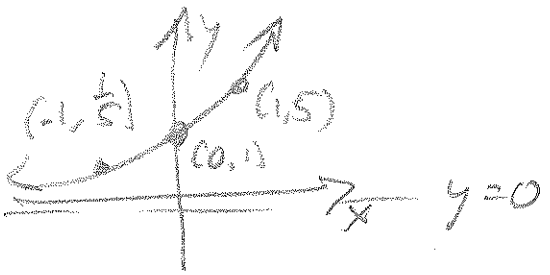
121 E4 Spring, 2016

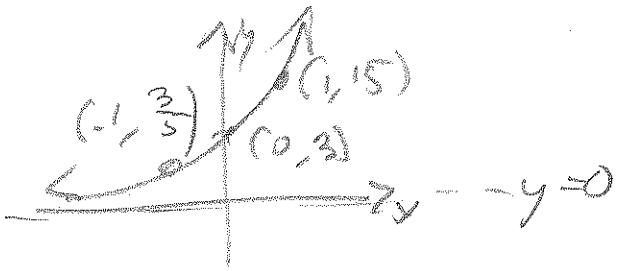
$g(x) = 3 \cdot 5^{-x+4} - 8$

20 pts

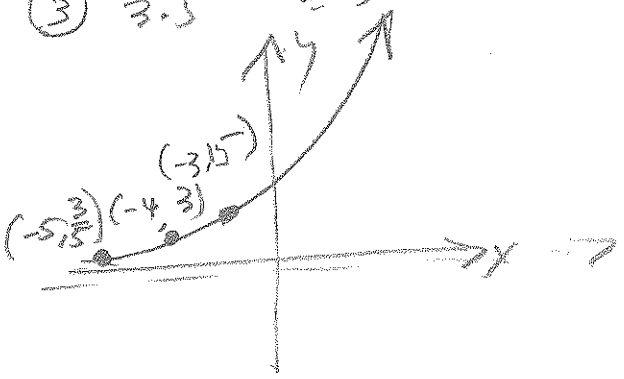
(1) $f(x) = 5^x$



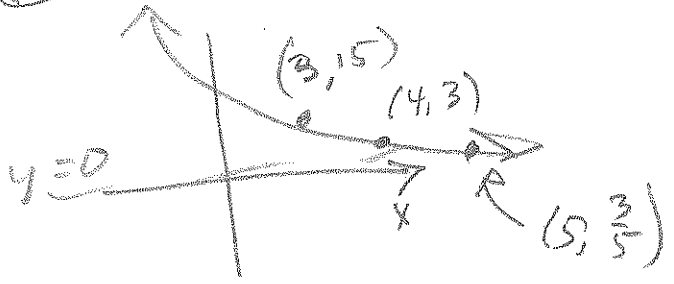
(2) $3 \cdot 5^x = 3f(x)$



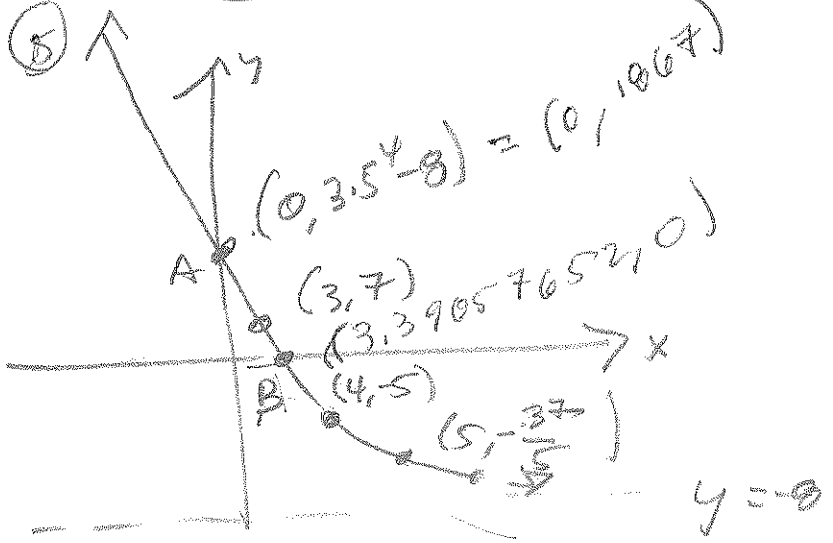
(3) $3 \cdot 5^{x+4} = 3f(x+4)$



(4) $3 \cdot 5^{-x+4} = 3f(-x+4)$



(5)



(1) $g(0) = 3 \cdot 5^4 - 8$ is BIG!

$A = (0, 3 \cdot 5^4 - 8) = (0, 1867)$

$g(x) = 3 \cdot 5^{-x+4} - 8 = 0$

10 pts

$3 \cdot 5^{-x+4} = 8$

$5^{-x+4} = \frac{8}{3}$

$\log_5(\ ) = \log(\ )$

$-x+4 = \log_5(8/3)$

$-x = \log_5(8/3) - 4$

$x = 4 - \log_5(8/3)$

$B = (4 - \log_5(8/3), 0)$

$\approx (3.3905765240, 0)$

121 E4

(3) $f(x) = \sqrt{x+2}$ $g(x) = \frac{x-1}{x+5}$ (2)

(a) $D(f)$: $x+2 \geq 0$

$\{x \mid x \geq -2\} = [-2, \infty)$

(b) $D(g)$: $x+5 = 0$

$\{x \mid x \neq -5\} = (-\infty, -5) \cup (-5, \infty)$

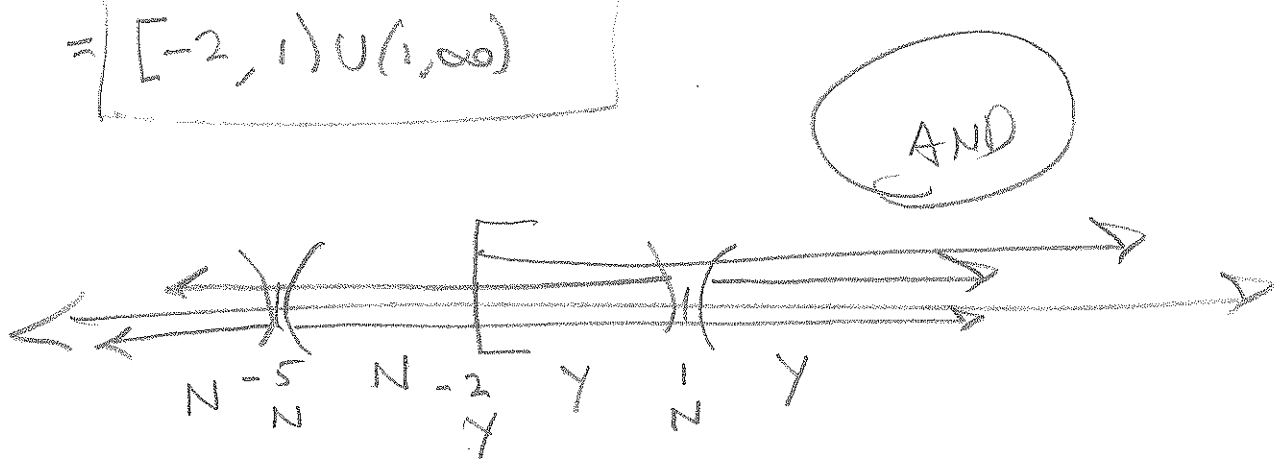
(c) $\frac{f}{g} = \frac{\sqrt{x+2}}{\frac{x-1}{x+5}}$

(d) $D\left(\frac{f}{g}\right) = \{x \mid x \in D(f) \text{ and } x \in D(g) \text{ and } g(x) \neq 0\}$

$= \{x \mid x \geq -2 \text{ and } x \neq -5 \text{ and } x-1 \neq 0\}$

$= \{x \mid x \geq -2 \text{ and } x \neq -5 \text{ and } x \neq 1\}$

$= \boxed{[-2, 1) \cup (1, \infty)}$



121 E4

(3a) $f \circ g = \sqrt{\frac{x-1}{x+5}+2}$

(3b) $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$ (3)

$= \{x \mid x \geq -2 \text{ and } \frac{x-1}{x+5} \geq -2\}$

$= \{x \mid x \geq 2 \text{ and } (x \leq -3 \text{ or } x > -5)\}$

Scratch:

$\frac{x-1}{x+5} \geq -2$

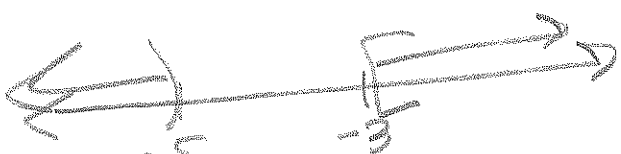
$\frac{x-1}{x+5} + 2 \geq 0$

$\frac{x-1}{x+5} + \frac{2(x+5)}{x+5} \geq 0$

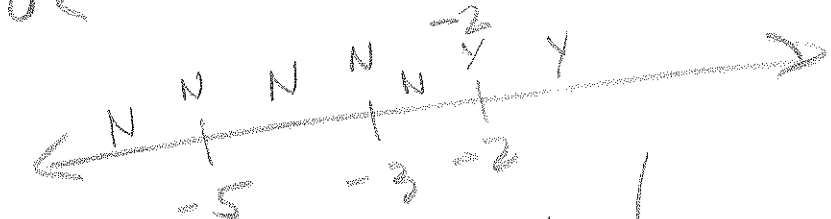
$\frac{x-1+2x+10}{x+5} \geq 0$

$\frac{3x+9}{x+5} \geq 0$

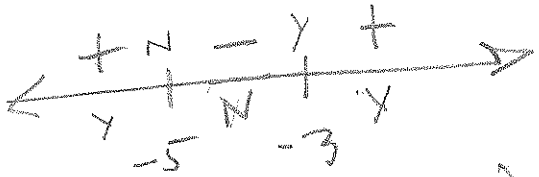
$\frac{3(x+3)}{x+5} \geq 0$



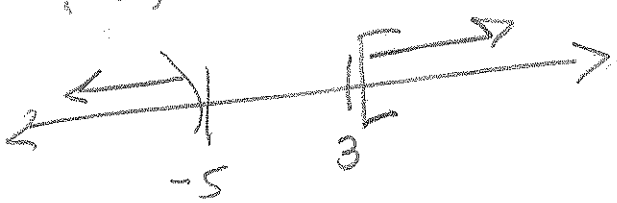
AND



$= [-2, \infty)$!



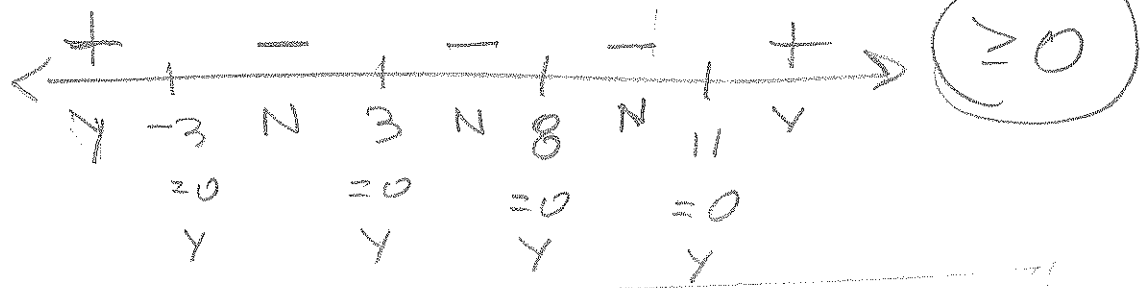
$(-\infty, -5) \cup [-3, \infty)$



12.1 #4

(4) $\sqrt{(x-3)^2(x+3)(x-8)^2(x-11)} = f(x)$ (4)

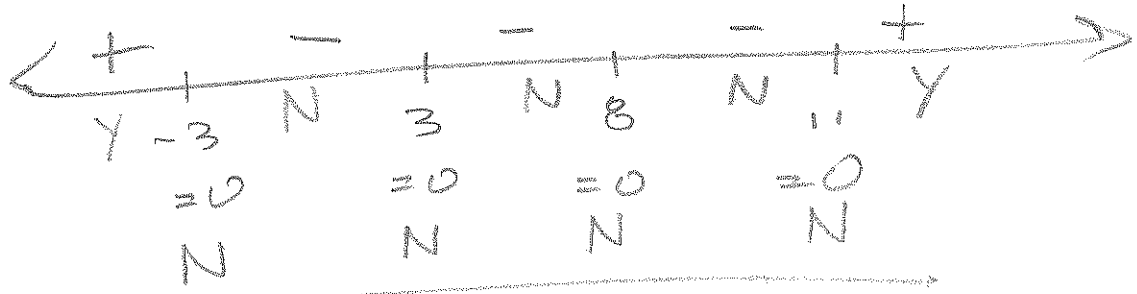
(a) $D(f)$: Need $(x-3)^2(x+3)(x-8)^2(x-11) \geq 0$



$D(f) = (-\infty, -3] \cup \{3, 8\} \cup [11, \infty)$

(b) $\log_3\left[(x-3)^2(x+3)(x-8)^2(x-11)\right] = g(x)$

$D(g)$: Need $(x-3)^2(x+3)(x-8)^2(x-11) > 0$



$D(g) = (-\infty, -3) \cup (11, \infty)$

-4

(5) $\ln(x-4) + \ln(x+2) = \ln(7)$

(b) $\ln((x-4)(x+2)) = \ln 7$

$(x-4)(x+2) = 7$

$x^2 - 2x - 8 = 7$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x \in \{-3, 5\}$ → $x \in \{5\}$

(a) $D = \{x / x > 4 \text{ and } x > -2\}$

$= \{x / x > 4\}$

$= (4, \infty)$

(5)

(6) $7^{2x-1} = 3^{-x+5}$

$\log_7(7^{2x-1}) = \log_7(3^{-x+5})$

$2x - 1 = (-x+5)\log_7(3) = (-x+5)a = -ax + 5a$

a

$\Rightarrow 2x + ax = 1 + 5a$

$(2+a)x = 5a + 1$

$x = \frac{5a+1}{a+2} = \boxed{\frac{\log_7(3) + 1}{\log_7(3) + 2} = \frac{\ln(7) + 5\ln(3)}{\ln(7) + \ln(3)}}$

≈ 1.490446685

10 pts

121 E4

10 pts

6

(7) $\frac{1}{2}$-life is 6,000 yrs

$A(t) = A_0 e^{6000k} = \frac{1}{2} A_0$

$\Rightarrow e^{6000k} = \frac{1}{2}$

$\Rightarrow \ln(e^{6000k}) = \ln(\frac{1}{2})$

$\Rightarrow 6000k = -\ln(2)$

$\Rightarrow \boxed{k = -\frac{\ln(2)}{6000}}$

Now

$A(t) = A_0 e^{kt} = .15 A_0$

$\Rightarrow e^{kt} = .15$

$\Rightarrow kt = \ln(.15)$

$\Rightarrow \boxed{t = \frac{\ln(.15)}{k}}$

$\boxed{\approx 16{,}421.79356 \text{ yrs}}$

$k \approx -1.155245301 \times 10^{-4}$

$= -.0001155245301$

(B1) $2\pi^{x+3} = 5e^{-x-5}$

$\ln(2\pi^{x+3}) = \ln(5e^{-x-5})$

$\ln 2 + \ln(\pi^{x+3}) = \ln 5 + \ln e^{-x-5}$

$a + (\ln \pi)(x+3) = b + (-x-5)$

$a + c(x+3) = b - x - 5$

$a + cx + 3c = b - x - 5$

$cx + x = b - 5 - a - 3c$

$x(c+1) = b - a - 5 - 3c \approx$ ~~-2.5745476598~~ -3.505289398

$x = \frac{b - a - 5 - 3c}{c+1} = \boxed{\frac{\ln 5 - \ln 2 - 5 - 3\ln \pi}{\ln \pi + 1}}$

12.1 E4

B2 $|7-3x| \geq 8$

ONLINE STUDENTS: Skip to page 10

Bonus for #27 (7)

$7-3x \geq 8$ OR $7-3x \leq -8$

$-3x \geq 1$ $\quad$ $-3x \leq -15$

$\{x \mid x \leq -\frac{1}{3}$ OR $x \geq 5\}$

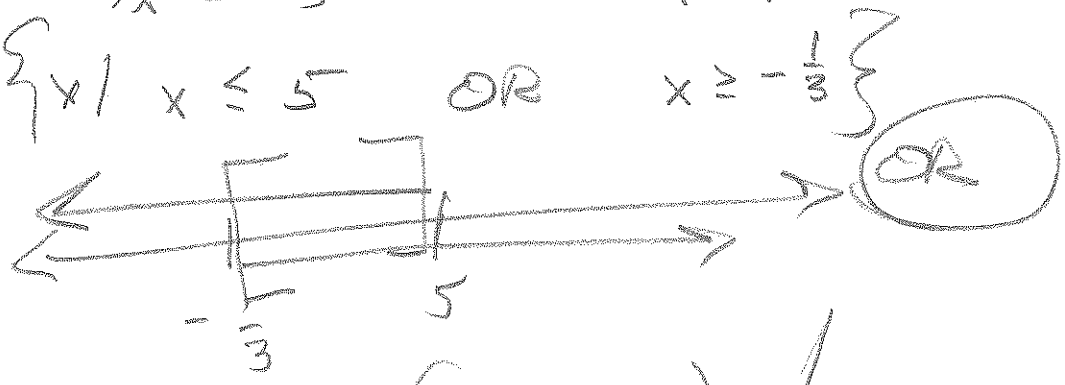
$= (-\infty, -\frac{1}{3}] \cup [5, \infty)$

B3 $|7-3x| \geq -8$

$7-3x \geq -8$ OR $7-3x \leq 8$

$-3x \geq -15$ $\quad$ $-3x \leq 1$

$\{x \mid x \leq 5$ OR $x \geq -\frac{1}{3}\}$

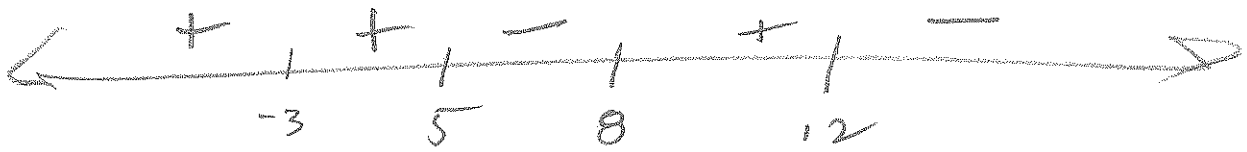


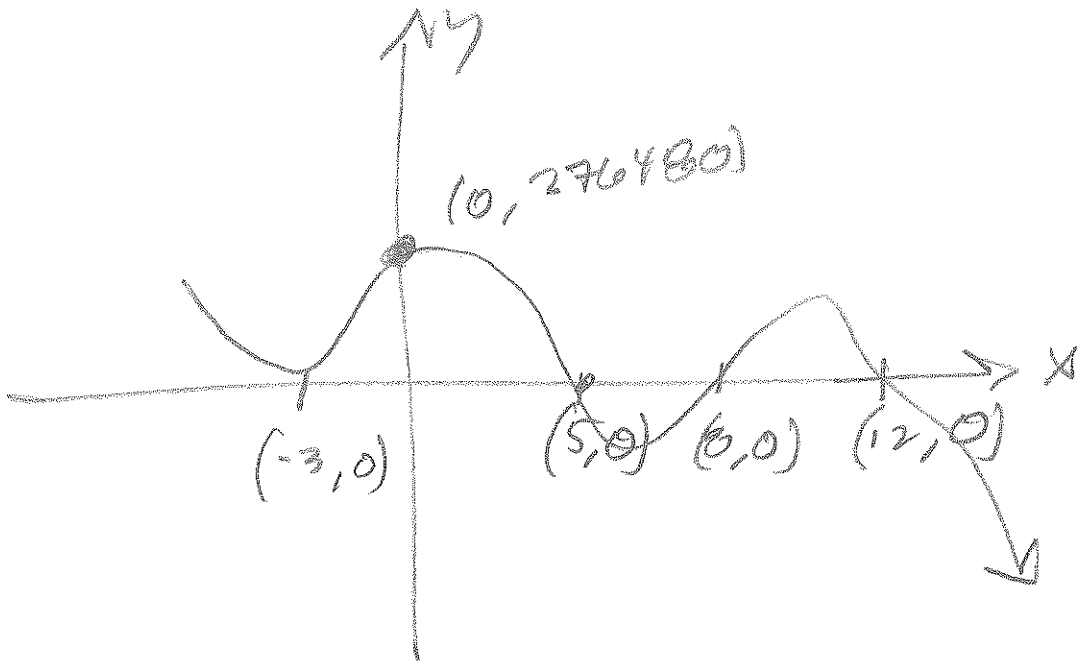
$= (-\infty, \infty)$!

121 E4

B4 $f(x) = (x+3)^2(5-x)(x-8)^3(x-12)$ (8)

$= -x^7 + \cdots + 276480$



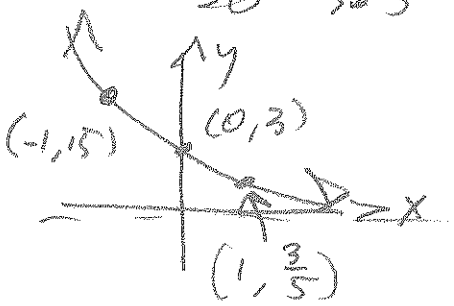


Other versions of #1 & #2

(1)

(3) $-x+4 = -(x-4)$

So $3 \cdot 5^{-x}$



(2) Using ln:

$5^{-x+4} = \frac{3}{5}$

$\ln(5^{-x+4}) = \ln(\frac{3}{5})$

$(\ln 5)(-x+4) = \ln(\frac{3}{5})$

$a(-x+4) = b$

$-ax + 4a = b$

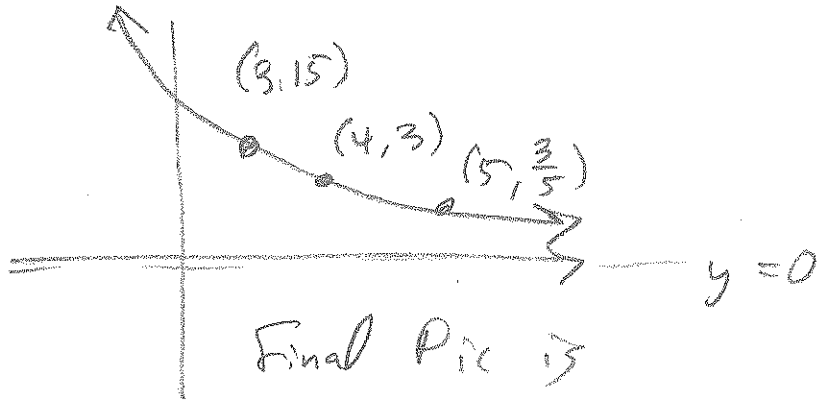
$-ax = b - 4a$

$x = \frac{b-4a}{-a} = \frac{4a-b}{a}$

$= \frac{4\ln 5 - \ln(3/5)}{\ln 5}$

(4)

$3 \cdot 5^{-(x-4)}$



Final Pic is same.

Other Version of #6

$\ln(7^{2x-1}) = \ln(3^{-x+5})$

$(\ln 7)(2x-1) = (\ln 3)(-x+5)$

$a(2x-1) = b(-x+5)$

$2ax - a = -bx + 5b$

$2ax + bx = 5b + a$

$(2a+b)x = 5b + a$

$x = \frac{5b+a}{2a+b}$

$= \frac{5\ln 3 + \ln 7}{2\ln 7 + \ln 3}$

≈ 1.490646645

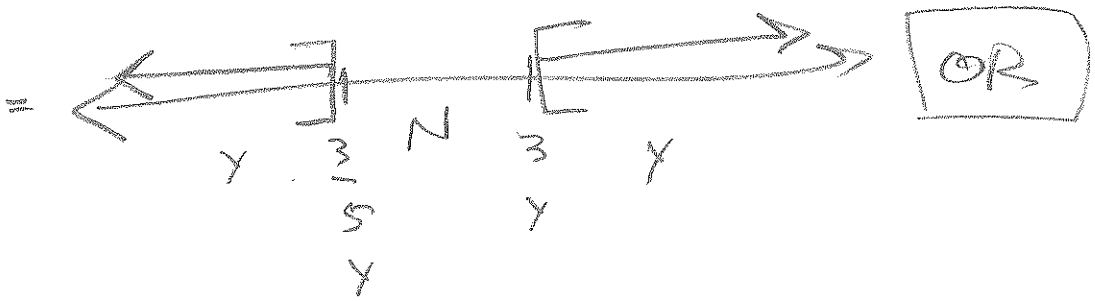
ONLINE VERSIONS OF BONUS PAGE #1 WERE THE SAME (10)

B2 $|9-5x| \geq 6$

$9-5x \geq 6$ OR $9-5x \leq -6$

$-5x \geq -3$ $\quad -5x \leq -15$

$\{x \mid x \leq \frac{3}{5}$ OR $x \geq 3\}$



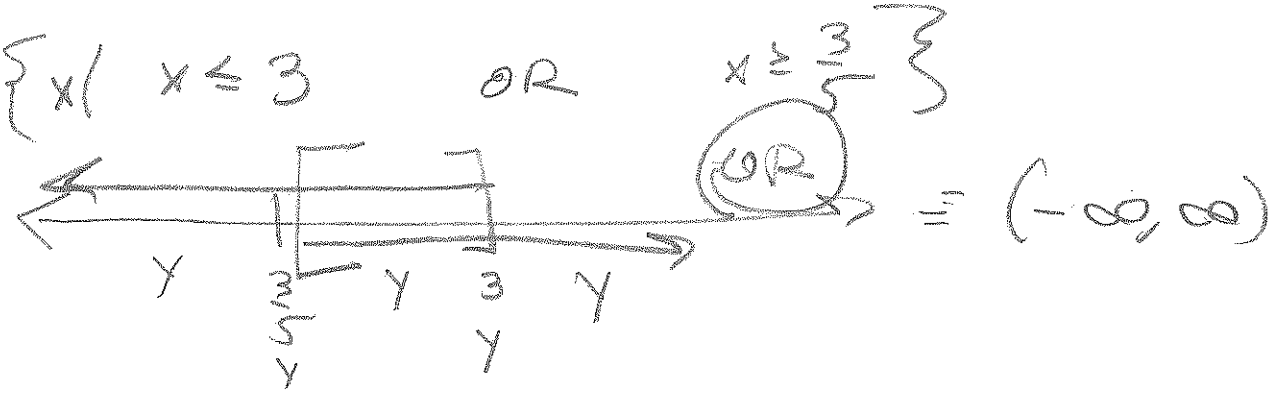
$= (-\infty, \frac{3}{5}] \cup [3, \infty)$

B3 $|9-5x| \geq -6$

$9-5x \geq -6$ OR $9-5x \leq 6$

$-5x \geq -15$ $\quad -5x \leq -3$

$\{x \mid x \leq 3$ OR $x \geq \frac{3}{5}\}$



$= (-\infty, \infty)$

12) E4 ONLINE'S BONUS PAGE (11)

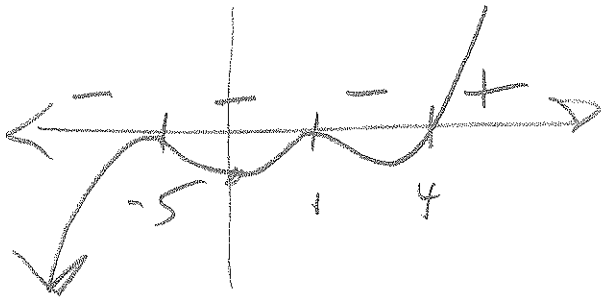
(4) $f(x) = (x-1)^2(x+5)^2(x-4) = x^5 + \cdots - 100$

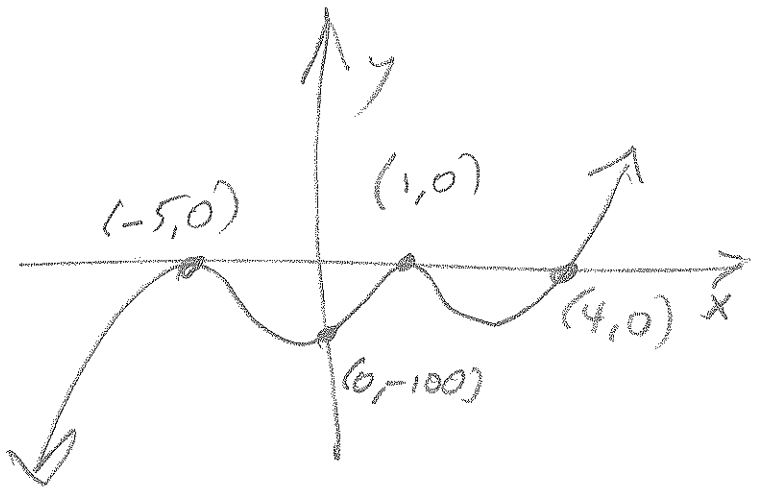
$f(0) = (-1)^2(5)^2(-4)$

$= -100$

$(0, -100)$

x-int	m	
1	2	touch
-5	2	"
4	1	cross





121 54

B5 (12)

$$R(x) = \frac{x^2 - x - 6}{x^2 - 3x - 10} = \frac{(x-3)(x+2)}{(x-5)(x+2)}$$

$$= \frac{x-3}{x-5} = R^*(x) \text{ when } x \neq -2, \text{ where there's a hole.}$$

$D: \mathbb{R} \setminus \{-2, 5\}$

$R(0) = \frac{3}{5}$

y-int: $(0, \frac{3}{5})$

HOLE: $x = -2$

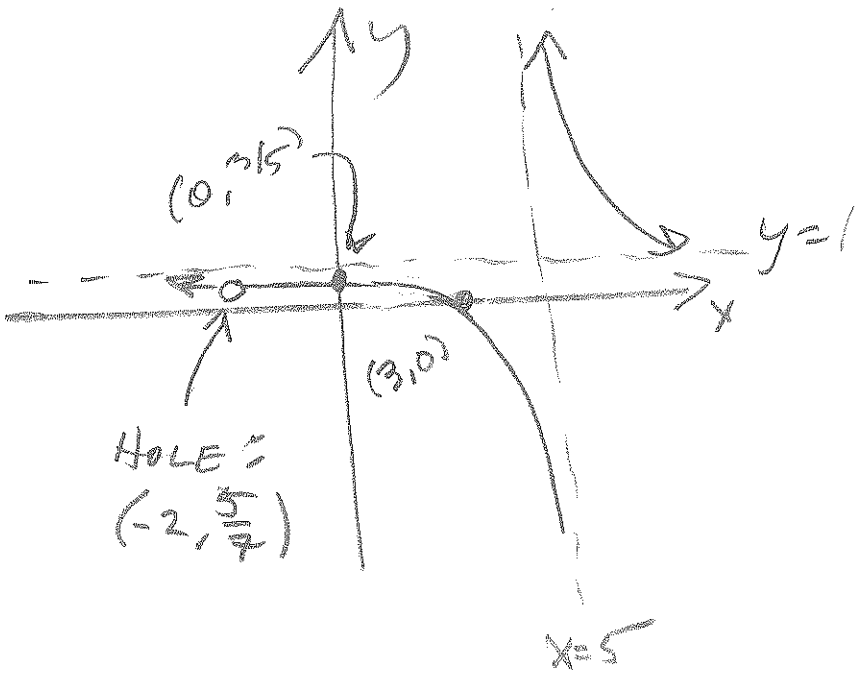
$R^*(-2) = \frac{-2-3}{-2-5} = \frac{-5}{-7} = \frac{5}{7}$

HOLE: $(-2, \frac{5}{7})$

V.A.: $x = 5$

H.A.: $y = \frac{x}{x} = 1$

$y = 1$



$\frac{x-3}{x-5}$: + | − | + (3, 5)

121 E4 BONUS ONLINE (13)

B6

$$\frac{x+2}{(x+1)(x-2)} < 1$$

LCD = $(x+1)(x-2)$

Subtract 1 both sides
Find common denom

$$\frac{x+2}{(x+1)(x-2} - \left(\frac{1}{1}\right)\frac{(x+1)(x-2)}{(x+1)(x-2)} < 0$$

$$\frac{x+2-(x+1)(x-2)}{(x+1)(x-2)} < 0$$

$$\frac{x+2-(x^2-x-2)}{(x+1)(x-2)} < 0$$

$$\frac{x+2-x^2+x+2}{(x+1)(x-2)} < 0$$

$$\frac{-x^2+2x+4}{(x+1)(x+2)} < 0 \Rightarrow \frac{-(x-(1+\sqrt{5}))(x-(1-\sqrt{5}))}{(x+1)(x+2)}$$

(< 0)

Y N N + N Y N N + N Y

-2 $1-\sqrt{5}$ -1 $1+\sqrt{5}$

☆ =0 ☆ =0

$x^2-2x-4=0$

$x^2-2x+1^2=4+1$

$(x-1)^2=5$

$x-1=\pm\sqrt{5}$

$x=1\pm\sqrt{5}$

$= (-\infty,-2)\cup(1-\sqrt{5},-1)\cup(1+\sqrt{5},\infty)$

121 E4

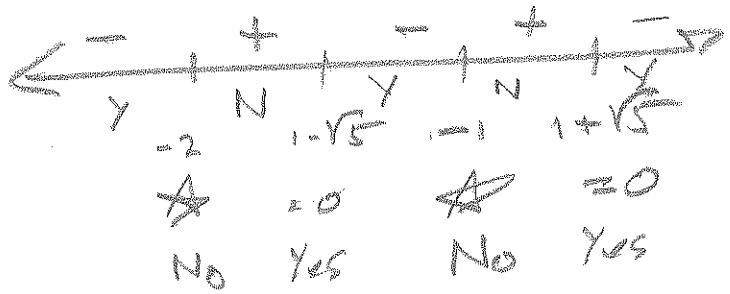
(87)

(14)

$$\frac{x+2}{(x+1)(x-2)} \leq 1$$

Same as (B6) only $\neq 0$

So



$\Rightarrow x \in (-\infty, -2) \cup [1-\sqrt{5}, -1) \cup [1+\sqrt{5}, \infty)$

Only difference in answers is square brackets