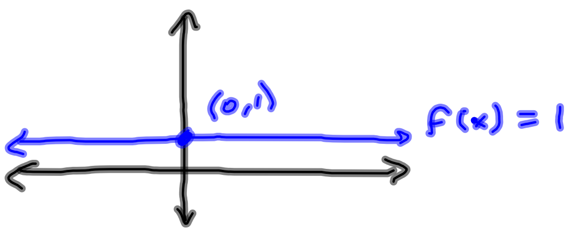
Basic Functions Quick Primer

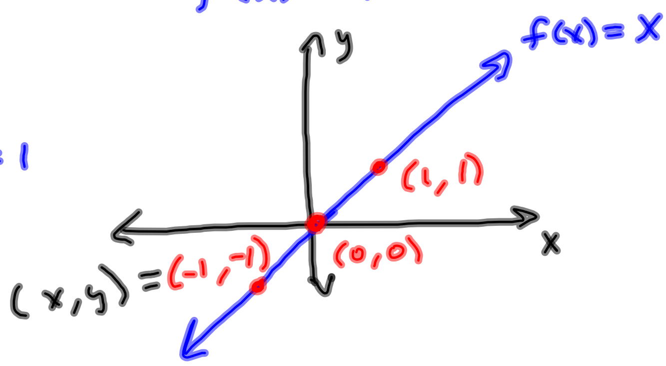
Constant

$f(x) = 1$



Identity

$f(x) = x$



Our goal is to see

$f(x) = 3x - 7$ as variations

OR

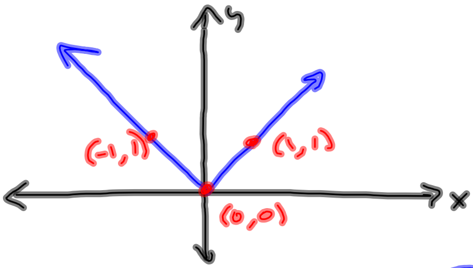
$f(x) = 2(x+5) - 11$ on the $f(x) = x$ theme.

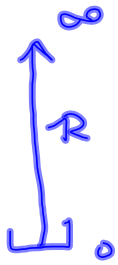
Down the road! If we can do that, we can graph a large family of OTHER functions, pretty easily.

See $2(x-5)^2 + 4$ as a variation on $f(x) = x^2$

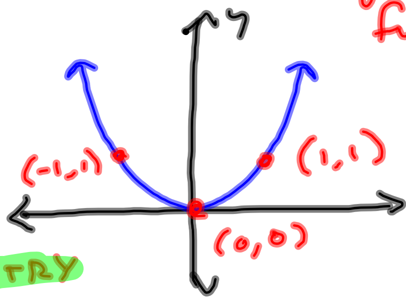
First, we need to KNOW the basic functions.

$f(x) = |x|$





$f(x) = x^2$ Square function



$\mathcal{D} = \{ x \mid f(x) \text{ is legal} \}$

$= \text{Domain}$

$= \mathbb{R} = (-\infty, \infty) =$

$= \{ x \mid x \text{ is real} \}$

$\mathcal{R} = \{ y \mid y = f(x) \text{ for some } x \in \mathcal{D} \}$

$= \text{Range} = [0, \infty)$

SYMMETRY ABOUT THE Y-AXIS

$f(-x) = f(x)$

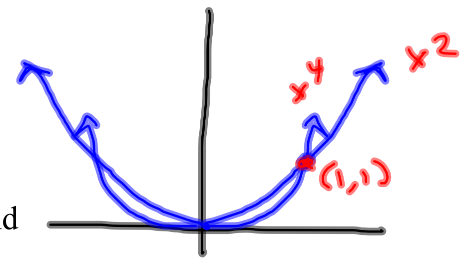
$\boxed{\text{EVEN}}$

$\mathcal{D}, \mathcal{R}$ same as $f(x) = |x|$.

x^2 is real, when x is.

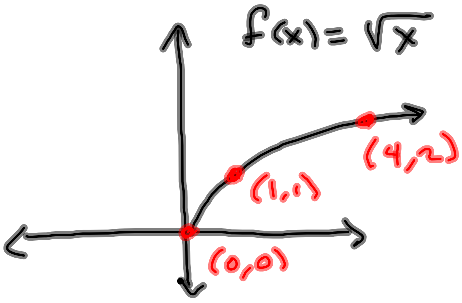
$x^4, x^6, x^8, x^{10}, \ldots$ all have pretty much the same shape/graph.

For $-1 < x < 1$, x^4 is below x^2. outside that domain, $x^4 \geq x^2$



But when you're graphing x^4 by itself, go ahead and just make the x^2 shape.

$f(x) = \sqrt{x}$



$\mathcal{D} = [0, \infty) = \{x \mid x \geq 0\}$

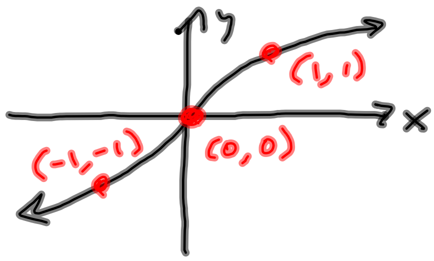
$\mathcal{R} = [0, \infty) = \{y \mid y \geq 0\}$

Same picture for

$f(x) = \sqrt[4]{x}, \sqrt[6]{x}, \sqrt[8]{x}, \ldots$

$x^{\frac{1}{4}}, x^{\frac{1}{6}}, x^{\frac{1}{8}}, \ldots$

$f(x) = \sqrt[3]{x}$



$\mathcal{D} = \mathbb{R}$

$\mathcal{R} = \mathbb{R}$

It's symmetric through/about the origin

It's ODD.

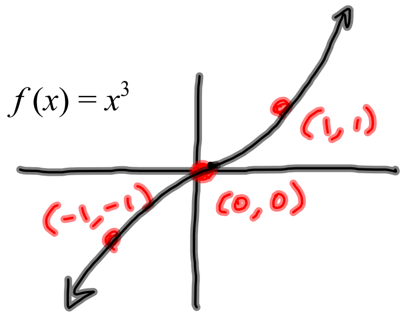
$f(-x) = \sqrt[3]{-x} = \sqrt[3]{(-1)x}$

$= \sqrt[3]{-1}\,\sqrt[3]{x} = -1\sqrt[3]{x}$

$= -\sqrt[3]{x} = -f(x).$

Looks like I cheated you of the basic graph of x^3

Let's do that on the next page.



$\mathcal{D} = \mathbb{R}$

$\mathcal{R} = \mathbb{R}$

It's symmetric through/about the origin

It's ODD.

$f(x) = x^5, x^7, x^9, \ldots$ have essentially the same graph, symmetry, domain and range as x^3.

I really want to see the basic graph, with 3 points labeled, and each successive graph to show where those points were moved to.

Although we have other techniques for graphing lines, or *because* we have other techniques for graphing lines, it's useful to see how you can graph any line by shifting/stretching the basic function $f(x) = x$.

$$g(x) = 3(x-2) + 5$$

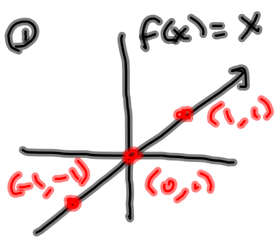
① $f(x) = x$

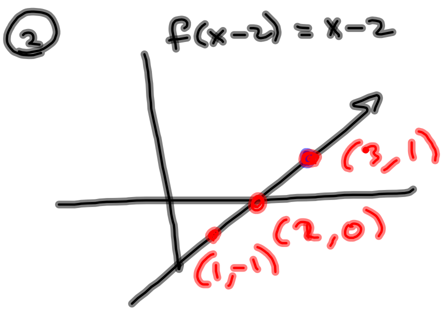
HORIZONTAL REFLECTION None

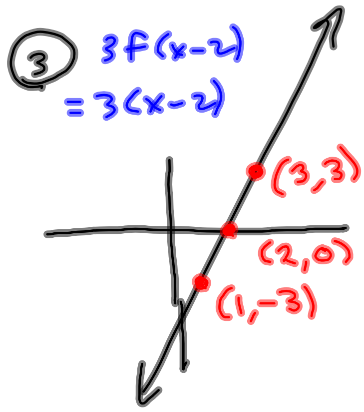
② HORIZONTAL SHIFT $f(x-2) = x-2$ RIGHT 2

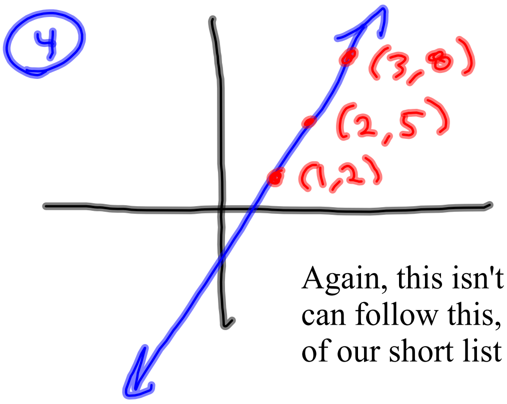
③ VERTICAL STRETCH $3f(x-2) = 3(x-2)$ Vertical stretch by a factor of 3, i.e., multiply y-values by 3.

④ VERTICAL SHIFT. $3f(x-2)+5$ Up 5.









$3f(x-2)+5$

$= 3(x-2)+5 = g(x) =$ our goal.

Again, this isn't our typical method for graphing lines, but if you can follow this, you can do this for ANY function that's built out of our short list of basic functions.

I'll generate some more examples.

A sequence of moves that will keep you on the right track:

1. Horizontal stretch/compress/reflect	$f(ax)$	Multiply x's by $1/a$
2. Horizontal shift	$f(x+a)$	Left a (Subtract a from x's)
3. Vertical stretch/compress/reflect	$a\,f(x)$	Multiply y's by a.
4. Vertical shift.	$f(x)+a$	Up a units.

If you stick to this sequence of moves, you will be OK. If you mix up #3 and #4 you will NOT be OK. If you mix up #1 and #2 you will NOT be OK.

You could do the vertical stuff first and then the horizontal, but why not just stick with what we have?