1121 E3 Spring '16

(1) $x=13, m=1$; $x=3-7i, m=1$; $x=-5, m=3$

$(x-13)(x-(3-7i))(x-(3+7i))(x+5)^3$

10 pts

(2) $P(x) = x^6 - 7x^4 + 3x^3 - 10x^2 + 5x - 5$

$P(3)$?

	1	0	−7	3	−10	5	−5
3		3	9	6	27	51	168
	1	3	2	9	17	56	163 = P(3)

10 pts

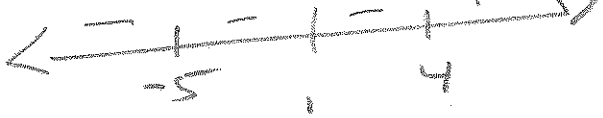
(3) The above says

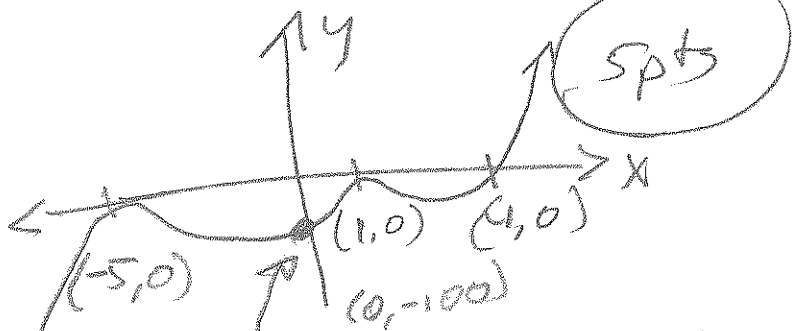
$P(x) = (x-3)(x^5 + 3x^4 + 2x^3 + 9x^2 + 17x + 56) + 163$

5 pts

(4) $f(x) = (x-1)^2(x+5)^2(x-4) = x^5 + 4x^4 - 26x^3 - 64x^2 + 185x - 100$

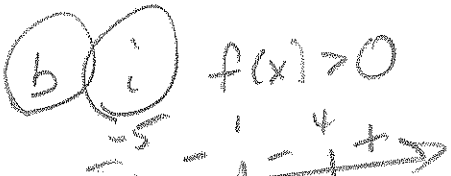
(a)





5 pts

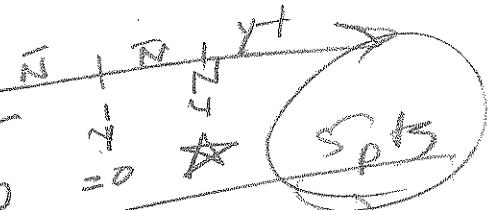
(b) (i) $f(x) > 0$



5 pts

$(4, \infty)$

(b) (ii) $\frac{(x-1)^2(x+5)^2}{x-4} \geq 0$



5 pts

$[-5, 4) \cup (4, \infty)$

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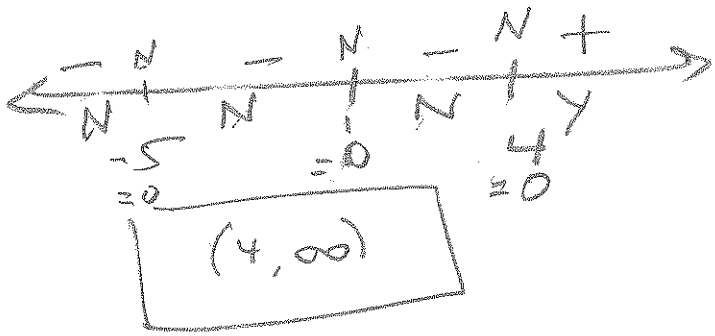
#46 Re-write

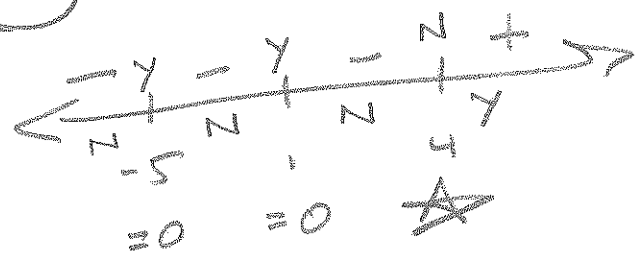
(i) $(x+5)^2(x-1)^2(x-4) > 0$



$(4, \infty)$

5pts

(ii) $(x+5)^2(x-1)^2(x-4) \geq 0$



$\{-5\} \cup [1, \infty)$

OR $\{-5, 1\} \cup [4, \infty)$

5pts

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(5) $f(x) = 4x^5 - 12x^4 - x^3 + 28x^2 - 9x - 22$

Descartes: 3 or 1 positive

$f(-x) = -4x^5 - 12x^4 + x^3 + 28x^2 + 9x - 22$

2 or 0 negative

Rational Zeros:

p's: ± 22 — $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}$

q's: 4 — $\pm 2, \pm \frac{2}{2}, \pm \frac{2}{4}$

$\pm 11, \pm \frac{11}{2}, \pm \frac{11}{4}$

My guesses: $\pm 22, \pm \frac{22}{2}, \pm \frac{22}{4}$

$\pm 1, \pm 2$. After some work, $x = -1$ works twice, $x = 2$ works once. We present the tidied-up work, here.

	4	−12	−1	28	−9	−22
2		8	−8	−18	20	22
	4	−4	−9	10	11	0
−1		−4	8	1	−11	
	4	−8	−1	11	0	
−1		−4	12	−11		
	4	−12	11	0		

10 pts

So, $f(x) = (x+1)^2(x-2)(4x^2 - 12x - 11)$

(6) $a = 4, b = -12, c = -11$ ⟹

$b^2 - 4ac = (-12)^2 - 4(4)(-11) = 144 - 176 = -32$

$\sqrt{-32} = i\sqrt{2^5} = i\sqrt{2^4}\sqrt{2^1} = 2^2 i\sqrt{2} = 4i\sqrt{2}$

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(6) cont'd

So $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{12 \pm 4i\sqrt{2}}{2(4)}$

$= \frac{4(3 \pm i\sqrt{2})}{8} = \frac{3 \pm i\sqrt{2}}{2}$

6pts

$\therefore \boxed{f(x) = 4(x+1)^2(x-2)\left(x-\left(\frac{3+i\sqrt{2}}{2}\right)\right)\left(x-\left(\frac{3-i\sqrt{2}}{2}\right)\right)}$

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(7) $R(x) = \dfrac{x^2-3x-10}{x^2-x-6} = \dfrac{(x-5)(x+2)}{(x-3)(x+2)} = \dfrac{x-5}{x-3} = R^*$ w/ a hole @ $x=-2$

Domain: $\mathbb{R} \setminus \{-2, 3\}$

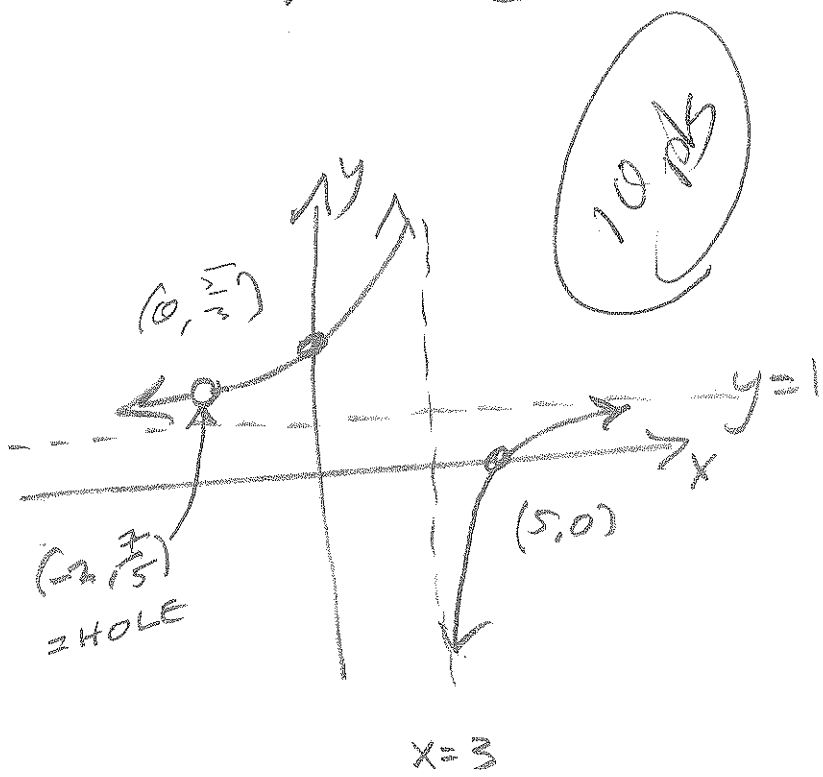
$x=3$ is V.A.

$x=-2$ is hole:

$\left(\dfrac{-2-5}{-2-3}\right) = R^*(-2)$

$= \dfrac{-7}{-5} = \dfrac{7}{5}$

$\left(-2, \dfrac{7}{5}\right)$ = HOLE



$R^*(x) = \dfrac{x-5}{x-3}$ SET 0

$\Rightarrow x-5=0$

$x=5$

$\rightsquigarrow$ (5,0) = x-int

$R^*(0) = \dfrac{0-5}{0-3} = \dfrac{5}{3}$

$\rightsquigarrow \left(0, \dfrac{5}{3}\right)$ = y-int

End behavior

deg Top = Deg Bottom

Look at b.y stuff:

$\dfrac{x-5}{x-3} \xrightarrow{|x|\to\infty} \dfrac{x}{x} = 1 = y$

H.A.

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(50) $(x-(3+2i))(x-(3-2i))$

$= x^2 - x(3-2i) - (3+2i)x + (3+2i)(3-2i)$

$= x^2 - 3x + 2ix - 3x - 2ix + 3^2 + 2^2$

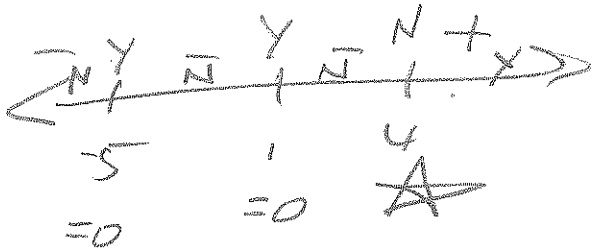
$= \boxed{x^2 - 6x + 13}$

5R

B1 Domain of $\sqrt{\frac{(x-1)^2(x+5)^2}{x-4}}$:5

$\{x \mid \frac{(x-1)^2(x+5)^2}{x-4} \geq 0\}$

$= \boxed{\{-5, 1\} \cup (4, \infty)}$



B2 $|-2x+3| > 5$

$-2x+3 > 5$ OR $-2x+3 < -5$

$-2x > 2$ $\quad -2x < -8$

$x < \frac{2}{-2} = -1$ $\quad x > \frac{-8}{-2} = 4$

$\{x \mid x < -1 \text{ OR } x > 4\} = \boxed{(-\infty, -1) \cup (4, \infty)}$

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B3 $f(x) = \sqrt{x+1}$ & $g(x) = \frac{x-2}{x+5}$

$\mathcal{D}(f) = [-1, \infty)$ (Need $x+1 \geq 0 \Rightarrow x \geq -1$)

$\mathcal{D}(g) = \mathbb{R} \setminus \{-5\}$ ($x+5$ in denom.)

$f \circ g = f(g(x)) = \sqrt{g(x)+1} = \sqrt{\frac{x-2}{x+5} + 1} = f \circ g$

$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \neq -5 \text{ and } \frac{x-2}{x+5} \geq -1\}$

$= \{x \mid x \neq -5 \text{ and } (x < -5 \text{ or } x \geq -\frac{3}{2})\}$

$= (-\infty, -5) \cup [-\frac{3}{2}, \infty) = \mathcal{D}(f \circ g)$

Scratch:

$\frac{x-2}{x+5} \geq -1$

$\frac{x-2}{x+5} + 1 \geq 0$

$\frac{x-2}{x+5} + \frac{x+5}{x+5} \geq 0$

$\frac{x-2+x+5}{x+5} \geq 0$

$\frac{2x+3}{x+5} \geq 0$

$2x+3 = 0$, $2x = -3$, $x = -\frac{3}{2}$

$x+5 = 0$, $x = -5$

	-5		$-\frac{3}{2}$	
	+ N	− Y	+ Y	
	Y	N	Y	

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B4

$2x^2 - 3x - 4 = 0$

$2x^2 - 3x = 4$

$2\left(x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2\right) = 4 + 2\left(\frac{3}{4}\right)^2 = 4 + 2\left(\frac{9}{16}\right)$

$= 4 + \frac{9}{8}$

$= \frac{32+9}{8} = \frac{41}{8}$

$2\left(x - \frac{3}{4}\right)^2 = \frac{41}{8}$

$\left(x - \frac{3}{4}\right)^2 = \frac{41}{16}$

$x - \frac{3}{4} = \pm\frac{\sqrt{41}}{4}$

$$x = \frac{3 \pm \sqrt{41}}{4}$$

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B5 $g(x) = \frac{x-2}{x+5}$ is 1-to-1 ?

If $g(x_1) = g(x_2)$. We show $x_1 = x_2$.

$$\frac{x_1-2}{x_1+5} = \frac{x_2-2}{x_2+5}$$

$$(x_1-2)(x_2+5) = (x_2-2)(x_1+5)$$

$$x_1x_2 + 5x_1 - 2x_2 - 10 = x_2x_1 + 5x_2 - 2x_1 - 10$$

$$5x_1 - 2x_2 = 5x_2 - 2x_1$$

$$7x_1 = 7x_2$$

$$x_1 = x_2$$ ∎

B6 Let x = # of hours Jenny works
y = " " " Jim "

Jenny takes 8 hrs, alone $\frac{1}{8}x$ = How much of job Jenny does

Jim " 10 " , alone $\frac{1}{10}y$ = How much of job Jim does

Jenny

Now, Jenny starts 3 hours early, so either $x = y+3$ or $y = x-3$.

$$\frac{1}{8}x + \frac{1}{10}y = 1 \longrightarrow \left[\frac{1}{8}x + \frac{1}{10}(x-3) = 1\right] \quad (80)$$

$$\longrightarrow 10x + 8(x-3) = 80$$

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B10 cont'd

$10x + 8(x-3) = 80$

$10x + 8x - 24 = 80$

$18x = 104$

$x = \frac{104}{18} = \frac{52}{9}$ hours

$\therefore y = x - 3 = \frac{52}{9} - 3 = \frac{52-27}{9} = \frac{25}{9} = y$

Check: Want 1 job done =

$\left(\frac{1}{8}\frac{\text{job}}{\text{hr}}\right)\left(\frac{52}{9}\text{ hr}\right) + \left(\frac{1}{10}\frac{\text{job}}{\text{hr}}\right)\left(\frac{25}{9}\text{ hr}\right)$

$= \frac{52}{72}\text{ job} + \frac{25}{90}\text{ job}$

$= \frac{13}{18}\text{ job} + \frac{5}{18}\text{ job} = \frac{18}{18}\text{ job} = 1\text{ job}!$

Cool!

Jenny works about 5.8 hrs

Jim " 2.8 hrs