

Use separate paper to do the work on this take-home test. Start a fresh sheet of paper to show work on #4. Use paper without lines. Use only one side of each sheet of paper. *I will not grade work written on the backs of pages.* Write clearly and make sure your pencil work is *dark*. It's a struggle for me to read faint print.

Let $f(x) = 2x^5 - 9x^4 + 58x^3 - 10x^2 - 128x + 87$. We will say everything we can about this polynomial.

1. (2 pts) Describe the end behavior of the graph of f with a simple graphic.
2. (2 pts) Use Descartes' Rule of Signs to determine the *possible* number of positive and negative zeroes of f .
3. (2 pts) Use the Rational Zeroes Theorem to determine the possible rational zeroes of f .
4. (2 pts) Informed by your work, above, use synthetic division to find the zeroes. Each time you find a zero, it *should* reduce (depress) the question by one degree. Each time you find a zero, you should thereafter be working with a *depressed polynomial* that is of lesser degree.
5. (2 pts) From your work, above, factor f over the real numbers. This will involve an irreducible quadratic factor.
6. (2 pts) From your work above, factor f over the complex numbers. This should split f into linear factors.
7. (2 pts) Give a rough sketch of f that shows all intercepts.
8. (2 pts) Sketch the graph of $\frac{x^2 + 2x - 3}{x^2 + 3x - 10} = \frac{(x+3)(x-1)}{(x+5)(x-2)}$. Factored and expanded form given for convenience. Show all asymptotes, intercepts and any holes.
9. (2 pts) The graph of $g(x) = \frac{x^3 + 4x^2 + x - 6}{x^3 + 5x^2 - 4x - 20} = \frac{(x+3)(x-1)(x+2)}{(x+5)(x-2)(x+2)}$ differs from the graph of f , in #8, in only one small detail. Sketch the graph of g , showing all asymptotes, intercepts and holes.
10. (2 pts) Sketch the graph of $h(x) = \frac{x^2 + 5x + 6}{x-1}$, showing all asymptotes, intercepts and holes.