

1 a (10 pts)

$$3x + 2y = 12$$

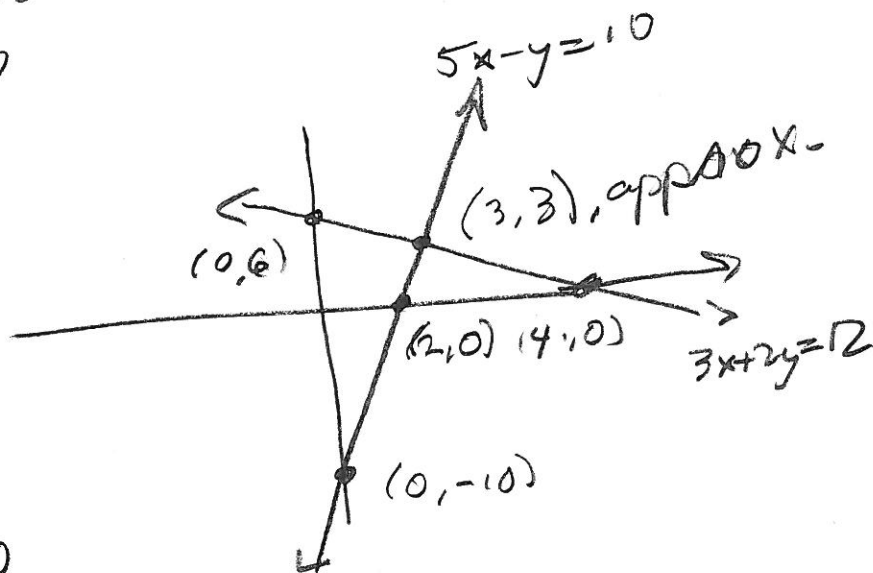
$$5x - y = 10$$

$$3x + 2y = 12$$

x	y
0	6
4	0

$$5x - y = 10$$

x	y
0	-10
2	0



b (10 pts)

$$5x - y = 10$$

$$-y = -5x + 10$$

$$y = 5x - 10$$

$$\Rightarrow 3x + 2y = 3x + 2(5x - 10) = 12$$

$$\Rightarrow 3x + 10x - 20 = 12$$

$$\Rightarrow 13x = 32$$

$$\Rightarrow x = \frac{32}{13}$$

$$\Rightarrow y = 5x - 10 = 5\left(\frac{32}{13}\right) - 10$$

$$= \frac{160}{13} - \frac{10}{1} = \frac{13}{13}$$

$$= \frac{160 - 130}{13} =$$

$$\frac{30}{13} = y$$

$$\textcircled{10} \quad \begin{aligned} 3x + 2y &= 12 \\ 5x - y &= 10 \end{aligned}$$

$$-5E1 \quad -15x - 10y = -60$$

$$3E2 \quad 15x - 3y = 30$$

$$-5E1 + 3E2 \quad -13y = -30$$

$$\Rightarrow \boxed{y = \frac{30}{13}}$$

$$3x + 2y = 12$$

$$3x + 2\left(\frac{30}{13}\right) = 12$$

$$3x + \frac{60}{13} = 12$$

$$3x = -\frac{60}{13} + \frac{12}{1} \cdot \frac{13}{13} = \frac{-60 + 156}{13} = \frac{96}{13}$$

$$\Rightarrow x = \frac{96}{13} \cdot \frac{1}{3} = \boxed{\frac{32}{13} = x}$$

121

WP 44

(3)

2 (10 pts)

$$\begin{array}{r} 339 \\ 4 \\ \hline 156 \end{array}$$

$$\begin{aligned} 5x + 11y + z &= 39 \\ 4x + 9y &= 25 \\ -2x - 4y - z &= -21 \end{aligned}$$

$$\begin{array}{r} 2E1 \quad 10x + 22y + 2z = 78 \\ 5E3 \quad -10x - 20y - 5z = -105 \\ \hline 2E1 + 5E3 \quad 2y - 3z = -27 \end{array}$$

$$\begin{array}{r} -4E1 \quad -20x - 44y - 4z = -156 \\ 5E2 \quad 20x + 45y = 125 \\ \hline -4E1 + 5E2 \quad y - 4z = -31 \end{array}$$

~~$$2y - 3z = -27$$~~

$$\begin{aligned} y - 4z &= -31 \\ 2y - 3z &= -27 \end{aligned}$$

$$\begin{array}{r} -2E1 \quad -2y + 8z = 62 \\ E2 \quad 2y - 3z = -27 \\ \hline 5z = 35 \\ \boxed{z = 7} \end{array}$$

$(13, -3, 7) = (x, y, z)$

$$\begin{aligned} y - 4z &= -31 \\ y = 4z - 31 &= 4(7) - 31 = 28 - 31 = -3 = y \\ \boxed{-3 = y} \end{aligned}$$

$$\begin{aligned} 5x + 11y + z &= 39 \\ 5x &= -11y - z + 39 = -11(-3) - 7 + 39 \\ 5x &= 33 - 7 + 39 = 72 - 7 = 65 \\ \Rightarrow x &= \frac{65}{5} = 13 = x \\ \boxed{13 = x} \end{aligned}$$

$$5x + 11y + z = 39 \quad E1$$

$$4x + 9y = 25 \quad E2$$

$$-2x - 4y - z = -21 \quad E3$$

$$E1 \quad 5x + 11y + z = 39$$

$$= 25$$

$$E2 \quad 4x + 9y$$

$$= 18$$

$$E1 + E3 \quad 3x + 7y$$

$$-3E2 \quad -12x - 27y = -75$$

$$4E3 \quad 12x + 28y = 72$$

$$-3E2 + 4E3$$

$$y = -3$$

So $3x + 7y = 3x + 7(-3) = 18$, from $E3$

$$3x - 21 = 18$$

$$3x = 39$$

$$x = 13$$

Finally $5x + 11y + z = 5(13) + 11(-3) + z = 39$ $E1$

$$\Rightarrow 65 - 33 + z = 39$$

$$32 + z = 39$$

$$z = 7$$

$$(x, y, z) = (13, -3, 7)$$

$$\text{or } (x, y, z) \in \{(13, -3, 7)\}$$

I'd prefer
you be
 $E1$ systematic
 $E2$ than be
clever.

$E3$ This is

the
clever
solution.

③ $x + 2y + 3z = 5$
 $2x + 7y + 8z = 6$

② $2x + y + 4z = 14$
 $-2E1 \quad -2x - 4y - 6z = -10$
 $E2 \quad 2x + 7y + 8z = 6$

$3y + 2z = -4$

$-2E1 \quad -2x - 4y - 6z = -10$
 $E3 \quad 2x + y + 4z = 14$

$-3y - 2z = 4$

New System:
 $x + 2y + 3z = 5$
 $3y + 2z = -4$
 $-3y - 2z = 4$

$E2$
 $+ E3$

$0 = 0$

$\Rightarrow 3y = -2z - 4$
 $\Rightarrow y = \frac{-2z - 4}{3}$

$x + 2y + 3z = 5$
 $x + 2\left(\frac{-2z - 4}{3}\right) + 3z = 5$
 $x + \frac{-4z - 8}{3} + \frac{3z}{1} \cdot \frac{3}{3} = 5$
 $x + \frac{-4z - 8 + 9z}{3} = 5$
 $x + \frac{5z - 8}{3} = \frac{5}{1} \cdot \frac{3}{3}$
 $x = \frac{-5z + 8}{3} + \frac{15}{3}$

$z = \text{ANY}$

$x = \frac{-5z + 23}{3}$

3b) $z=0 \rightarrow$

$$x = \frac{-5(0) + 23}{3} = \frac{23}{3} = x$$

$$y = \frac{-2(0) - 4}{3} = \frac{-4}{3} = y$$

$z=1 \rightarrow$

$$x = \frac{-5(1) + 23}{3} = \frac{23-5}{3} = \frac{18}{3} = x = 6$$

$$y = \frac{-2(1) - 4}{3} = \frac{-6}{3} = -2 = y$$

$z=-1 \rightarrow$

$$x = \frac{-5(-1) + 23}{3} = \frac{5+23}{3} = \frac{28}{3} = x$$

$$y = \frac{-2(-1) - 4}{3} = \frac{2-4}{3} = \frac{-2}{3} = y$$



There are 3 particular solutions:

$$\left(\frac{23}{3}, -\frac{4}{3}, 0\right), (6, -2, 1), \left(\frac{28}{3}, -\frac{2}{3}, -1\right)$$

(4)

$$x + 2y + 3z = 5$$

$$2x + 7y + 8z = 6$$

$$2x + y + 4z = 10$$

$$-2E1 \quad -2x - 4y - 6z = -10$$

$$E2 \quad 2x + 7y + 8z = 6$$

$$-2E2 \quad -2x - 4y - 6z = -10$$

$$E3 \quad 2x + y + 4z = 10$$

$$-3y - 2z = 0$$

$$3y + 2z = -4$$

New System:

$$x + 2y + 3z = 5$$

$$3y + 2z = -4$$

$$-3y - 2z = 0$$

~~E3~~
E2
E1

$$0 = -4 ? !$$

Absurd!

∴ No solution (assuming my arithmetic is correct.)

(Therefore, there is not a solution, assuming my arithmetic is correct.)

LEFT-TO-RIGHT
TOP-TO-BOTTOM

(I) $ax + by + cz = e$
 $fx + gy + hz = i$
 $jx + ky + lz = m$

(II) $ax + by + cz = e$
 $ny + oz = p$
 $qy + rz = s$

(III) $ax + by + cz = e$
 $ny + oz = p$
 $tz = u$

So, $z = \frac{u}{t}$, etc.