

① $x^2 + 8x - 33 = 0$
 $a=1, b=8, c=-33 \rightarrow$

$b^2 - 4ac = 8^2 - 4(1)(-33)$
 $= 64 + 132$
 $= 196 = 14^2$

\rightarrow Factors by ac method

$\rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{196}}{2(1)}$
 $= \frac{-8 \pm 14}{2} = -4 \pm 7$
 $\rightarrow -4 + 7 = 3$
 $\rightarrow -4 - 7 = -11$

$\rightarrow x \in \{-11, 3\}$

② $16.12x^2 + 5.85x - 2.47 = 0$
 $\Rightarrow 1612x^2 + 585x - 247 = 0$

$\Rightarrow a=1612, b=585, c=-247$

$\Rightarrow b^2 - 4ac = 585^2 - 4(1612)(-247)$
 $= 342225 + 1592656$
 $= 1934881 = 1391^2$

\rightarrow factors "over the rationals," like #1!

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-585 \pm \sqrt{1934881}}{2(1612)}$
 $= \frac{-585 \pm 1391}{3224}$
 $\rightarrow \frac{1}{4} = 0.25$
 $\rightarrow -\frac{19}{31} \approx -0.6129032258$

$x \in \left\{ \frac{1}{4}, -\frac{19}{31} \right\}$ is ALSO the bonus!

BONUS

Scratch

2 1976	2 3224
2 988	2 1612
2 494	2 806
13 247	13 403
19	31

$\frac{1976}{3224} = \frac{2^3 \cdot 13 \cdot 19}{2^3 \cdot 13 \cdot 31}$
 $= \frac{19}{31}$

#2
 $x \in \sum_{n=2}^{2500} \frac{1}{n} = 6.1295$

$$(3) \quad 6x^2 + 5x + 3 = 0$$

$$\Rightarrow a=6, b=5, c=3$$

$$\Rightarrow b^2 - 4ac = 5^2 - 4(6)(3)$$

$$= 25 - 72$$

$$= -47 \Rightarrow 2 \text{ nonreal solutions}$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{-47}}{2(6)}$$

$$= \frac{-5 \pm i\sqrt{47}}{12}$$

$$x \in \left\{ \frac{-5 \pm i\sqrt{47}}{12} \right\}$$

$$(4) \quad ax^2 + 2rx + 7w = 0$$

$$\Rightarrow a=a, b=2r, c=7w$$

$$\Rightarrow b^2 - 4ac = (2r)^2 - 4(a)(7w)$$

$$= 4r^2 - 28aw$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2r \pm \sqrt{4r^2 - 28aw}}{2a}$$

$$= \frac{-2r \pm \sqrt{4(r^2 - 7aw)}}{2a}$$

$$= \frac{-r \pm \sqrt{r^2 - 7aw}}{a} = x$$

$$x \in \left\{ \frac{-r \pm \sqrt{r^2 - 7aw}}{a} \right\}$$

Nope

17w

$$b^2 - 4ac = (2r)^2 - 4(a)(7w)$$

$$= 4r^2 - 4(7aw)$$

$$= 4(r^2 - 7aw)$$

$$x = \frac{-2r \pm \sqrt{4(r^2 - 7aw)}}{2a}$$

$$= \frac{-2r \pm 2\sqrt{r^2 - 7aw}}{2a}$$

$$= \frac{-r \pm \sqrt{r^2 - 7aw}}{a}$$

(5) $x^2 + 8x - 33$
 $= x^2 + 11x - 3x - 33$
 $= x(x+11) - 3(x+11)$
 $= (x+11)(x-3) = 0$
 $\Rightarrow x \in \{-11, 3\}$

M1

(6) $33x^2 + 4x - 12$
 $= 33x^2 + 22x - 18x - 12$
 $= 11x(3x+2) - 6(3x+2)$
 $= (3x+2)(11x-6) = 0$
 $\Rightarrow x \in \{-\frac{2}{3}, \frac{6}{11}\}$

$-(12)(33)$
 $= (-2)(2)(3)(3)(11)$
 want sum of +4

$(-(2)(3)(3))(2)(11)$
 $-18 + 22$ ✓ M2

Another way:
 $4 = 4 - 0$
 $= 5 - 1$
 $= 13 - 14$
 $= 20 - 16$
 $= 21 - 17$
 $= 22 - 18$
 0
 -5
 -252
 -320
 -357
 -396
 sweet!
 $-(2)(33) = \text{magic} = 396$

M3 Finish: Quadratic gave us answers, now write factored form:
 $33(x - \frac{6}{11})(x + \frac{2}{3})$
 Now fool the teacher =
 $3(11)(x - \frac{6}{11})(x + \frac{2}{3})$
 $= 11(x - \frac{6}{11})(3)(x + \frac{2}{3})$
 $= (11x - 6)(3x + 2)$
 cool!

M3: $a = 33, b = 4, c = -12$
 $b^2 - 4ac = 4^2 - 4(33)(-12)$
 $= 16 + 1584 = 1600 = 40^2$
 $\Rightarrow x = \frac{-4 \pm 40}{2(33)} = \frac{-2 \pm 20}{33}$
 $\swarrow \searrow$
 $\frac{18}{33} = \frac{6}{11}$
 $\frac{-22}{33} = -\frac{2}{3}$

(7)

$$x^2 + 7x - 33 =$$

Scratch =

$$= x^2 + 7x + \left(\frac{7}{2}\right)^2 - \frac{49}{4} - \left(\frac{33}{1}\right)\left(\frac{4}{4}\right)$$

$$\frac{-49 - 132}{4}$$

$$= \left(x + \frac{7}{2}\right)^2 - \frac{181}{4} \stackrel{\text{SET}}{=} 0$$

$$= \frac{-181}{4}$$

$$\Rightarrow \left(x + \frac{7}{2}\right)^2 = \frac{181}{4}$$

$$\sqrt{181}$$

$$\Rightarrow x + \frac{7}{2} = \pm \sqrt{\frac{181}{4}} = \pm \frac{\sqrt{181}}{2}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{181}}{2} \Rightarrow x \in \left\{ \frac{-7 \pm \sqrt{181}}{2} \right\}$$

(7) Done the way it's taught. I like the 1st way, above, because it's useful, later.

$$x^2 + 7x - 33 = 0$$

$$\Rightarrow x^2 + 7x = 33$$

$$\Rightarrow x^2 + 7x + \left(\frac{7}{2}\right)^2 = 33 + \frac{49}{4} = \frac{132 + 49}{4} = \frac{181}{4}$$

$$\Rightarrow \left(x + \frac{7}{2}\right)^2 = \frac{181}{4}$$

$$\Rightarrow x + \frac{7}{2} = \pm \sqrt{\frac{181}{4}} = \pm \frac{\sqrt{181}}{2}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{181}}{2} \Rightarrow x \in \left\{ \frac{-7 \pm \sqrt{181}}{2} \right\}$$

121

WP #1

$$\textcircled{8} \quad 2x^2 - 4x - 17 = 0$$

METHOD 1

$$\Rightarrow 2x^2 - 4x = 17$$

$$\Rightarrow 2(x^2 - 2x) = 17$$

$$\Rightarrow 2(x^2 - 2x + 1^2) = 17 + 2(1) = 19$$

$$\Rightarrow 2(x-1)^2 = 19$$

$$\Rightarrow (x-1)^2 = \frac{19}{2}$$

$$\Rightarrow x-1 = \pm \sqrt{\frac{19}{2}} = \pm \sqrt{\frac{19 \cdot 2}{2 \cdot 2}} = \pm \sqrt{\frac{38}{4}} = \pm \frac{\sqrt{38}}{2}$$

$$\Rightarrow x \in \left\{ 1 \pm \frac{\sqrt{38}}{2} \right\}$$

$\textcircled{8}$ METHOD 2

$$2x^2 - 4x - 17 = 0$$

$$x^2 - 2x - \frac{17}{2} = 0$$

$$x^2 - 2x = \frac{17}{2}$$

$$x^2 - 2x + 1^2 = \frac{17}{2} + 1 = \frac{17+2}{2} = \frac{19}{2}$$

$$(x-1)^2 = \frac{19}{2}$$

$$x-1 = \pm \sqrt{\frac{19}{2}} = \pm \sqrt{\frac{19}{2}} \sqrt{\frac{2}{2}} = \pm \frac{\sqrt{38}}{\sqrt{4}} = \pm \frac{\sqrt{38}}{2}$$

$$x = 1 \pm \frac{\sqrt{38}}{2}$$

$$\Rightarrow x \in \left\{ 1 \pm \frac{\sqrt{38}}{2} \right\}$$

$$(9) \quad 3x^2 + 2x + 7 = 0$$

$$3\left(x^2 + \frac{2}{3}x\right) = -7$$

$$3\left(x^2 + \frac{2}{3}x + \left(\frac{1}{3}\right)^2\right) = -7 + 3\left(\frac{1}{9}\right) = -\frac{21}{3} + \frac{1}{3}$$

$$3\left(x + \frac{1}{3}\right)^2 = -\frac{20}{3}$$

$$\left(x + \frac{1}{3}\right)^2 = -\frac{20}{9}$$

$$x + \frac{1}{3} = \pm \sqrt{-\frac{20}{9}} = \pm i \sqrt{\frac{4 \cdot 5}{3 \cdot 3}} = \pm i \frac{2\sqrt{5}}{3}$$

$$\Rightarrow x \in \left\{ \frac{-1 \pm 2i\sqrt{5}}{3} \right\}$$

$$(10) \quad 23x^2 - 4x + 5 = 0$$

$$23\left(x^2 - \frac{4}{23}\right) = -5$$

$$23\left(x^2 - \frac{4}{23} + \left(\frac{2}{23}\right)^2\right) = -5 + 23\left(\frac{4}{23^2}\right)$$

$$= -5 + \frac{4}{23} = \frac{-115 + 4}{23} = \frac{-111}{23}$$

$$\Rightarrow 23\left(x - \frac{2}{23}\right)^2 = -\frac{111}{23}$$

$$\Rightarrow \left(x - \frac{2}{23}\right)^2 = -\frac{111}{23^2}$$

$$3 \overline{) 111} \\ \underline{37}$$

$$\Rightarrow x - \frac{2}{23} = \pm i \sqrt{\frac{-111}{23^2}} = \pm i \frac{\sqrt{111}}{23}$$

$$\Rightarrow x \in \frac{2 \pm i\sqrt{111}}{23}$$