoring to solve the following quadratic equation:

$x^2 + 7x - 18 = 0$

$x^2 + 9x - 2x - 18$

$= x(\underline{x+9}) - 2(\underline{x+9})$

$= \boxed{(x+9)(x-2) = 0}$

$x+9=0$ OR $x-2=0$

$x=-9$ OR $x=2$

$\boxed{x \in \{-9, 2\}}$

2 | 18
3 | 9
3

$AB = 0 \Rightarrow$
$A=0$ OR $B=0$

FACTORING CHEAT

SLEDGEHAMMER!

$a=1, b=7, c=-18$

$b^2 - 4ac = 7^2 - 4(1)(-18)$

$= 49 + 72 = 121 = 11^2$

$x = \frac{-7 \pm \sqrt{121}}{2(1)} = \frac{-7 \pm 11}{2}$

$\frac{-7+11}{2} = \frac{4}{2} = 2$

$\frac{-7-11}{2} = \frac{-18}{2} = -9$

are zeros, so

$(x-2)(x-(-9))$ is how it factors.

$(x-2)(x+9) = 0 \Rightarrow$

$x \in \{-9, 2\}$

Use Factoring to solve the following quadratic equation:

$56x^2 - 11x - 12 = 0$

Need factoring

Partial Fractions!

Mills Method for old-school factoring.

Magic Number is $ac = (56)(-12)$

$= -672$

$$\begin{array}{r} 56 \\ 12 \\ \hline 112 \\ 560 \\ \hline 672 \end{array}$$

Now, use #s that add up to -11 to get a product equal to -672.

	sum	product
$-11 =$	$-12+1$	-12
	$= -13+2$	-26
	$= -20+9$	-180
	$= -21+10$	-210
	$= -30+19$	-570 "under"
	$= -35+24$	-840 "over"
	$= -33+22$	
	$= -34+23$	
	$= -32+21$	$= -672$ Sweet!

$56x^2 - 11x - 12 =$

$= 56x^2 - 32x + 21x - 12$

$= 8x(7x-4) + 3(7x-4)$

$= (7x-4)(8x+3) = 0$

$7x - 4 = 0$ $\quad 8x + 3 = 0$

$7x = 4$ or $8x = -3$

$x = \frac{4}{7}$ or $x = -\frac{3}{8}$

$$\boxed{x \in \left\{-\frac{3}{8}, \frac{4}{7}\right\}}$$

THE CHEAT!

$\boxed{x \in \left\{-\frac{3}{8}, \frac{4}{7}\right\}}$ from Quadratic Formula got it for us, say.

Reverse-Engineer the factorization

$56\left(x + \frac{3}{8}\right)\left(x - \frac{4}{7}\right)$

$= (8)(7)\left(x + \frac{3}{8}\right)\left(x - \frac{4}{7}\right)$

$= 8\left(x + \frac{3}{8}\right)(7)\left(x - \frac{4}{7}\right)$

$$\begin{array}{r} 24 \\ -35 \\ \hline \end{array}$$

$= (8x+3)(7x-4)$ Sweet!

$= 0$, etc.

$\rightarrow \boxed{x \in \left\{-\frac{3}{8}, \frac{4}{7}\right\}}$

Special Product : Binomial Squared:

$(x+b)^2 = (x+b)(x+b) = x^2 + bx + bx + b^2 = x^2 + 2bx + b^2$

$(x-b)^2 = x^2 - 2bx + b^2$

Use Completing the Square to solve the following quadratic equation:

$x^2 + 4x - 7 = 0$

$x^2 + 4x = x^2 + 2bx$

$4x = 2bx$ FIND b!

$\frac{4x}{2x} = \frac{2bx}{2x}$ Use b^2!

$2 = b$

$4 = 2^2 = b^2$

Square Root of the square is the ABSOLUTE VALUE of the principle square root

$\sqrt{3^2} = 3$

$\sqrt{(-3)^2} = \sqrt{9} = 3$

$\sqrt{x^2}$ Behaves like $|x|$

$|-3| = 3$

$|3| = 3$

Short way:

$x^2 + 4x - 7 = 0$

$\frac{4}{2} = 2 \rightarrow 2^2 = b^2$

continuing: ...

$x^2 + 4x + 2^2 - 7 = 0 + 4$

$(x+2)^2 - 7 = 4$

$\boxed{(x+2)^2 = 11}$

$\sqrt{(x+2)^2} = \sqrt{11}$

$|x+2| = \sqrt{11}$

$x + 2 = \sqrt{11}$ OR $x + 2 = -\sqrt{11}$

compresses to

Should be able to go straight to

$\boxed{x + 2 = \pm\sqrt{11}}$

$\Rightarrow x = -2 \pm \sqrt{11}$

$x \in \{-2 \pm \sqrt{11}\}$

THIS SAYS

$x^2 + 4x - 7 = (x - (2+\sqrt{11}))(x - (2-\sqrt{11}))$

Use Completing the Square to solve the following quadratic equation:

$x^2 - x - 6 = 0$ $(x-3)(x+2)=0$

$x^2 - x + \left(\frac{1}{2}\right)^2 = 6 + \frac{1}{4} = \frac{25}{4}$

$\frac{1}{2} \rightsquigarrow \left(\frac{1}{2}\right)^2$

$\left(x-\frac{1}{2}\right)^2 = \frac{25}{4}$

$x - \frac{1}{2} = \pm\sqrt{\frac{25}{4}}$

$x = \frac{1}{2} \pm \frac{5}{2}$

$\frac{1+5}{2} = \frac{6}{2} = 3$ $\frac{1-5}{2} = \frac{-4}{2} = -2$

$\sqrt{x^2} = |x|$ is in there.

$\sqrt{\frac{25}{4}} = \frac{\sqrt{25}}{\sqrt{4}} = \frac{5}{2}$

$\boxed{x \in \{-2, 3\}}$

Completing the square cheat.

$ax^2 + bx + c$

$= a\left(x^2 + \frac{b}{a}x \qquad\right) + c$

$= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + c - a\left(\frac{b^2}{4a^2}\right)$

$= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$

$= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$

OR

$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$

Use Completing the Square to solve the following quadratic equation:

$3x^2 + 5x - 3 = 0$

$$3x^2 + 5x = 3$$

$$3\left(x^2 + \frac{5}{3}x + \left(\frac{5}{6}\right)^2\right) = 3 + 3\left(\frac{25}{36}\right)$$

$$\frac{5}{6} \rightarrow \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

$$3\left(x + \frac{5}{6}\right)^2 = 3 + \frac{3(25)}{36}$$

$$\left(x + \frac{5}{6}\right)^2 = \frac{3}{3} + \frac{3(25)}{3(36)} = 1 + \frac{25}{36} = \frac{36+25}{36} = \frac{61}{36}$$

$$x + \frac{5}{6} = \pm\sqrt{\frac{61}{36}} = \pm\frac{\sqrt{61}}{6}$$

$$x = \frac{-5 \pm \sqrt{61}}{6}$$

$$x \in \left\{\frac{-5 \pm \sqrt{61}}{6}\right\}$$

CHEAT for Completing the square

Use Completing the Square to solve the following quadratic equation:

$3x^2 + 5x - 3 = 0$

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + c + \text{Stuff}$$

$a = 3,\ b = 5,\ c = -3$

Now, expand to see what was added

$$3x^2 + 5x + ? = 3\left(x + \frac{5}{6}\right)^2 = 3\left(x^2 + \frac{5}{3}x + \frac{25}{36}\right) = 3x^2 + 5x + \frac{25}{12}$$

$$\frac{b}{2a} = \frac{5}{2(3)} = \frac{5}{6}$$

So add this to the OTHER side!

$$\text{So} \quad 3x^2 + 5x - 3 = 0$$

$$3x^2 + 5x = 3$$

$$3\left(x + \frac{5}{6}\right)^2 = 3 + \frac{25}{12} = \frac{36 + 25}{12} = \frac{61}{12}$$

$$3\left(x + \frac{5}{6}\right)^2 = \frac{61}{12}$$

Hard part done

Now solve

$$\left(x + \frac{5}{6}\right)^2 = \frac{61}{36}$$

$$x + \frac{5}{6} = \pm\sqrt{\frac{61}{36}} = \frac{\pm\sqrt{61}}{6}$$

$$x = \frac{-5 \pm \sqrt{61}}{6}$$