1. (10 pts) What is the domain of the function $f(x) = \sqrt{3x+2}$?

Need $3x+2 \geq 0$

$3x \geq -2$

$\mathcal{D} = \left\{ x \mid x \geq -\frac{2}{3} \right\} = \left[-\frac{2}{3}, \infty \right)$

2. (5 pts) Let $f(x) = \dfrac{x^2 + 2x}{2x-1}$. Find the following values:

a. $f(2) = \dfrac{2^2 + 2(2)}{2(2)-1} = \dfrac{4+4}{4-1} = \boxed{\dfrac{8}{3}}$

b. $f(-3) = \dfrac{(-3)^2 + 2(-3)}{2(-3)-1} = \dfrac{9-6}{-6-1} = \dfrac{3}{-7} = \boxed{-\dfrac{3}{7}}$

3. (10 pts) What is the average rate of change of the function $f(x) = \sqrt{x-1}$ from $x=5$ to $x=10$?

$\dfrac{f(10)-f(5)}{10-5} = \dfrac{\sqrt{10-1} - \sqrt{5-1}}{5} = \dfrac{\sqrt{9} - \sqrt{4}}{5} = \dfrac{3-2}{5} = \boxed{\dfrac{1}{5}}$

4. (10 pts) Find and simplify the difference quotient for $f(x) = x^2 - 5x$, that is, simplify $\dfrac{f(x+h)-f(x)}{h}$.

$= \dfrac{(x+h)^2 - 5(x+h) - (x^2 - 5x)}{h} = \dfrac{x^2 + 2xh + h^2 - 5x - 5h - x^2 + 5x}{h}$

$= \dfrac{2xh + h^2 - 5h}{h} = \dfrac{h(2x+h-5)}{h} = \boxed{2x+h-5}$

In MAT 201, they let $h \to 0$ at the end, and say "$f'(x) = 2x-5$," which is the DERIVATIVE of $f(x)$ and tells us the slope at any point on a CURVE!

5. Let $f(x) = \dfrac{x-2}{x-3}$ and $g(x) = \sqrt{3x+2}$.

   a. (5 pts) What is the domain of f?

Need $x - 3 \neq 0$

$\mathcal{D} = \{x \mid x \neq 3\}$

$= (-\infty, 3) \cup (3, \infty)$

   b. (5 pts) What is the domain of g?

$\{x \mid x \geq -\frac{2}{3}\} = [-\frac{2}{3}, \infty)$

   c. (5 pts) Write the function $(f - g)(x)$. Do not simplify. What is its domain?

$\dfrac{x-2}{x-3} - \sqrt{3x+2}$

$\mathcal{D} = \mathcal{D}(f) \cap \mathcal{D}(g) = [-\frac{2}{3}, 3) \cup (3, \infty)$

   d. (5 pts) Write the function $\left(\dfrac{f}{g}\right)(x)$. Do not simplify. What is its domain?

$\dfrac{\dfrac{x-2}{x-3}}{\sqrt{3x+2}}$ Stop!

$= \dfrac{x-2}{(x-3)\sqrt{3x+2}}$

Domain is same as part c, except $x = -2/3$ makes the denominator $= 0$, so

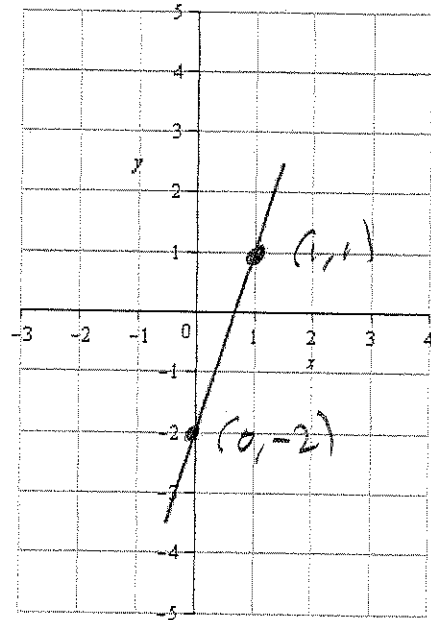$\mathcal{D} = (-\frac{2}{3}, 3) \cup (3, \infty)$

   e. (5 pts) Write the function $(f \circ g)(x)$. Do not simplify. What is its domain?

$f(g(x)) = \dfrac{\sqrt{3x+2} - 2}{\sqrt{3x+2} - 3}$

Scratch:

$\sqrt{3x+2} = 3$

$3x + 2 = 9$

$3x = 7$

$x = \dfrac{7}{3}$

$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \geq -\frac{2}{3} \text{ and } \sqrt{3x+2} \neq 3\}$

$= \{x \mid x \geq -\frac{2}{3} \text{ and } x \neq \frac{7}{3}\}$

$= [-\frac{2}{3}, \frac{7}{3}) \cup (\frac{7}{3}, \infty)$

6. (10 pts) Determine the equation of the line from its graph. Give
the equation in...

   a. ... point-slope form and

   b. ... slope-intercept form.

$$m = \frac{1-(-2)}{1-0} = \frac{3}{1} = 3$$
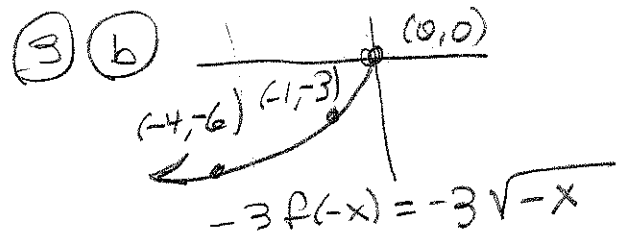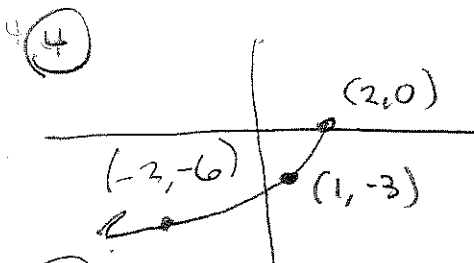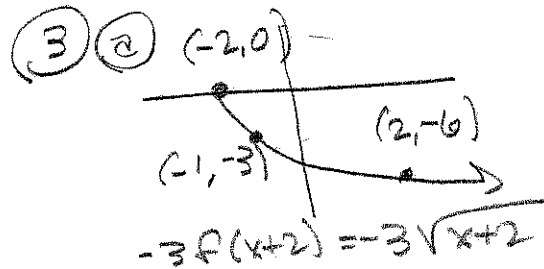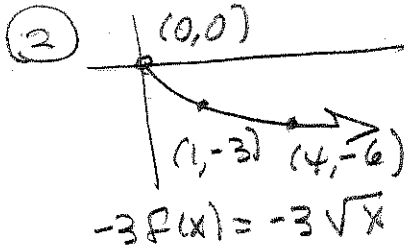
(a) $y = m(x-x_1) + y_1$

$$\boxed{y = 3(x-1) + 1}\quad \text{OR}\quad y = 3x - 2$$
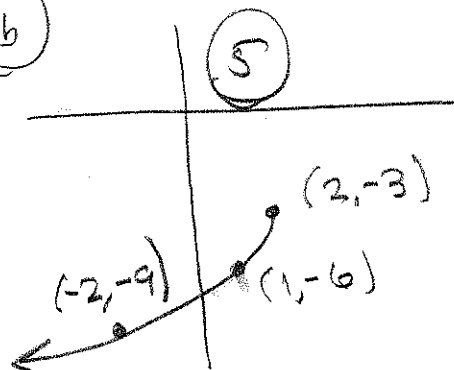
(b) $\boxed{y = 3x - 2}$

7. Graph each of the following by the techniques of shifting, stretching, compressing or reflecting. Start with the graph of a basic function and show all steps. I expect to see 3 points labeled in the first sketch, and to see where those points are moved to in each subsequent step.
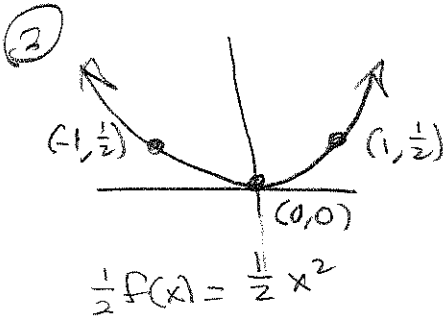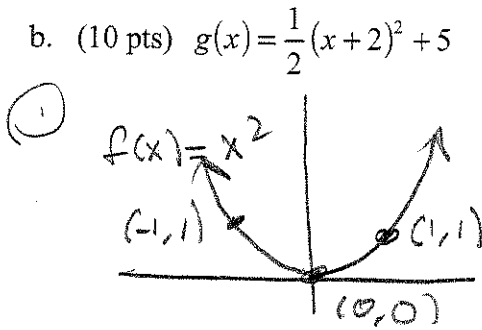
   a. (10 pts) $g(x) = -3\sqrt{-x+2} - 3$

① $f(x) = \sqrt{x}$ points $(0,0), (1,1), (4,2)$
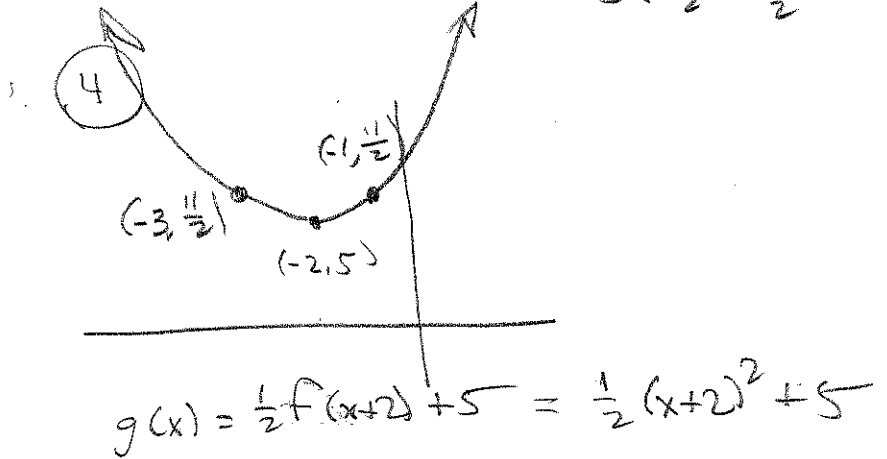
② $-3f(x) = -3\sqrt{x}$ $(0,0), (1,-3), (4,-6)$

③ⓐ $-3f(x+2) = -3\sqrt{x+2}$ $(-2,0), (-1,-3), (2,-6)$

④ $(2,0), (-2,-6), (1,-3)$

③ⓑ $-3f(-x) = -3\sqrt{-x}$ $(0,0), (-4,-6), (-1,-3)$

④ⓐ $-3\sqrt{-x+2} = -3\sqrt{-(x-2)}$ ④ⓑ

$$\mathcal{D} = (-\infty, 2]$$
$$\mathcal{R} = (-\infty, -3]$$

⑤ $(2,-3), (1,-6), (-2,-9)$

b. (10 pts) $g(x) = \frac{1}{2}(x+2)^2 + 5$

① $f(x) = x^2$

$(-1,1)$ $(1,1)$

$(0,0)$

② $(-1,\frac{1}{2})$ $(1,\frac{1}{2})$

$(0,0)$

$\frac{1}{2} f(x) = \frac{1}{2} x^2$

$5 + \frac{1}{2} = \frac{11}{2}$

③ $(-3,\frac{1}{2})$ $(-1,\frac{1}{2})$

$(-2,0)$

$\frac{1}{2} f(x+2) = \frac{1}{2}(x+2)^2$

④ $(-1,\frac{11}{2})$

$(-3,\frac{11}{2})$ $(-2,5)$
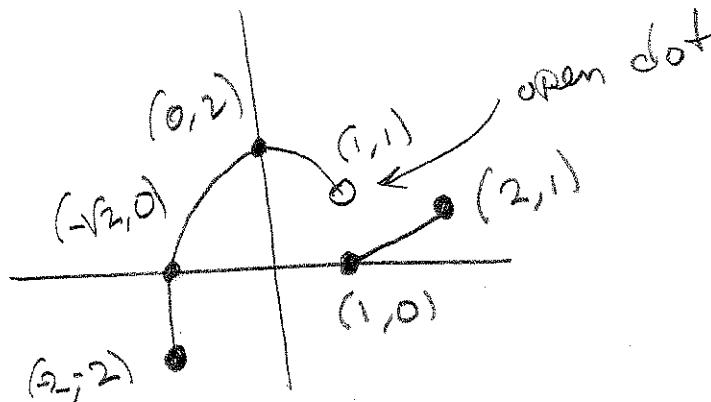
$g(x) = \frac{1}{2} f(x+2) + 5 = \frac{1}{2}(x+2)^2 + 5$
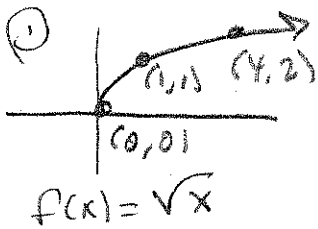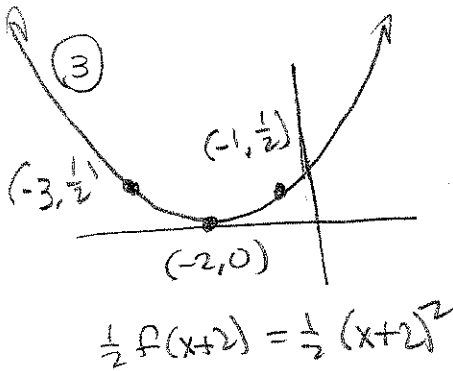
8. (10 pts) Sketch the graph of the piecewise-defined function $f(x) = \begin{cases} 2-x^2 & \text{if } -2 \le x < 1 \\ x-1 & \text{if } 1 \le x \le 2 \end{cases}$

$2-x^2$;

$2-(-2)^2 = 2-4 = -2$

$2-1^2 = 1$

$1-1 = 0$

$2-1 = 1$

$(0,2)$ $(2,1)$

$(-2,-2)$

$(2,1)$ $(1,0)$

open dot for "$x < 1$"

$(0,2)$ $(1,1)$

$(-\sqrt{2},0)$ $(2,1)$

$(1,0)$

$(-2,-2)$