2. For $f(x) = x^2$ -1 and $g(x) = \sqrt{2x-6}$, determine the following composite functions, *simplify them*, and state their domains:

a. (5 pts) $(f \circ g)(x)$

b. (5 pts) $(g \circ f)(x)$

3. (5 pts) What is the domain of $g(x) = \ln(2x-6)$?

4. (5 pts) What is the domain of $\sqrt{\dfrac{(x+5)^2}{(x-4)(x-1)^3}}$? (This is like a Chapter 3 question!)

$\mathcal{D}(f) = \mathbb{R}$ (Polynomial) $\quad \mathcal{D}(g) = \{x \mid 2x-6 \geq 0\}$

$f(x) = x^2-1, \quad g(x) = \sqrt{2x-6} \quad = \{x \mid 2x \geq 6\}$

$f(g(x)) = \left(\sqrt{2x-6}\right)^2 - 1 \quad = \{x \mid x \geq 3\}$

$= 2x-6-1 = 2x-7 \quad = [3, \infty)$

$\mathcal{D} = \{x \mid x \in \mathcal{D}(g) \text{ AND } g(x) \in \mathcal{D}(f)\}$

Since $\mathcal{D}(f) = \mathbb{R}$, this comes down

to $\mathcal{D}(g) = \{x \mid x \geq 3\} = [3, \infty) = \mathcal{D}(f \circ g)$

$g(f(x)) = \sqrt{2(x^2-1)-6} = \sqrt{2x^2-2-6}$

$= \sqrt{2x^2-8} = \sqrt{2(x^2-4)}$

Need

$\mathcal{D}(g \circ f) = \{x \mid x \in \mathcal{D}(f) \text{ AND } f(x) \in \mathcal{D}(g)\}$

Since $\mathcal{D}(f) = \mathbb{R}$, it comes down to

needing $f(x) \in \mathcal{D}(g)$, i.e.

$\sqrt{2(x^2-1)-6}$ needs to be real.

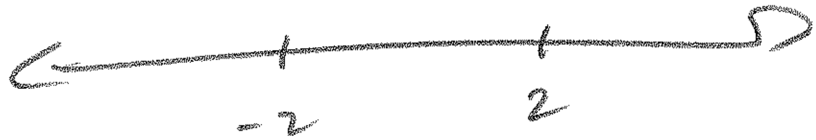
so $2(x^2-1)-6 \geq 0$ is needed.

so $2(x^2-4) \geq 0$ (see above)

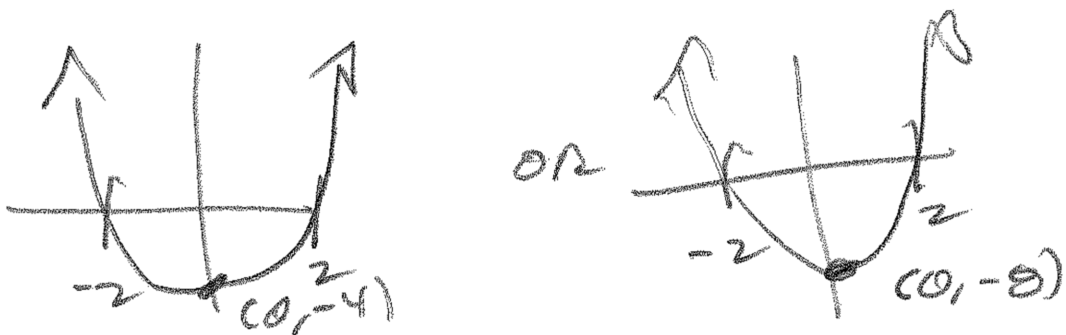
$2(x-2)(x+2) \geq 0$

So, critical pts are $x = \pm 2$

Lay out on # line:

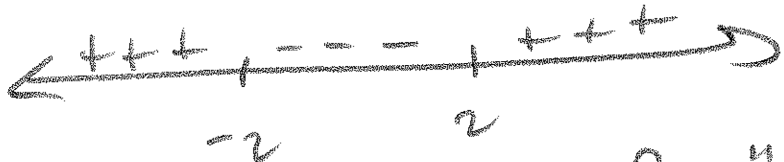


If you can Graph $x^2 - 4$ or $2(x^2-4)$:



$x^2 - 4$ $2(x^2-4)$

Then you can see the sign pattern

is 

We want the "+" for "≥ 0"

So $\mathcal{D}(g \circ f) = (-\infty, -2] \cup [2, \infty)$

$= \{x \mid x \leq -2 \text{ OR } x \geq 2\}$

Now you don't HAVE to graph it, to discern its sign pattern (positive/negative). You can also just use a test value in each interval, for instance, plug in x = 0 to test the sign in the interval (- 2, 2). (0)^2 - 4 = - 4 is all you need to know that it's < 0 between -2 and + 2. That's the - - - - signs in the sign pattern, above.