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Use separate paper to do the work on this take-home test. Start a fresh sheet of paper to show work on \#4. Use paper without lines. Use only one side of each sheet of paper. I will not grade work written on the backs of pages. Write clearly and make sure your pencil work is dark. It's a struggle for me to read faint print.

Let $f(x)=3 x^{5}-10 x^{4}-22 x^{3}+88 x^{2}+56 x-160$. We will say everything we can about this polynomial.

1. (2 pts) Describe the end behavior of the graph of $f$ with a simple graphic.
2. (2 pts) Use Descartes' Rule of Signs to determine the possible number of positive and negative zeroes of $f$.
3. ( 2 pts ) Use the Rational Zeroes Theorem to determine the possible rational zeroes of $f$.
4. ( 2 pts ) Informed by your work, above, use synthetic division to find the zeroes. Each time you find a zero, it should reduce (depress) the question by one degree. Each time you find a zero, you should thereafter be working with a depressed polynomial that is of lesser degree.
5. ( 2 pts ) From you work, above, factor $f$ over the real numbers. This will involve an irreducible quadratic factor.
6. (2 pts) From your work above, factor $f$ over the complex numbers. This should split $f$ into linear factors.
7. ( 2 pts ) Give a rough sketch of $f$ that shows all intercepts.
8. (2 pts) Sketch the graph of $\frac{x^{2}+3 x+2}{x^{2}+2 x-8}=\frac{(x+2)(x+1)}{(x+4)(x-2)}$. Factored and expanded form given for convenience. Show all asymptotes, intercepts and any holes.
9. (2 pts) The graph of $g(x)=\frac{x^{3}-x^{2}-4 x+4}{x^{3}-12 x+16}=\frac{(x+2)(x+1)(x-3)}{(x+4)(x-2)(x-3)}$ differs from the graph of $f$, in \#8, in only one small detail. Sketch the graph of $g$, showing all asymptotes, intercepts and holes.
10. (2 pts) Sketch the graph of $h(x)=\frac{x^{2}+3 x+2}{x+4}$, showing all asymptotes, intercepts and holes.
