121 WA#3 Fall, 2019

$f(x) = 25x^5 + 10x^4 - 66x^3 - 80x^2 - 47x - 18$

(1) $25x^5$ ↙ ↗ 2pts End Behavior.

(2) Descartes' Rule

$25x^5 + 10\underbrace{x^4 - 66}_{1}x^3 - 80x^2 - 47x - 18$

Exactly one sign change ⟹ 1 positive root.

$f(-x) = -25\underbrace{x^5 + 10}_{1}x^4 \underbrace{+66x^3 - 80}_{2}\underbrace{x^2 + 47}_{3}\underbrace{x - 18}_{4}$

(from $25(-x)^5 + 10(-x)^4 - 66(-x)^3 - 80(-x)^2 - 47(-x) - 18$)

2pts

4 sign changes ⟹

4, 2, or 0 negative roots.

(3) Rational zeros:

2 | 18 5 | 25
3 | 9 5
3

$p = -18$
$q = 25$

⟹ if $\frac{p}{q}$ is a root, then

p is a factor of 18: 1, 2, 3, 2·3, 3·3, 2·3·3
1, 2, 3, 6, 9, 18

q is a factor of 25:
1, 5, 25

3 cntd:

2pts

$\pm 1, \pm \frac{1}{5}, \pm \frac{1}{25}, \pm 2, \pm \frac{2}{5}, \pm \frac{2}{25},$

$\pm 3, \pm \frac{3}{5}, \pm \frac{3}{25}, \pm 6, \pm \frac{6}{5}, \pm \frac{6}{25},$

$\pm 9, \pm \frac{9}{5}, \pm \frac{9}{25}, \pm 18, \pm \frac{18}{5}, \pm \frac{18}{25}$

18 of 'em, by my count.

4 2pts Beg in search. Positives, because **one** is guaranteed.

1	25	10	−66	−80	−47	−18
		25	35	−31	111	53
	25	35	−31	111	53	No

2	25	10	−66	−80	−47	−18
		50	120	108	56	18
	25	60	54	28	9	0. sweet!

2	25	60	54	28	9
		50	220	BIG	2BIGGER
	25	110	BIG	BIGGR	Huge

So, $\boxed{x=2}$ is a root of multiplicity 1, and

$f(x) = (x-2)(25x^4 + 60x^3 + 54x^2 + 28x + 9)$

one down, 4 to go!

(4) <u>cnt'd</u> working with the depressed polynomial $25x^4+60x^3+54x^2+28x+9$ after using $x=2$ to split off a factor of $x-2$: Done making positive guesses—

$$\begin{array}{r|rrrrr}
-1 & 25 & 60 & 54 & 28 & 9 \\
 & & -25 & -35 & -19 & -9 \\
\hline
 & 25 & 35 & 19 & 9 & 0
\end{array}$$

Sweet!

$$\begin{array}{r|rrrr}
-1 & 25 & 35 & 19 & 9 \\
 & & -25 & -10 & -9 \\
\hline
 & 25 & 10 & 9 & 0
\end{array}$$

Sweet!

$$\begin{array}{r|rrr}
-1 & 25 & 10 & 9 \\
 & & -25 & 15 \\
\hline
 & 25 & -15 & 24
\end{array}$$

24 Nope

$\boxed{x=-1\ m=2}$

So we're looking at

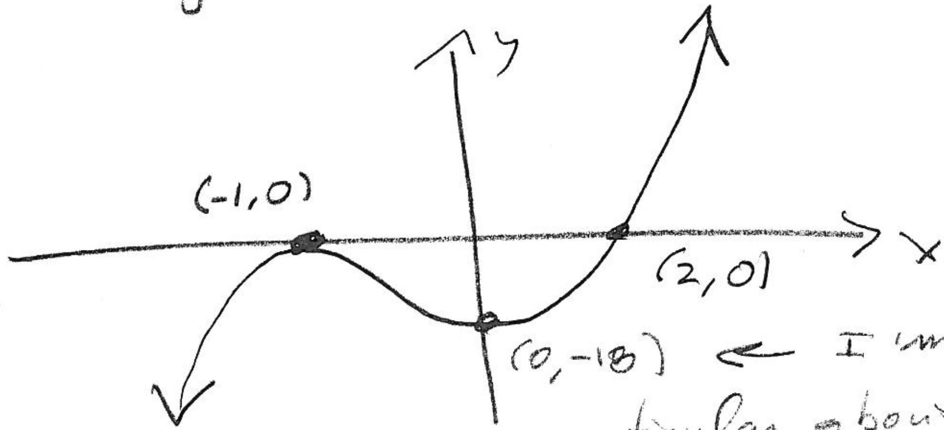
(5) $f(x)=(x-2)(x+1)^2(25x^2-15x+24)$

(2pts)

121 WP #3

(4)

(6) Rough Sketch



← I'm not too particular about exactly where the exact low point is, as long as the y-intercept is below the x-axis.

2pts

(7) We solve $25x^2+10x+9=0$ & factor $f(x)$.

M1

$a=25,\ b=10,\ c=9$ ⟶

$b^2-4ac = 10^2-4(25)(9)$

$= 100-900$

$= -800$

$$\begin{array}{r|l} 2 & 800 \\ 2 & 400 \\ 2 & 200 \\ 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ & 5 \end{array}$$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{-10 \pm \sqrt{-800}}{2(25)}$

$= \dfrac{-10 \pm 2\cdot 2\cdot 5\, i\sqrt{2}}{50}$

$= \dfrac{10(-1 \pm 2i\sqrt{2})}{50} = \boxed{\dfrac{-1 \pm 2i\sqrt{2}}{5} = x}$

M2

$25\left(x^2+\frac{10}{25}+\frac{9}{25}\right)=0$ $\frac{2}{5}$

$x^2+\frac{2}{5}x+\left(\frac{1}{5}\right)^2$

$= -\frac{9}{25}+\frac{1}{25} = \frac{-8}{25}$

$\left(x+\frac{1}{5}\right)^2 = -\frac{8}{25}$

$x+\frac{1}{5} = \pm\frac{2i\sqrt{2}}{5}$

$x = \dfrac{-1 \pm 2i\sqrt{2}}{5}$ ✓

$f(x) = 25(x+1)^2(x-2)\left(x-\frac{-1+2i\sqrt{2}}{5}\right)\left(x-\frac{-1-2i\sqrt{2}}{5}\right)$

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5

7 Ans: 2pts

$$\boxed{f(x) = 25(x+1)^2(x-2)\left(x-\left(\frac{-1+2i\sqrt{2}}{5}\right)\right)\left(x-\left(\frac{-1-2i\sqrt{2}}{5}\right)\right)}$$

Split into linear factors

8 5pts $R(x) = \dfrac{6x^2+x-15}{x^2+x-12} = \dfrac{(3x+5)(2x-3)}{(x+4)(x-3)}$

$(6)(-15) = -(2)(3)(3)(5) \;:\; -9+10 = 1$ ✓

$6x^2+10x-9x-15$

$= 2x(3x+5) - 3(3x+5) = (3x+5)(2x-3)$

$R(x) = \dfrac{(3x+5)(2x-3)}{(x+4)(x-3)}$

$D = \mathbb{R} \setminus \{-4, 3\}$

No cancellations →

V.A.: $\boxed{x=-4,\ x=3}$

y-int: $R(0) = \dfrac{-15}{-12} = +\dfrac{5}{4}$

$\boxed{\left(0, \frac{5}{4}\right)}$

x-int: $R(x)=0$

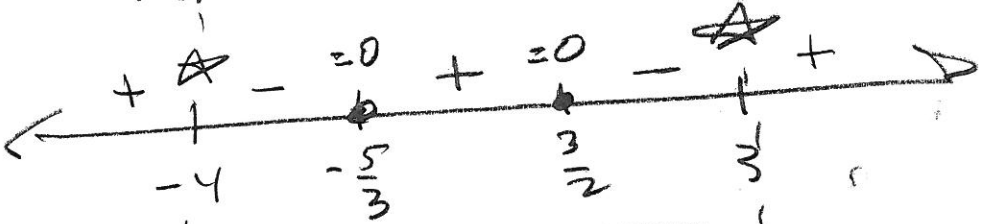
$(3x+5)(2x-3)=0$

$x = -\frac{5}{3}, \frac{3}{2} \rightarrow \boxed{\left(-\frac{5}{3}, 0\right), \left(\frac{3}{2}, 0\right)}$

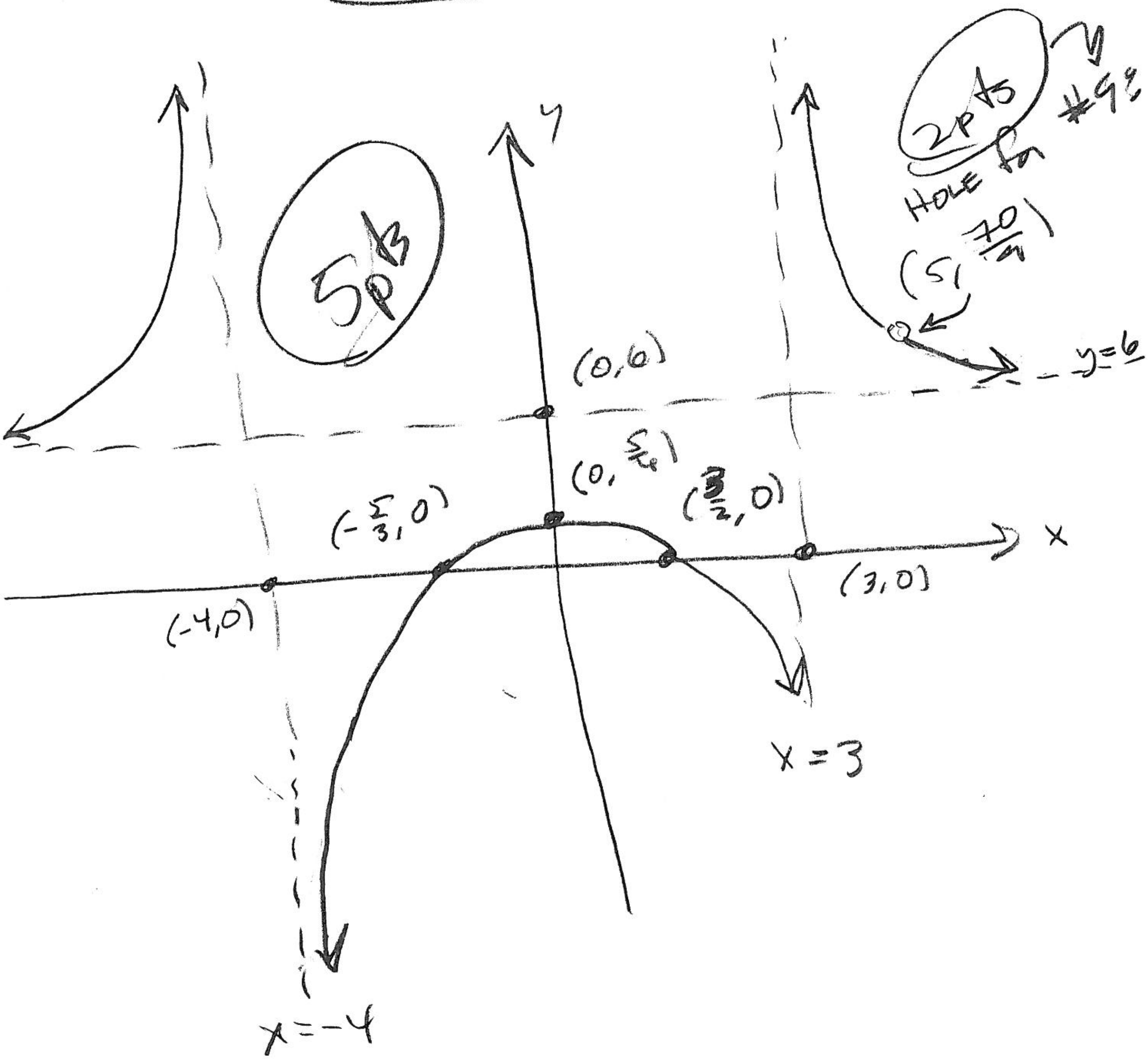
121 WP #3 (6)

(8) cnt'd

VA & x-int: $x = -4, 3, -\frac{5}{3}, \frac{3}{2}$



H.A.: $\frac{6x^2}{x^2} = \boxed{6 = y}$



120 WP #3 7

9 2pts $Q(x) = \dfrac{6x^3-29x^2-20x+75}{x^3-4x^2-17x+60}$

is built off #9, and has a hole.

$R(x) = \dfrac{(3x+5)(2x-3)}{(x+4)(x-3)}$ $x = -\frac{5}{3}, \frac{3}{2}$

$$\begin{array}{r|rrrr} \frac{3}{2} & 6 & -29 & -20 & 75 \\ & & 9 & -30 & -75 \\ \hline -\frac{5}{3} & 6 & -20 & -50 & 0 \\ & & -10 & 50 & \\ \hline & 6 & -30 & 0 & \end{array}$$

$(6x-30)(x+\frac{5}{3})(x-\frac{3}{2})$

$= 6(x-5)(x+\frac{5}{3})(x-\frac{3}{2})$

$$\begin{array}{r|rrrr} 5 & 1 & -4 & -17 & 60 \\ & & 5 & 5 & -60 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

$(x-5)(x+4)(x-3)$

$Q(x) = \dfrac{6(x+\frac{5}{3})(x-\frac{3}{2})\cancel{(x-5)}}{(x+4)(x-3)\cancel{(x-5)}}$ hole @ $x=5$

$R(5)$: $$\begin{array}{r|rrr} 5 & 6 & 1 & -15 \\ & & 30 & 155 \\ \hline & 6 & 31 & 140 \end{array}$$

$$\begin{array}{r|rrr} 5 & 1 & 1 & -12 \\ & & 5 & 30 \\ \hline & 1 & 6 & 18 \end{array}$$

$\frac{140}{18} = \frac{70}{9} \rightsquigarrow (5, \frac{70}{9})$ is hole

12 1 WP #3 8

10 5pts

$$T(x) = \frac{6x^3 - 29x^2 - 20x + 75}{x^2 + x - 12}$$

$$= \frac{6(x-5)(x+\frac{5}{3})(x-\frac{3}{2})}{(x+4)(x-3)}$$ by previous work.

$\mathcal{D} = \mathbb{R} \setminus \{-4, 3\}$

VA: $\boxed{x = -4,\ x = 3}$

y-int: $\frac{75}{-12} = \frac{-25}{4} \rightsquigarrow (0, \frac{-25}{4})$

x-int: $\boxed{x = -\frac{5}{3}, \frac{3}{2}, 5}$

$\rightsquigarrow (-\frac{5}{3}, 0), (\frac{3}{2}, 0), (5, 0)$

Slant Asymptote (Oblique Asymptote)

O.A.:

$$\begin{array}{r l}
 & 6x - 35 \\
x^2 + x - 12 & \overline{)\,6x^3 - 29x^2 - 20x + 75} \\
 & -(6x^3 + 6x^2 - 72x) \\
\hline
 & -35x^2 + 52x
\end{array}$$

$y = 6x - 35$

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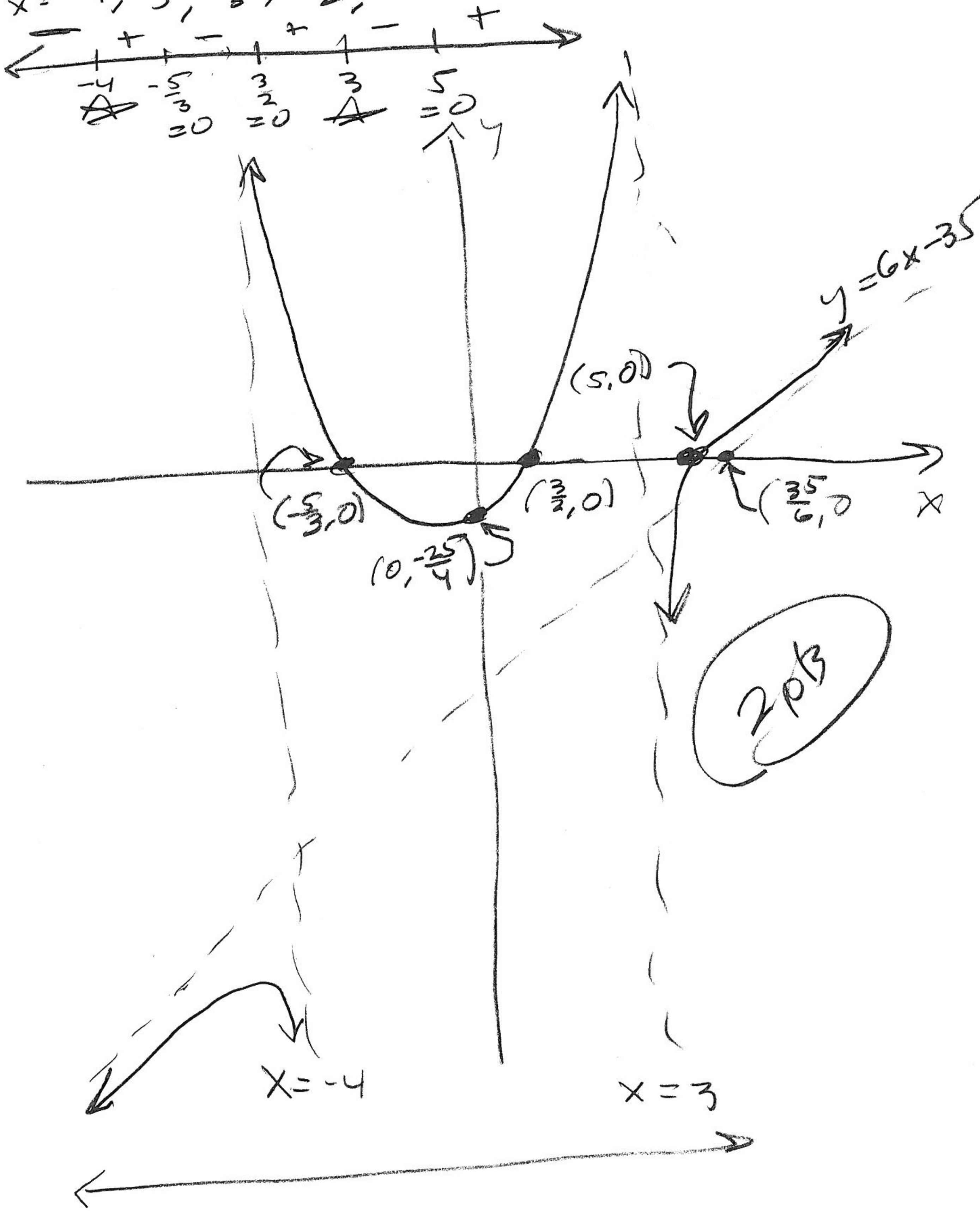
WP #3

10 cnt'd

$x = -4,\ 3,\ -\frac{5}{3},\ \frac{3}{2},\ 5$



121 WP #3 -10

(11) $w(x) = \sqrt{(x+3)(x-3)^2(x+7)(x-8)} = \sqrt{\text{stuff}}$

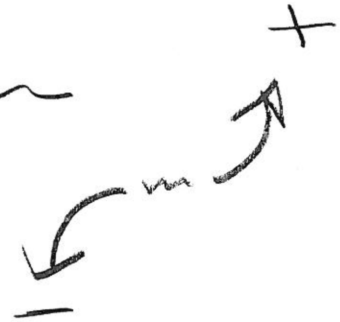
$\mathcal{D}$: Need stuff ≥ 0

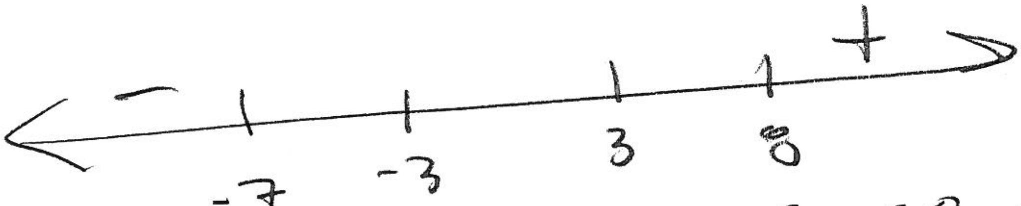
$\Rightarrow$ $x = -3, 3, -7, 8$ matter

$-7, -3, 3, 8$ in order

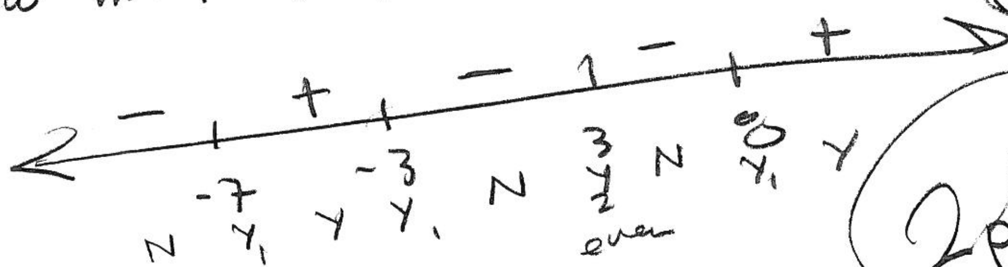
Stuff $= (x\cdot)(x\cdot)^2(x\cdot)(x\cdot) + \text{mm}$

$= x^5 + \text{smaller} \therefore$





Now $m = 1$ except $x = 3$, $m = 2$, so ≥ 0



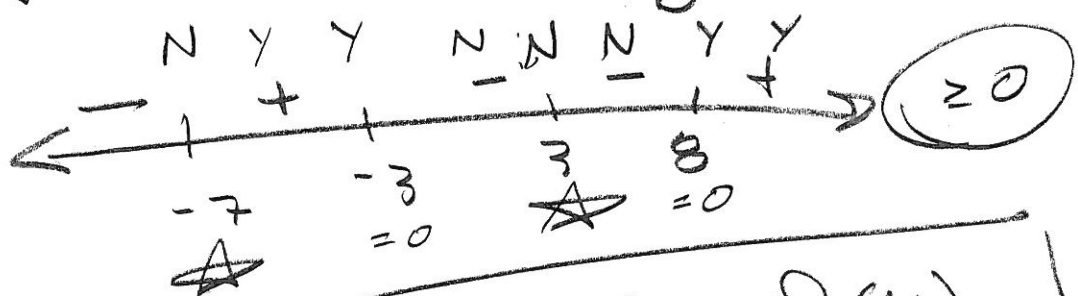
even $\Rightarrow$ no change

2 pts

$\Rightarrow \boxed{\mathcal{D}(w) = [-7, -3] \cup \{3\} \cup [8, \infty)}$

12 2pts $k(x) = \sqrt{\dfrac{(x+3)(x-8)}{(x-3)^2(x+7)}} = \sqrt{\dfrac{A}{B}}$

$\mathcal{D}(k(x))$: Need $\dfrac{A}{B} \geq 0$ and $B \neq 0$



$$\boxed{(-7,-3] \cup [8,\infty) = \mathcal{D}(k)}$$

2pts