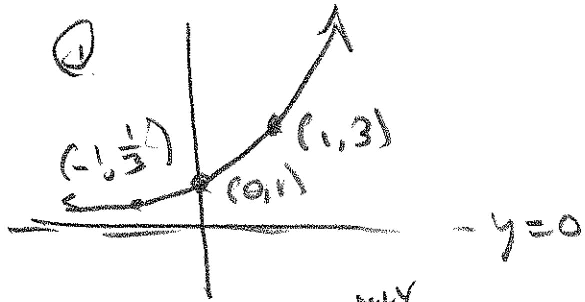
MAT 121 G81 TEST 4 Spring '14

(1) $f(x) = 3^x$, $g(x) = 2 \cdot 5^{x+4} - 3$ What?! Teacher's an idiot.

I'll do both $2 \cdot 5^{x+4} - 3$ and $2 \cdot 3^{x+4} - 3$

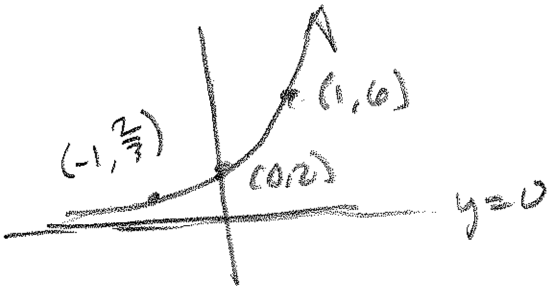
(1)



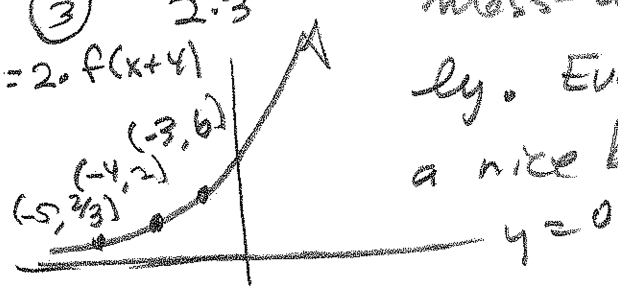
$2 \cdot 3^{x+4} - 3$ version

Every student dealt with my mess-up differently. Everyone got a nice bump in score.

(2) $2 \cdot 3^x = 2f(x)$



(3) $2 \cdot 3^{x+4}$
$= 2 \cdot f(x+4)$

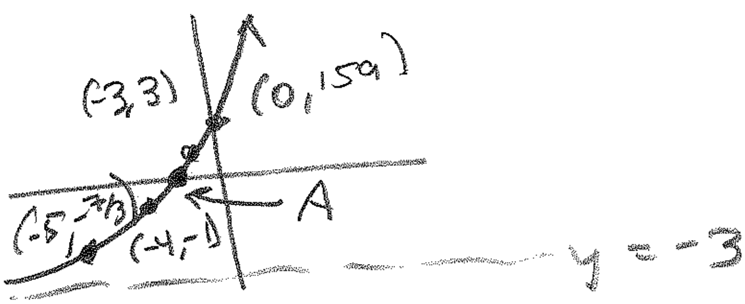


(4) $2 \cdot 3^{x+4} - 3 = 2 \cdot f(x+4) - 3$

$g(0) = 2 \cdot 3^4 - 3$
$= 2(81) - 3$
$= 162 - 3$
$= 159$

$\frac{2}{3} - 3 =$

$\frac{2-9}{3} = -\frac{7}{3}$



$$\boxed{A \approx (-3.6309, 0)}$$
5 Bonus

$g(x) = 0$

$2 \cdot 3^{x+4} - 3 = 0$

$2 \cdot 3^{x+4} = 3$

$3^{x+4} = \frac{3}{2}$

$x+4 = \log_3(3/2)$

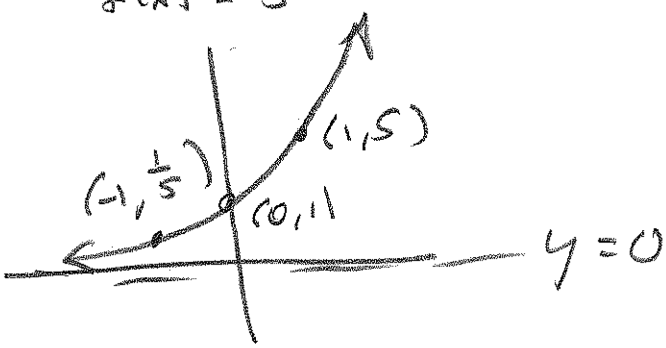
$x = \log_3(3/2) - 4$

$= \frac{\ln(3/2)}{\ln(3)} - 4$

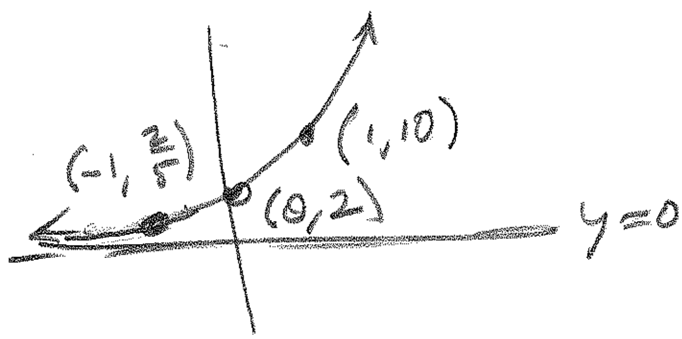
≈ -3.63092975

① $f(x) = 5^x$, $g(x) = 2 \cdot 5^{x+4} - 3$

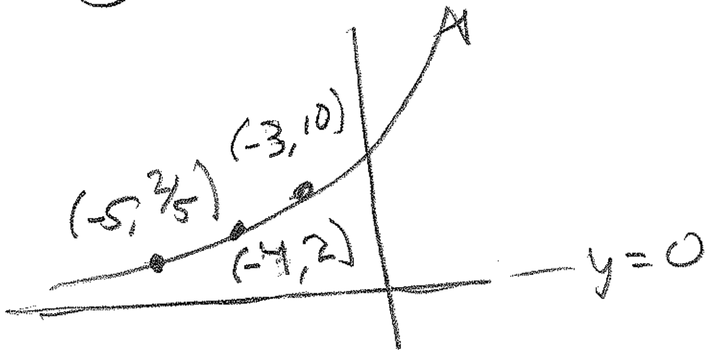
① $f(x) = 5^x$



② $2 \cdot 5^x = 2f(x)$

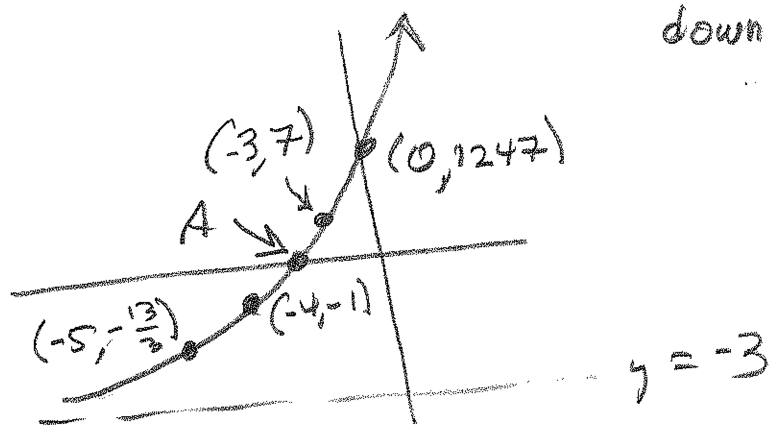


③ $2 \cdot 5^{x+4} = 2f(x+4)$



④ $2 \cdot 5^{x+4} - 3 = 2f(x+4) - 3$

(x, 2y) stretch (x−4, y) left (x, y−3) down



$\frac{2}{5} - 3 = \frac{2-15}{3} = -\frac{13}{3}$

$$g(0) = 2 \cdot 5^4 - 3$$
$$= 2(625) - 3$$
$$= 1250 - 3$$
$$= 1247$$

$\boxed{A \approx (-3.7481, 0) \quad \text{5 BONUS}}$

$$g(x) = 0$$
$$2 \cdot 5^{x+4} - 3 = 0$$
$$2 \cdot 5^{x+4} = 3$$
$$5^{x+4} = \frac{3}{2}$$
$$x + 4 = \log_5(3/2)$$

$$x = \log_5(3/2) - 4$$
$$= \frac{\ln(3/2)}{\ln(5)} - 4$$
$$\approx -3.748070364$$

A

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(3)

(2) $f(x)=\sqrt{2x+8}$, $g(x)=\dfrac{1}{x+2}$ $\Longrightarrow$

(a) Need $2x+8\geq 0$

$2x\geq -8$

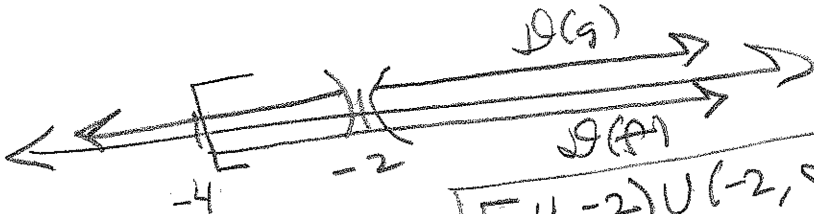
$\boxed{\begin{aligned}\mathcal{D}(f)&=\{x \mid x\geq -4\}\\ &=[-4,\infty)\end{aligned}}$

(b) Need $x+2\neq 0$

$\boxed{\begin{aligned}\mathcal{D}(g)&=\{x \mid x\neq -2\}\\ &=(-\infty,-2)\cup(-2,\infty)\end{aligned}}$

(c) $(f+g)(x)=\boxed{\sqrt{2x+8}+\dfrac{1}{x+2}}$

(d) $\mathcal{D}(f+g)=\mathcal{D}(f)\cap\mathcal{D}(g)$ = Intersection



Intersection is $\boxed{[-4,-2)\cup(-2,\infty)}$

$\boxed{=\{x \mid x\geq -4 \text{ and } x\neq -2\}}$

(e) $(f\circ g)(x)=f(g(x))=\sqrt{2g(x)+8}=\boxed{\sqrt{2\left(\frac{1}{x+2}\right)+8}}$

(f) Need $x\neq -2$ to keep $g(x)$ happy, inside of f.

Need $g(x)\geq -4$ to keep $f(g(x))$ happy.

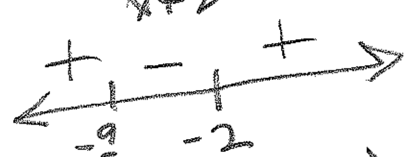
$\dfrac{1}{x+2}\geq -4$

$\dfrac{1}{x+2}+4\geq 0$

$\dfrac{1}{x+2}+\dfrac{4(x+2)}{x+2}\geq 0$

$\dfrac{4x+8+1}{x+2}\geq 0$

$\dfrac{4x+9}{x+2}\geq 0$



$\mathcal{D}(f\circ g)=(-\infty,-9/4]\cup[-2,\infty)$

$=\{x \mid x\leq -\frac{9}{4} \text{ OR } x\geq -2\}$

Kind of cramped at the bottom, there.

My "Hint" was totally misleading on #2f

$$\mathcal{D}(f \circ g) = \{ x \mid x \in \mathcal{D}(g) \ \underline{\text{AND}} \ g(x) \in \mathcal{D}(f) \}$$

$$= \{ x \mid x \neq -2 \ \text{AND} \ \frac{1}{x+2} \geq -4 \}$$

Handle $\frac{1}{x+2} \geq -4$ NEED <u>everything</u> on one side and 0 on the other. <u>THEN</u> need a good sign pattern:

$$\frac{1}{x+2} \geq -4$$

$$\frac{1}{x+2} + 4 \geq 0$$

$$\frac{1}{x+2} + \frac{4}{1} \cdot \frac{(x+2)}{(x+2)} \geq 0$$

$$\frac{1 + 4(x+2)}{x+2} \geq 0$$

$$\frac{1 + 4x + 8}{x+2} \geq 0$$

$$\frac{4x+9}{x+2} \geq 0$$

$4x+9=0$

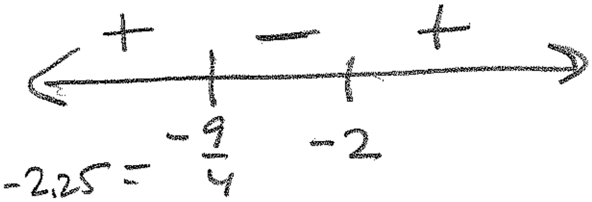
$4x=-9$

$x=-\frac{9}{4}$

$x+2=0$

$x=-2$

SIGN PATTERN



TEST: $x=-3$, $x=-2.2$, $x=0$

$$\frac{4(-3)+9}{-3+2} = \frac{-12+9}{-1} = \frac{-3}{-1} = 3 \ \text{Pos} \ +$$

$$\frac{4(-2.2)+9}{-2.2+2} = \frac{-8.8+9}{-.2} = \frac{.2}{-.2} = -1 \ \text{Neg} \ -$$

$$\frac{4(0)+9}{0+2} = \frac{9}{2} \ \text{POS} \ +$$

Want +. See SIGN PATTERN

$$\left(-\infty, -\tfrac{9}{4}\right] \cup (-2, \infty)$$

$x=-2$ is BAD, because of $x+2$ downstairs.

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Alternate for solving $\frac{1}{x+2} \geq -4$

via graph of $\frac{4x+9}{x+2}$

$\mathcal{D} = \{x \mid x \neq -2\}$

Vertical Asymptote $x=-2$ Change of sign @ $x=-2$

$\frac{4x+9}{x+2} = 0 \Longrightarrow 4x+9=0 \Longrightarrow \cdots\ x = -\frac{9}{4}$

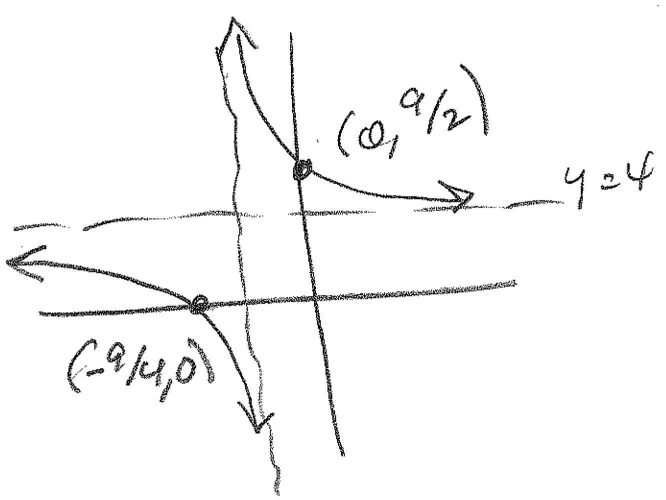
x-int: $(-\frac{9}{4}, 0)$ Change of sign at $x = -\frac{9}{4}$

y-int: $\frac{0+9}{0+2} = \frac{9}{2} \rightsquigarrow (0, \frac{9}{2})$ is y-int.

Horizontal asymptote

$\frac{4x+9}{x+2} \xrightarrow{x \to BIG} \frac{4x}{x} = 4$

$y=4$ is H.A.



By graph, $\frac{4x+9}{x+2} \geq 0$ when $x \leq -\frac{9}{4}$ OR

when $x \geq -2$

The y=4 tells me that it's positive (close to 4) when x is far left and far right. The y-int. tells me my work makes sense.

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(3) Domain of $\sqrt{\dfrac{(x+3)(x-2)^2}{(x-4)^3}}$

$$\frac{(x+3)(x-2)^2}{(x-4)^3} \ge 0$$

I get this "+" by observing everything is "+" once I make x BIG.

+		−		−		+
Yes	-3	No	2	No	4	Yes
	$=0$		$=0$			
			Yes			

$\mathcal{D} = (-\infty, -3] \cup \{2\} \cup (4, \infty)$

$= \{x \mid x \le -3$ OR $x = 2$ OR $x > 4$

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(3) Domain of $\sqrt{\dfrac{(x+3)(x-2)^2}{(x-4)^3}}$

1^{st}: Domain can't include $x=4$. Otherwise,

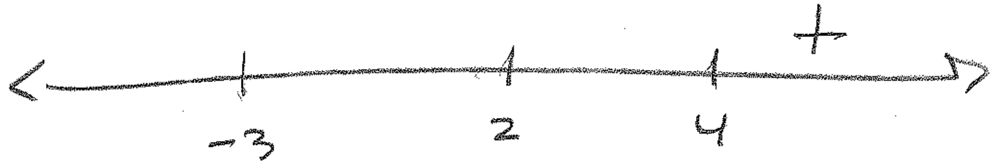
solve

$$\frac{(x+3)(x-2)^2}{(x-4)^3} \geq 0$$

$x=-3$, $m=1$ ODD CHANGES SIGN @ $x=-3$

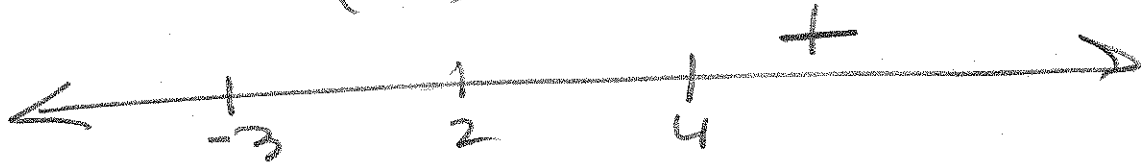
$x=2$, $m=2$ Doesn't CHANGE SIGN @ $x=2$

$x=4$, $m=3$ CHANGES SIGN @ $x=4$

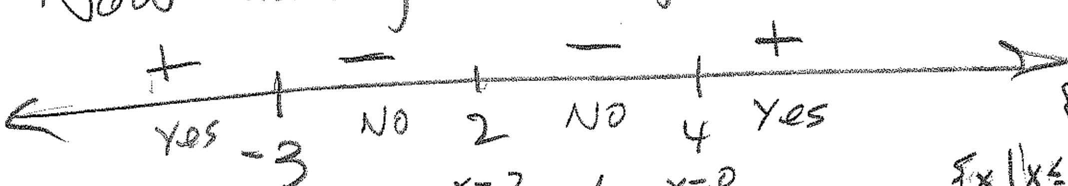


Need one spot for sign. Then manage sign changes

$x=5$: $\dfrac{(5+3)(5-2)^2}{(5-4)^3} = \dfrac{(POS)(POS)^2}{(POS)^3} = POS$



Now manage changes:



$x=2$ it $=0$!

$x=0$ Not included

FINAL ANS:

$\{x \mid x \leq -3 \text{ OR } x=2 \text{ OR } x>4\}$

$= (-\infty,-3] \cup \{2\} \cup (4,\infty)$

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4) $f(x) = 5^{2x-5} - 3$ Find $f^{-1}(x)$

$5^{2y-5} - 3 = x$

$5^{2y-5} = x + 3$

$2y - 5 = \log_5 (x+3)$

$2y = \log_5 (x+3) + 5$

$y = \boxed{\frac{1}{2}\left(\log_5 (x+3) + 5\right) = f^{-1}(x)}$

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5 (a) $1+3+9+27+\dots+19683$

$a=1,\ r=3$

$19683 = 3^9 = 3^{n-1}$

$9 = n-1$

$10 = n$

$S = a\left(\frac{1-r^n}{1-r}\right) = 1\left(\frac{1-3^{10}}{1-3}\right) = \frac{1-59049}{-2}$

or $a\left(\frac{r^n-1}{r-1}\right) = \frac{59048}{2} = \boxed{29{,}524}$

$3 \,|\, 19683$
$3 \,|\, 6561$
$3 \,|\, 2187$
$3 \,|\, 729$
$3 \,|\, 243$
$3 \,|\, 81$
$3 \,|\, 27$
$3 \,|\, 9$
3

(b) $\sum_{n=1}^{\infty} 2\left(\frac{2}{3}\right)^{n-1} = 2\left(\frac{1}{1-\frac{2}{3}}\right) = a\left(\frac{1}{1-r}\right) = 2\left(\frac{1}{\frac{1}{3}}\right) = \boxed{6}$

6 $\log_2(x-4)+\log_2(x+3)=3$

$2^{\log_2((x-4)(x+3))} = 2^3$

$(x-4)(x+3) = 2^3$

$x^2-x-12=8$

$x^2-x-20=0$

$(x-5)(x+4)=0$

$\boxed{x=5}$ or $\cancel{x=-4} \notin D$

$D =$
$\{x \mid x>4 \text{ and } x>-3\}$
$=\{x \mid x>4\}$

7 $\frac{1}{2}$-life is 5900 yrs

(a) $A(t) = A_0 e^{kt}$

$A(5900) = A_0 e^{5900k} = \frac{1}{2}A_0 = \frac{1}{2}$ is left in 5900 yrs

$e^{5900k} = \frac{1}{2}$

$5900k = \ln(1/2) = \overbrace{\ln(1) - \ln(2) = 0 - \ln(2)}^{\text{"STYLE"}}$

$5900k = -\ln(2)$

$\boxed{k = -\frac{\ln 2}{5900}} \approx -1.17482573 \times 10^{-4}$

$= -.000117482573 \approx k$

$\therefore A(t) = A_0 e^{-\frac{\ln 2}{5900}t}$

$\approx A_0 e^{-.000117482573t}$

(b) 43% of C-14 is gone $\longrightarrow$ 57% is left, i.e. $.57A_0$ is how much remains

Solve $A_0 e^{kt} = .57A_0$

$e^{kt} = .57$

$kt = \ln(.57)$

$t = \frac{\ln(.57)}{k} \approx \frac{\ln(.57)}{-.000117\ldots} \approx 4784.7 \approx \boxed{4785 \text{ yrs old}}$

A little more than $\frac{1}{2}$ is left after a little less than 5900 yrs!