MAT 121   Test 4 – Spring, 2016   Name_____
100 Points   Covers Chapter 4
Instructor: Harry S. Mills   No Graphing Calculator

Date, Time:

Do your own work on separate paper. Leave plenty of margin and plenty of room around your work. I'm not impressed if you squeeze more work into a smaller space. To the contrary. At the end, please make sure your problems are in order. I'm too old and ornery to want to go on a scavenger hunt to award you points.

1. (20 pts) Starting with $f(x)=5^x$, sketch the graph of $g(x)=3\cdot5^{-x+4}-8$ in 5 steps (counting $f(x)=6^x$ as the first step). Use $x=-1$, $x=0$, and $x=1$ to find 3 points in the first graph, and show how these 3 points are moved around by each step in the transformation to $g(x)$. Finding the x- and y-intercepts is #2, so don't worry about them, until #2. Label each sketch as some variation on $f(x)$, for instance, $7\cdot5^{x-11}-4$ would be $7f(x-11)-4$. Only the *first* graph is $f(x)$. Only the *last* graph is $g(x)$.

2. (10 pts) Let $g(x)=3\cdot5^{-x+4}-8$. Find the x- and y-intercepts for this function, rounded to 4 decimal places. For 5 bonus points, label these intercepts on your final graph for #1.

3. Let $f(x)=\sqrt{x+2}$ and $g(x)=\dfrac{x-1}{x+5}$.

   a. (5 pts) What is the domain of f?

   b. (5 pts) What is the domain of g?

   c. (5 pts) Write the function $\dfrac{f}{g}$. Do not simplify.

   d. (5 pts) What is the domain of $\dfrac{f}{g}$?

   e. (5 pts) Write the function $f\circ g$. Do not simplify.

   f. (5 pts) What is the domain of $f\circ g$?

4. Find the domain:

   a. (5 pts) $\sqrt{(x-3)^2(x+3)(x-8)^2(x-11)}$. To speed up your sign pattern, it should be helpful to know that
   $(x+3)^2(5-x)(x-8)^3(x-12)=-x^7+35x^6-423x^5+1625x^4+5132x^3-43680x^2+2304x+276480$.

   b. (5 pts) $\log_3\left((x-3)^2(x+3)(x-8)^2(x-11)\right)$ (Reinterpret previous sign pattern in the current context!)

5. Consider the equation $\ln(x-4)+\ln(x+2)=\ln(7)$.
   a. (5 pts) What is the domain of this equation?
   b. (5 pts) Solve the equation.

6. (10 pts) Solve $7^{2x-1}=3^{-x+5}$. Give an exact answer *and* a decimal answer, rounded to 4 decimal places.

7. (10 pts) Assuming the half-life of Carbon-14 is 6,000 years, and that charcoal from an ancient fire pit contains only 15% of the amount Carbon-14 found in living creatures. How old is the fire pit?

Bonus Section

1. BONUS (5 pts) Solve the equation $2\pi^{x+3} = 5e^{-x-5}$. Give an exact answer and a decimal answer, rounded to 4 places.

2. BONUS (5 pts) Solve the absolute value inequality $|7-3x| \geq 8$. Use a number line and either union or intersection ('and' or 'or') to find the solution.

3. BONUS (5 pts) The absolute value inequality $|7-3x| \geq -8$ is always true, since absolute value can never be negative. But show the steps and manage your and's and or's, with a number line graph at the end to interpret what the algebra is telling you.

4. BONUS (5 pts) Give a rough sketch of the function

$f(x) = (x+3)^2(5-x)(x-8)^3(x-12) = -x^7 + 35x^6 - 423x^5 + 1625x^4 + 5132x^3 - 43680x^2 + 2304x + 276480$