

121 WRITING PROJECT #4

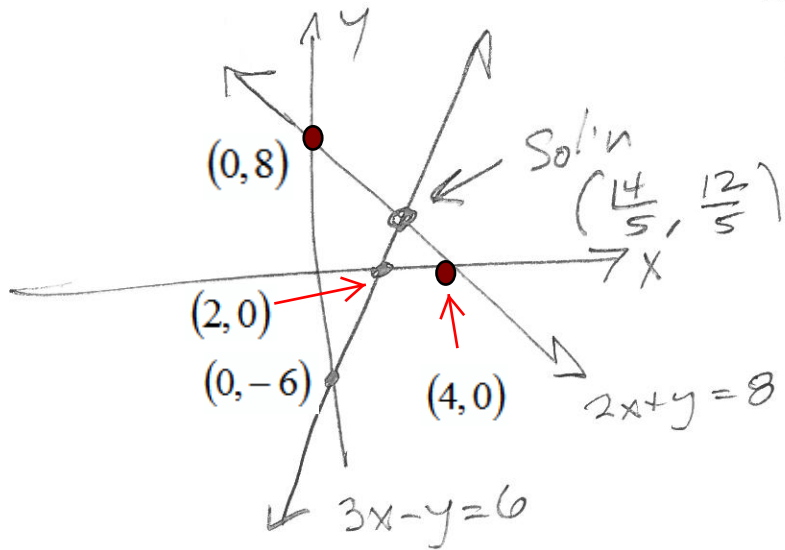
KEY

(1) a (10 pts)

$$\begin{aligned} 3x - y &= 6 \\ 2x + y &= 8 \end{aligned}$$

$$\begin{array}{c|c} x & y \\ \hline 0 & -6 \\ 2 & 0 \end{array}$$

$$\begin{array}{c|c} x & y \\ \hline 0 & 8 \\ 4 & 0 \end{array}$$



I supplied the exact coordinates after I solved, algebraically. The rough sketch got the location generally correct.

b (10 pts)

$$\begin{aligned} 3x - y &= 6 \quad (4, 0) \Rightarrow y = 3x - 6 \\ 2x + y &= 8 \end{aligned}$$

$$\Rightarrow 2x + (3x - 6) = 8$$

$$2x + 3x - 6 = 8$$

$$5x = 14$$

$$x = \frac{14}{5}$$

$$\begin{aligned} y &= 3\left(\frac{14}{5}\right) - 6 \\ &= \frac{42}{5} - \frac{30}{5} = \frac{12}{5} \end{aligned}$$

$$(x, y) = \left(\frac{14}{5}, \frac{12}{5}\right)$$

$$y = \frac{12}{5}$$

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① ② 10 pts

$$3x - y = 6$$

$$2x + y = 8$$

$$-2R1 \quad -6x + 2y = -12$$

$$3R2 \quad 6x + 3y = 24$$

$$5y = 12$$

$$y = \frac{12}{5}$$

$$(x, y) = \left(\frac{14}{5}, \frac{12}{5} \right)$$

$$3x - \frac{12}{5} = 6$$

$$\Rightarrow 3x = \frac{12 + 30}{5}$$

$$3x = \frac{42}{5}$$

$$x = \frac{14}{5}$$

2 10pts

$$3x + 7y - z = -6$$

$$x + 2y = -1$$

$$4x + 10y - 3z = -11$$

$$\Rightarrow \begin{aligned} x + 2y &= -1 \\ 3x + 7y - z &= -6 \\ 4x + 10y - 3z &= -11 \end{aligned}$$

$$\begin{array}{r} -3R1 \\ R2 \end{array} \quad \begin{array}{l} -3x - 6y = 3 \\ 3x + 7y - z = -6 \end{array}$$

$$R2 \quad 3x + 7y - z = -6$$

$$\begin{array}{r} -3R1 + R2 \\ \hline \end{array} \quad y - z = -3$$

$$x + 2y = -1$$

$$y - z = -3$$

$$2y - 3z = -7$$

$$\begin{array}{r} -2R2 \\ R3 \end{array} \quad \begin{array}{l} -2y + 2z = 6 \\ 2y - 3z = -7 \end{array}$$

$$R3 \quad 2y - 3z = -7$$

$$-z = -1$$

$$z = 1$$

$$y - 1 = -3$$

$$y = -2$$

$$x + 2(-2) = -1$$

$$x - 4 = -1$$

$$\begin{array}{r} -4R1 \\ R3 \end{array} \quad \begin{array}{l} -4x - 8y = 4 \\ 4x + 10y - 3z = -11 \end{array}$$

$$R3 \quad 4x + 10y - 3z = -11$$

$$2y - 3z = -7$$

$$(x, y, z) = (3, -2, 1)$$

OR write

$(x, y, z) \in \{(3, -2, 1)\}$ ←
set containing one point

$$x = 3$$

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$$\begin{aligned} \textcircled{3} \quad & x + 3y - 2z = 12 \\ & 3x + 11y - 5z = 34 \\ & 2x + 8y - 3z = 22 \end{aligned}$$

$$\begin{array}{r} \textcircled{2} \\ -3R1 \\ R2 \end{array} \begin{array}{l} -3x - 9y + 6z = -36 \\ 3x + 11y - 5z = 34 \end{array}$$

$$\begin{array}{r} -2R1 \\ R3 \end{array} \begin{array}{l} -2x - 6y + 4z = -24 \\ 2x + 8y - 3z = 22 \end{array}$$

$$\begin{array}{r} -3R1+R2 \\ -2R1+R3 \end{array} \quad \begin{array}{l} 2y + z = -2 \\ 2y + z = -2 \end{array}$$

$$\begin{array}{r} -2R1+R3 \\ \hline \end{array} \quad \begin{array}{l} 2y + z = -2 \end{array}$$

New system:

$$x + 3y - 2z = 12$$

$$2y + z = -2$$

$$\boxed{2y + z = -2} \leftarrow \text{REDUNDANT}$$

To see this:

$$\begin{array}{r} -R2 \\ R3 \end{array} \begin{array}{l} -2y - z = 2 \\ 2y + z = -2 \end{array}$$

$$\begin{array}{r} -R2+R3 \\ \hline \end{array} \quad 0 = 0$$

So the new system is

$$x + 3y - 2z = 12$$

$$2y + z = -2$$

$$2y + z = -2$$

$$2y = -z - 2$$

$$y = \frac{-z-2}{2}$$

$$\boxed{-\frac{z+2}{2} = y} \rightarrow$$

$$x + 3\left(-\frac{z+2}{2}\right) - 2z = 12$$

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$$\rightarrow x - \left(\frac{3(z+2)}{2} \right) - 2z = 12$$

$$x - \left[\frac{3z+6}{2} \right] - 2z = 12$$

$$x - \left[\frac{3z}{2} + \frac{6}{2} \right] - 2z = 12$$

$$x - \frac{3}{2}z - 3 - 2z = 12$$

$$x - \frac{7}{2}z = 15$$

$$x = \frac{7}{2}z + 15$$

$$\rightarrow \left\{ (x, y, z) \in \left(\left(\frac{3}{2}z + 15, -\frac{z+2}{2}, z \right) \mid z \in \mathbb{R} \right) \right\}$$

$-\left(\frac{z+2}{2}\right) = -\left(\frac{z}{2} + \frac{2}{2}\right) = -\frac{1}{2}z - 1$ is another way to write y .

$$\left\{ \left(\frac{3}{2}z + 15, -\frac{1}{2}z - 1, z \right) \mid z \in \mathbb{R} \right\}$$

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3b

10pts

$$z=0 \Rightarrow (x, y, z) = \left(\frac{3}{2}(0) + 15, -\frac{1}{2}(0) - 1, 0 \right)$$

$$= \boxed{(15, -1, 0)}$$

$$z=1 \Rightarrow (x, y, z) = \left(\frac{3}{2}(1) + 15, -\frac{1}{2}(1) - 1, 1 \right)$$

$$= \left(\frac{3}{2} + \frac{30}{2}, -\frac{1}{2} - \frac{2}{2}, 1 \right)$$

$$= \boxed{\left(\frac{33}{2}, -\frac{3}{2}, 1 \right)}$$

$$z=-1 \Rightarrow (x, y, z) = \left(\frac{3}{2}(-1) + 15, (-\frac{1}{2})(-1) - 1, -1 \right)$$

$$= \left(-\frac{3}{2} + \frac{30}{2}, \frac{1}{2} - \frac{2}{2}, -1 \right)$$

$$= \boxed{\left(\frac{27}{2}, -\frac{1}{2}, -1 \right)}$$

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(4)

$$x + 3y - 2z = 12$$

$$3x + 11y - 5z = 34$$

$$2x + 8y - 3z = 23$$

$$\begin{array}{r} -3R1 \\ R2 \end{array} \quad \begin{array}{l} -3x - 9y + 6z = -36 \\ 3x + 11y - 5z = 34 \end{array}$$

$$\hline 2y + z = -2$$

$$2y + z = -2$$

NEW SYSTEM:

$$x + 3y - 2z = 12$$

$$2y + z = -2$$

$$2y + z = -1$$

$$\begin{array}{r} -R2 \\ R3 \end{array} \quad \begin{array}{l} -2y - z = 2 \\ 2y + z = -1 \end{array}$$

$$\hline 0 = 1 \text{ ? ABSURD!}$$

0 = 1 ? ABSURD!

there is no solution.