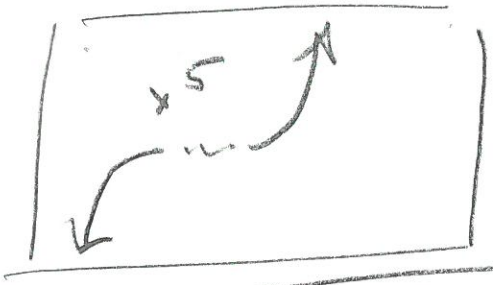
Writing Project #3 — Solns

(1) $f(x) = x^5 - 10x^4 + 231x^2 - 612x + 540$



2pts

(2) 4, 2, or 0 positive zeros

$f(-x) = -x^5 - 10x^4 + 231x^2 + 612x + 540$

1 negative zero

2pts

(3) $2 \mid 540$ $\quad a_n = 1$, $a_0 = 540$ & we're looking for $\frac{p}{q}$'s whose p is a factor of 540 & whose q is a factor of 1. So we're only looking for integers.

$2 \mid 270$

$3 \mid 135$

$3 \mid 45$

$3 \mid 15$

5

2pts

$\pm 5, \pm 1, \pm 2, \pm 4, \pm 6, \pm 12, \pm 18, \pm 36, \pm 54, \pm 108, \pm 10,$

$\pm 20, \pm 15, \pm 45, \pm 135, \pm 90, \pm 60, \pm 180, \pm 540$

Did I miss any? 270,

(4)

2pts

(6)

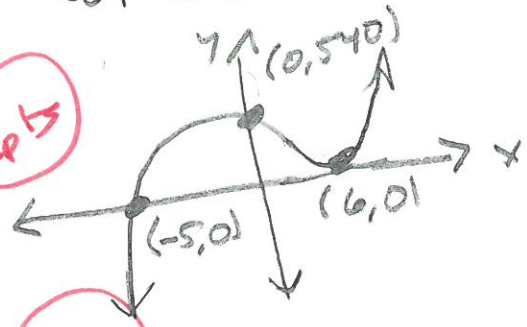
-5	1	-10	0	231	-612	540
		-5	75	-375	720	-540
	1	-15	75	-144	108	0
		6	-54	126	-108	
6	1	-9	21	-18	0	
		6	-18	18		
	1	-3	3	0		

(5) $(x+5)(x-6)^2(x^2-3x+3)$

$b^2-4ac = 9-4(3) = -3 < 0$ irreducible

12) WP #3

(6) 2pts



(7) 2pts $x^2-3x+3 \Rightarrow b^2-4ac = -3$ by #4 eds

$\Rightarrow x = \frac{3 \pm \sqrt{-3}}{2} = \frac{3 \pm i\sqrt{3}}{2} = x$

$f(x) = (x-6)^2(x+5)\left(x - \frac{3+i\sqrt{3}}{2}\right)\left(x - \frac{3-i\sqrt{3}}{2}\right)$

(8) 5pts $R(x) = \frac{6x^2+5x-14}{x^2+3x-10} = \frac{6x^2+12x-7x-14}{(x+5)(x-2)} =$

214
6
84 $(12)(-7)$, $12-7$

$= \frac{6x(x+2)-7(x+2)}{(x+5)(x-2)} = \frac{(x+2)(6x-7)}{(x+5)(x-2)}$

Domain: $\{x \mid x \neq -5, 2\}$

V.A.: $x = -5, x = 2$

H.A.: $\frac{6x^2}{x^2} = 6 \rightsquigarrow \boxed{y = 6}$

x-int: $\left(\frac{7}{6}, 0\right), (-2, 0)$

$\frac{6x^2+5x-14}{x^2+3x-10} = 6$

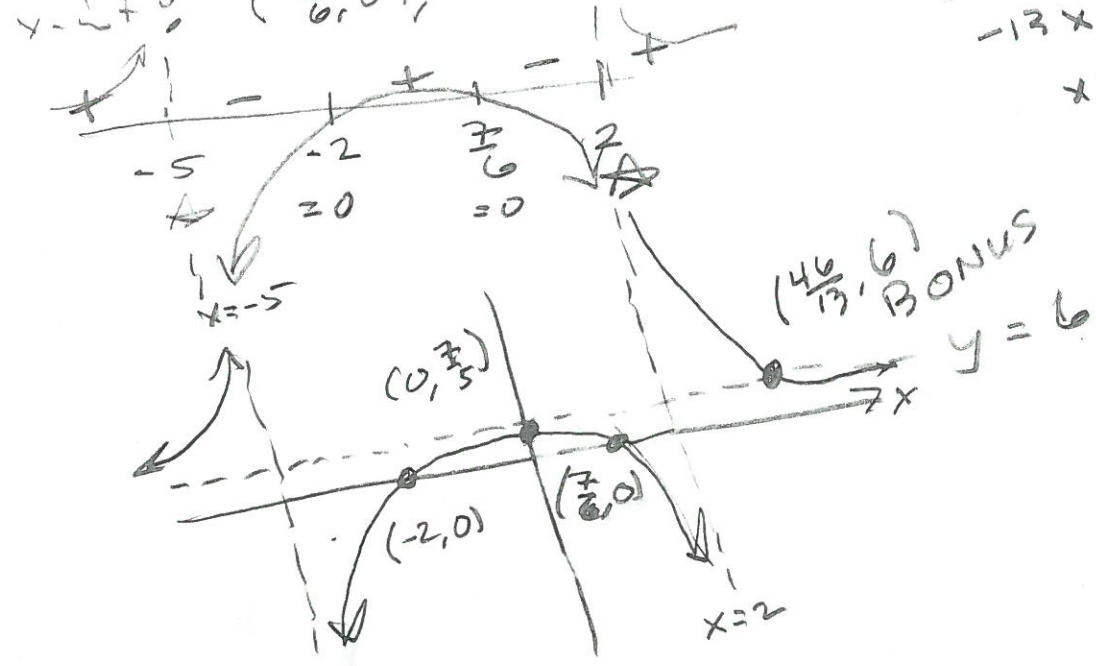
$6x^2+5x-14 = 6(x^2+3x-10)$

$R(x) = 6 \quad 6x^2+5x-14 = 6x^2+18x-60$

$5x-14 = 18x-60$

$-13x = -46$

$x = \frac{46}{13}$



121 WP #3

9) $Q(x) = \dfrac{6x^3+47x^2+21x-98}{x^3+10x^2+11x-70}$

We know $x=2$ & $x=-5$ are zeros of the denominator, so

$$\begin{array}{r|rrr} -5 & 1 & 10 & 11 & -70 \\ & & -5 & -25 & 70 \\ \hline & 1 & 5 & -14 & 0 \end{array}$$

$x^2+5x-14 = (x+7)(x-2)$

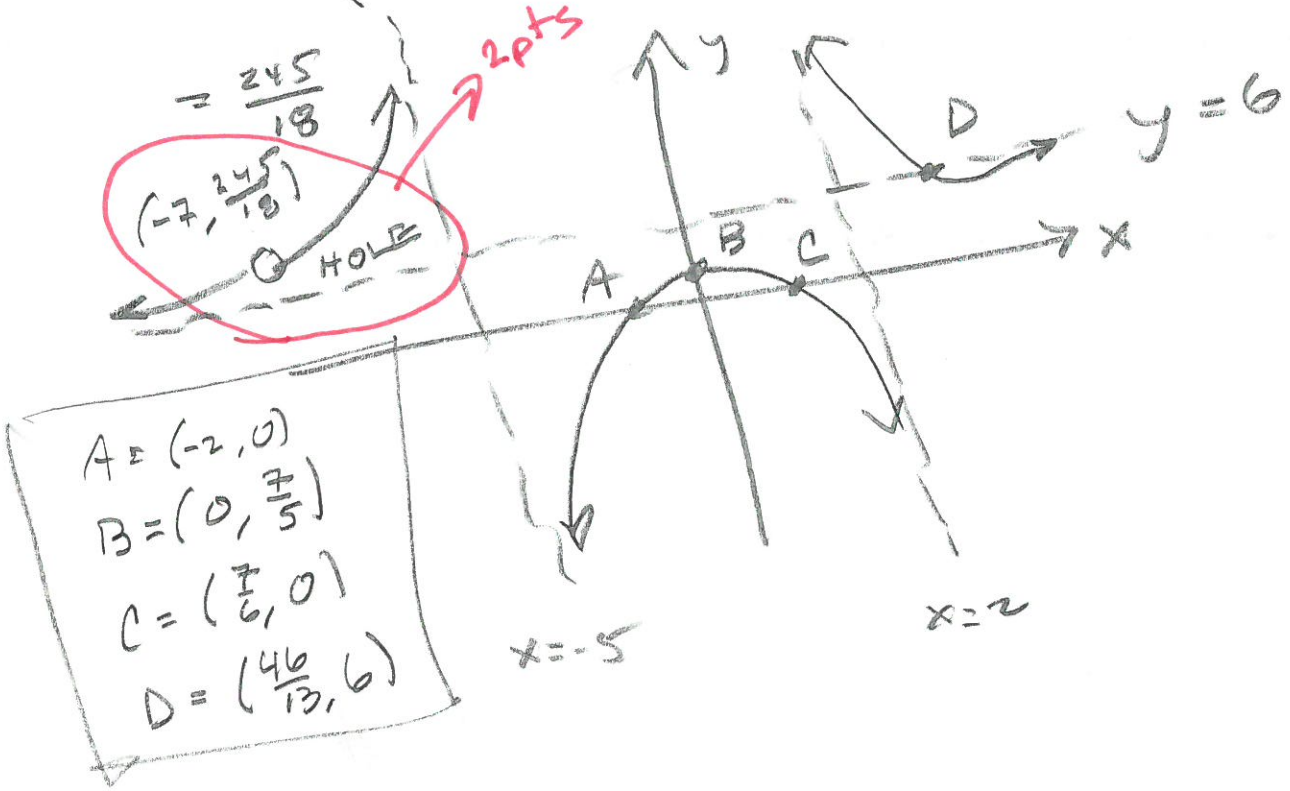
This gives $\dfrac{\sim\sim}{(x-2)(x+5)(x+7)}$ so $x+7$ is the added factor & $x=-7$ is where the hole is @

$Q(x) = \dfrac{(x+7)(x+2)(6x-7)}{(x+2)(x-2)(x+5)}$

Same graph. Hole @ $(-7, R(-7))$

$R(-7) = \dfrac{(-7+2)(6(-7)-7)}{(-7-2)(-7+5)} = \dfrac{(-5)(-42-7)}{(-9)(-2)} = \dfrac{-5(-49)}{18}$

$= \frac{245}{18}$



A = (-2, 0)
B = (0, 7/5)
C = (7/6, 0)
D = (46/13, 6)

121 WP#3

(10) 5pts $\dfrac{6x^3+47x^2+21x-98}{x^2+3x-10} = \dfrac{(6x-7)(x+2)(x+7)}{(x+5)(x-2)}$

by previous work. Same vertical asymptotes with oblique asymptote.

$$x^2+3x-10 \;\overline{)\;6x^3+47x^2+21x-98}$$

$6x$ (quotient)

$-(6x^3+18x^2-60x)$

$29x^2$

$+29$ (quotient)

$-(29x^2+87x-290)$

$-66x+192$

$y = 6x+29$

x	y
0	29
$-\frac{29}{6}$	0

$6x+29=0$

$6x=-29$

$x=-\frac{29}{6}$

-4.8

$6\overline{)-29.000}$

24

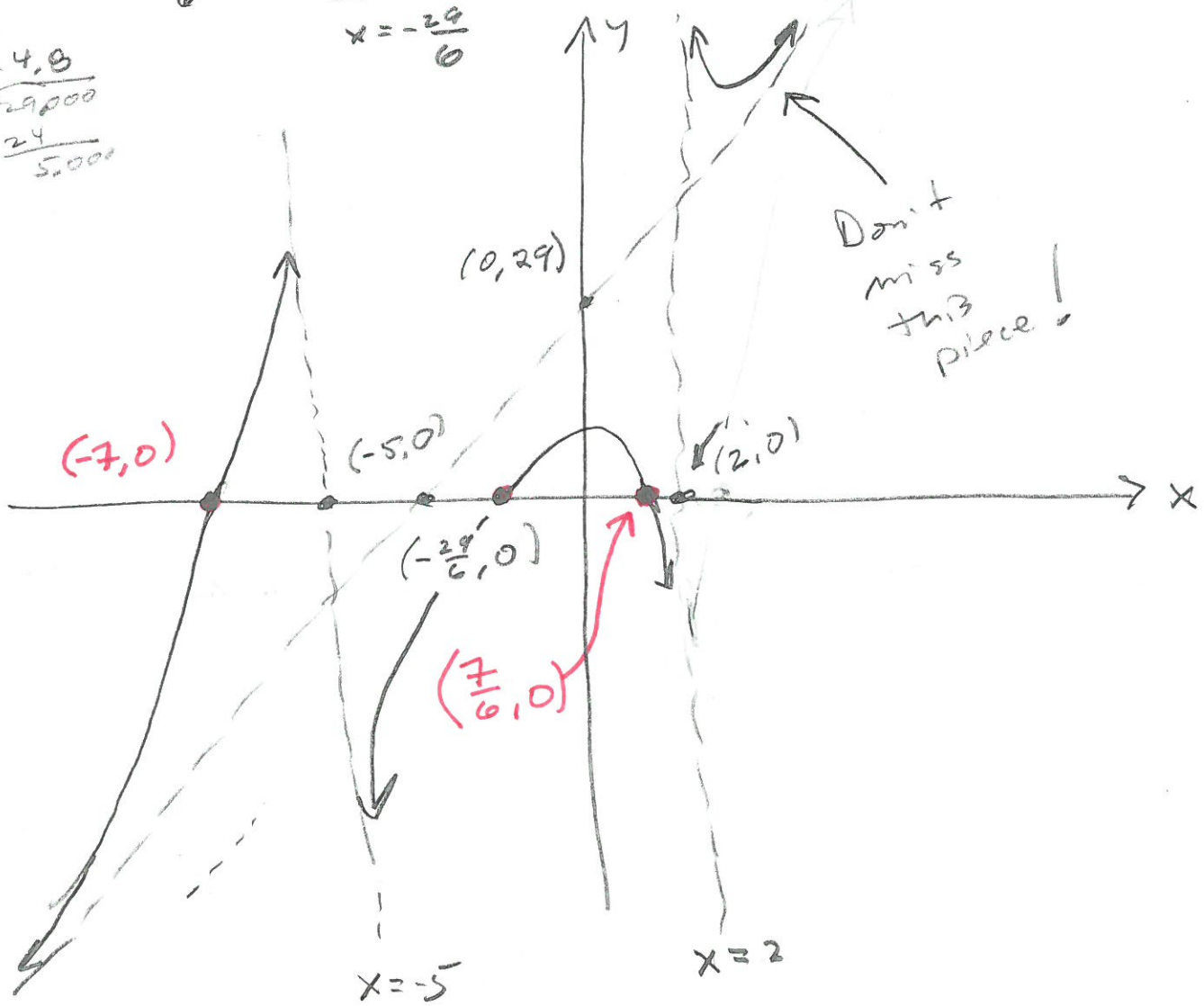
5.00

$-$ $+$ $-$ $+$ $-$ $+$

-7 -5 -2 $\frac{7}{6}$ 2

$=0$ ★ $=0$ $=0$ ★

y-int: $(0, \frac{49}{5})$



12' WP #3

(11) (2pts) $w(x) = \sqrt{(x-3)(x+5)^2(x-7)(x+3)}$

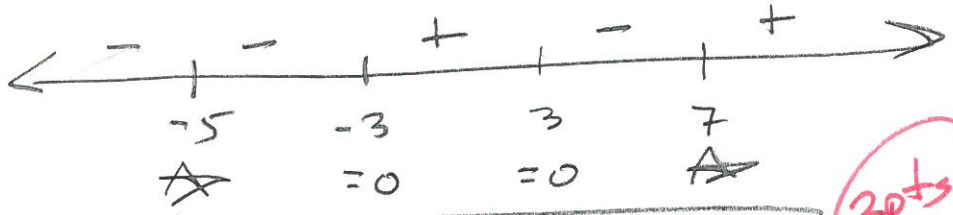
2pts

→ $\mathcal{D}(w) = \{-5\} \cup [-3, 3] \cup [7, \infty)$

1

(12) (2pts) $k(x) = \sqrt{\dfrac{(x-3)(x+3)}{(x+5)^2(x-7)}}$

Same Sign Pattern, but 2 points thrown out



2pts

→ $\mathcal{D}(k) = [-3, 3] \cup (7, \infty)$

(Threw out $x = -5$ & $x = 7$)