MAT 121-G81-G82 Test 3 – Fall, 2018 Name KEY

70 Points Covers Chapter 3 NO GRAPHING CALCULATORS!!!

Remember to use separate paper for everything except your name. Leave a margin in the top left corner. Spread out your work. Use only one column for your work. *Submit problems in order!!!*

1. (10 pts) Form a polynomial of *minimal degree* in *factored form* that has real coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong!

$x = 5 - 2i$, multiplicity 1; $x = -5$, multiplicity 4; $x = 2$, multiplicity 2.

2. (10 pts) Use synthetic division to find $P(3)$ if $P(x) = 3x^5 - 4x^4 + 8x^2 - 10x + 24$

3. (5 pts) Represent the work you just did on the previous problem by writing $P(x)$ in the form *Dividend* = *Divisor* • *Quotient* + *Remainder* .

4. Suppose $f(x) = (x-1)(x+3)^2(x-5)(x+2) = x^5 + 2x^4 - 22x^3 - 68x^2 - 3x + 90$. I'm showing you both factored and expanded form to help you answer the following:

   a. (10 pts) Solve the inequality $f(x) < 0$. Your sign pattern for this one will be helpful in the next two. You just have to interpret what you're seeing.

   b. (10 pts) Provide a rough sketch of *f*, using its zeros, their respective multiplicities and its end behavior. Include *x*- and *y*-intercepts. Your graph should be smooth. Un-exaggerate the vertical for a better quality graph.

   c. (5 pts) What is the domain of $g(x) = \sqrt{\dfrac{(x+3)^2(x-5)}{(x-1)(x+2)}}$?

5. Let $f(x) = 4x^5 + 32x^4 + 83x^3 + 55x^2 - 75x - 99$

   a. (5 pts) Use Descartes' Rule of Signs to determine the possible number of positive and negative zeros of *f*.

   b. (5 pts) List all possible rational zeros of *f*.

   c. (Bonus 5 pts) Find the smallest possible integer bounds on positive and negative zeros.

6. (10 pts) Find the *real* zeros of $f(x) = 4x^5 + 32x^4 + 83x^3 + 55x^2 - 75x - 99$. Then factor *f* over the set of **real numbers**. This should involve an irreducible quadratic factor.

(If things go haywire, come up with a *plausible*-looking polynomial, in factored form, with the right number of real roots and 2 nonreal roots. The 2 nonreal roots will still be living inside the irreducible quadratic factor, so you'll have to make up a quadratic factor with nonreal zeros).

7. (5 pts) Find the remaining (nonreal) zeros of f and factor f over the set of **complex numbers**. This step requires breaking down the quadratic piece that's irreducible over the real numbers. The fundamental theorem tells us that *nothing* is irreducible over the complex numbers.

   (You can still get full credit for this one, even if things went haywire, in #6, if you solve the depressed equation, correctly, and display your *plausible*-looking follow-up to your *plausible*-looking answer to #6. The more you know about what you're pointing towards, the more points you'll earn.)

8. (5 pts) You don't need to graph $R(x)=\dfrac{2x^3+7x^2-24x-45}{3x^2+x-10}$, here, but I do want to see its asymptotes.

   Hints: This function has no holes. Also, do not expect nice integer coefficients in your result.

9. (10 pts) Sketch the graph of $F(x)=\dfrac{2x^2-3x-9}{3x^2+x-10}$. Show all asymptotes and intercepts.

ANSWER ANY TWO (2) OF THE FOLLOWING, FOR UP TO 20 BONUS POINTS!!!

B1 (10 pts) Form a polynomial of *minimal degree* in *factored form* that has ***rational*** coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong.

Zeros: $x=2+\sqrt{2}$, multiplicity 2;
$x=1-4i$, multiplicity 1;
$x=7$, multiplicity 5.

B2 Solve both of the following absolute value inequalities.

a. (5 pts) $|2x+7|+8<9$

b. (5 pts) $|3x+11|+19>10$

B3 (10 pts) Sketch the graph of $R(x)=\dfrac{2x^3+7x^2-24x-45}{3x^2+x-10}$

Hints:

a. You already found $R(x)$'s asymptotes in #8.

b. One of $R(x)$'s x-intercepts is $(3,0)$.

B4 (10 pts) Sketch the graph of $G(x)=\dfrac{2x^3+7x^2-24x-45}{3x^3+16x^2-5x-50}$. Hint: $G(x)$ looks exactly like $F(x)$, from #9, except it has a hole.

B5 If $f(x)=\sqrt{x+11}$ and $g(x)=\dfrac{2}{x-6}$, what is the domain of $f\circ g$?

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1 10 pts

$$\boxed{(x-(5-2i))(x-(5+2i))(x+5)^4(x-2)^2}$$

2 10 pts

3	3	−4	0	8	−10	24
		9	15	45	159	447
	3	5	15	53	149	$\boxed{471 = P(3)}$

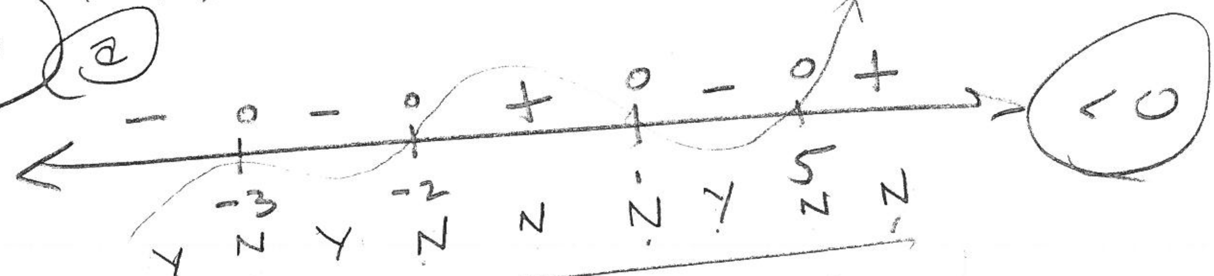
149
3
247

3 5 pts

$$\boxed{P(x) = (x-3)(3x^4+5x^3+15x^2+53x+149)+271}$$

4 $(x-1)(x+3)^2(x-5)(x+2) = x^5 \dots +90$

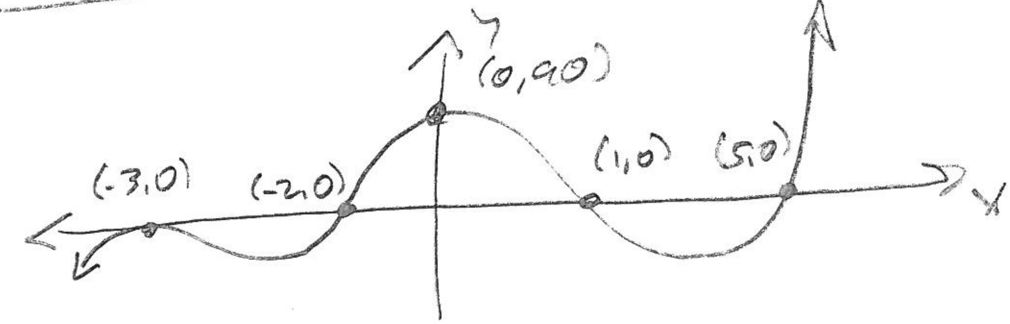
10 pts a



< 0

$$\boxed{x \in (-\infty,-3) \cup (-3,-2) \cup (1,5)}$$

b 10 pts



12, T3

4c 5pts $g(x) = \sqrt{\dfrac{(x+3)^2(x-5)}{(x-1)(x+2)}} = \sqrt{\text{STUFF}}$

$\mathcal{D}(g)$: Need STUFF ≥ 0

$-\quad 0 \quad - \quad ☆ \quad + \quad ☆ \quad - \quad 0 \quad +$ $\quad \geq 0$

$-3 \qquad -2 \qquad 1 \qquad 5$

N Y N N Y N N Y Y

$\boxed{\mathcal{D} = \{-3\} \cup (-2, 1) \cup [5, \infty)}$

(5) $f(x) = 4x^5 + 32x^4 + 83x^3 + 55x^2 - 75x - 99$

1

(a) 5pts Descartes $\boxed{\text{1 positive zero}}$

$f(-x) = -4x^5 + 32x^4 - 83x^3 + 55x^2 + 75x - 99$

1 2 3 4

$\boxed{\text{4, 2, or 0 negative zeros}}$

$3\,|\,99$
$3\,|\,33$
11

(b) 5pts p's: 99, q's: 4

$\pm 1, \pm 3, \pm 9, \pm 11, \pm 33, \pm 99$

$\pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}, \pm \frac{11}{2}, \pm \frac{33}{2}, \pm \frac{99}{2}$

$\pm \frac{1}{4}, \pm \frac{3}{4}, \pm \frac{9}{4}, \pm \frac{11}{4}, \pm \frac{33}{4}, \pm \frac{99}{4}$

18 of 'em

'21 T3

5c Bonus

$\underline{1}\rfloor$	4	32	83	55	-75	-99
		4	36	119	174	99
	4	36	119	174	99	0

ALL POSITIVE

By Theorem:

$\underline{-4}\rfloor$	4	32	83	55	-75	-99
		-16				
	4	NO				

$\underline{-8}\rfloor$	4	32	83	55	-75	-99
		-32	0	-664	HUGE	-BIG
	4	0	83	-609	HUGE	-REAL BIG

x = -8 is <u>A</u> L.B. on negative roots, by theorem.

By analysis, x = -3 is, in fact, the GLB.

The theorem doesn't necessarily find the actual greatest lower bound or least upper bound

121 T3

6 10pts $f(x) = 4x^5 + 32x^4 + 83x^3 + 55x^2 - 75x - 99$

$$\begin{array}{r|rrrrrr}
1 & 4 & 32 & 83 & 55 & -75 & -99 \\
 & & 4 & 36 & 119 & 174 & 99 \\
\hline
-3 & 4 & 36 & 119 & 174 & 99 & 0 \\
 & & -12 & -72 & -141 & -99 & \\
\hline
-3 & 4 & 24 & 47 & 33 & 0 & \\
 & & -12 & -36 & -33 & & \\
\hline
 & 4 & 12 & 11 & 0 & &
\end{array}$$

0 Sweet

0 Sweet!

0 Sweet!

$\begin{array}{r} 2\,47 \\ 3 \\ \hline 141 \end{array}$ $\begin{array}{r} 16 \\ 160 \\ \hline 176 \end{array}$

$b^2 - 4ac = 12^2 - 4(4)(11) = 144 - 176$

$= -32 < 0 \longrightarrow$ No real roots.

$\longrightarrow$ $\boxed{x = 1;\ x = -3, m = 2}$

8 $\boxed{f(x) = (x-1)(x+3)^2(4x^2 + 12x + 11)}$

$a = 4, b = 12, c = 11$

7 5pts Continuing:

$b^2 - 4ac = -32$

$\sqrt{-32} = 4i\sqrt{2}$

$\begin{array}{r|l} 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-12 \pm 4i\sqrt{2}}{2(4)} = \dfrac{-3 \pm i\sqrt{2}}{2}$

$\longrightarrow$ $\boxed{f(x) = 4(x-1)(x+3)^2\left(x - \frac{-3+i\sqrt{2}}{2}\right)\left(x - \frac{-3-i\sqrt{2}}{2}\right)}$

121 T3

8 5 pts $R(x) = \dfrac{2x^3+7x^2-24x-45}{3x^2+x-10}$

$= \dfrac{2x^3+7x^2-24x-45}{(3x-5)(x+2)}$

$\boxed{x = \frac{5}{3},\ x = -2}$ V.A.

S.A. 4.

$$\begin{array}{r} \frac{2}{3}x + \frac{19}{9} \\ 3x^2+x-10 \overline{\big) 2x^3+7x^2-24x-45} \\ -\left(2x^3+\frac{2}{3}x^2-\frac{20}{3}x\right) \\ \hline \frac{19}{3}x^2 \end{array}$$

$\dfrac{2x^3}{3x^2} = \frac{2}{3}x$

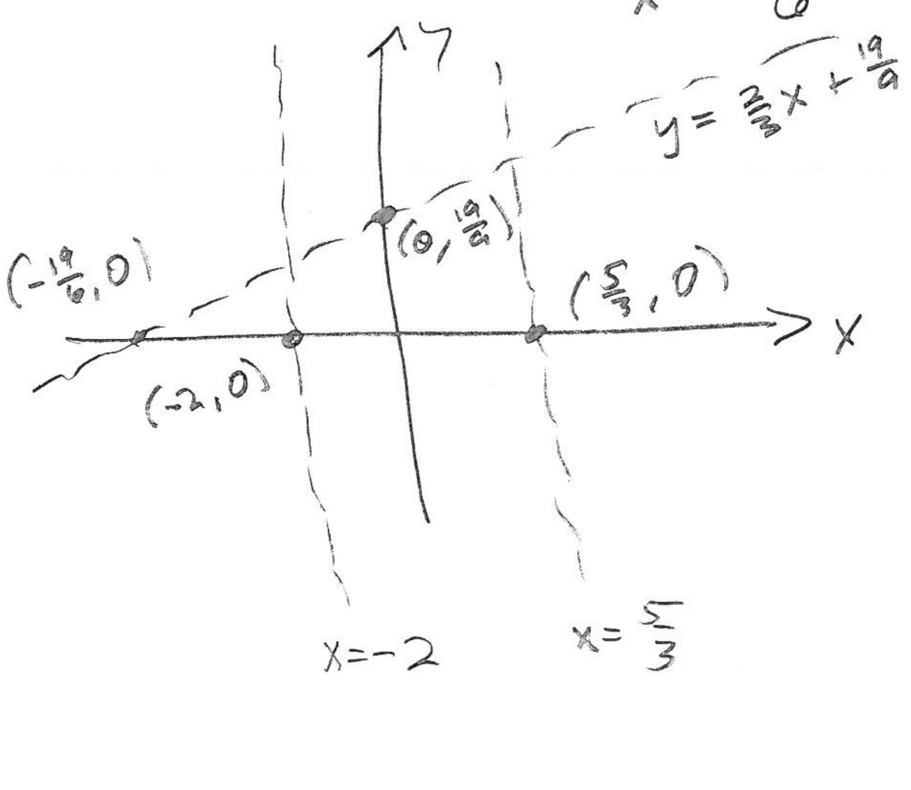
$7 - \frac{2}{3} = \frac{21-2}{3} = \frac{19}{3}$

$\dfrac{\frac{19}{3}x^2}{3x^2} = \frac{19}{9}$

$\frac{2}{3}x + \frac{19}{9} \overset{\text{SET}}{=} 0$

$\frac{2}{3}x = -\frac{19}{9}$

$x = -\frac{19}{6} \rightsquigarrow \left(-\frac{19}{6}, 0\right)$



121 T3

9 10 pts $F(x) = \dfrac{2x^2 - 3x - 9}{3x^2 + x - 10} = \dfrac{(2x+3)(x-3)}{(3x-5)(x+2)}$

$D = \mathbb{R} \setminus \{-2, \frac{5}{3}\}$

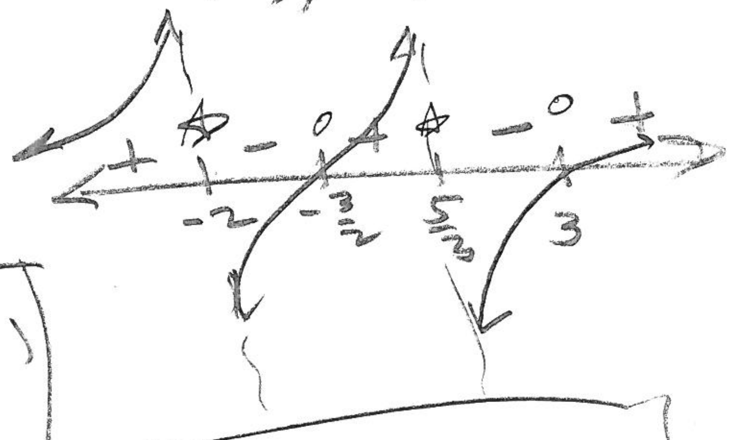
$\boxed{\text{V.A.: } x = -2,\ x = \frac{5}{3}}$

$\boxed{\text{H.A.: } y = \frac{2}{3}}$

x-int: $(-\frac{3}{2}, 0), (3, 0)$

y-int: $(0, \frac{9}{10})$

$-2, \frac{5}{3}, -\frac{3}{2}, 3$



A = (−2, 0)

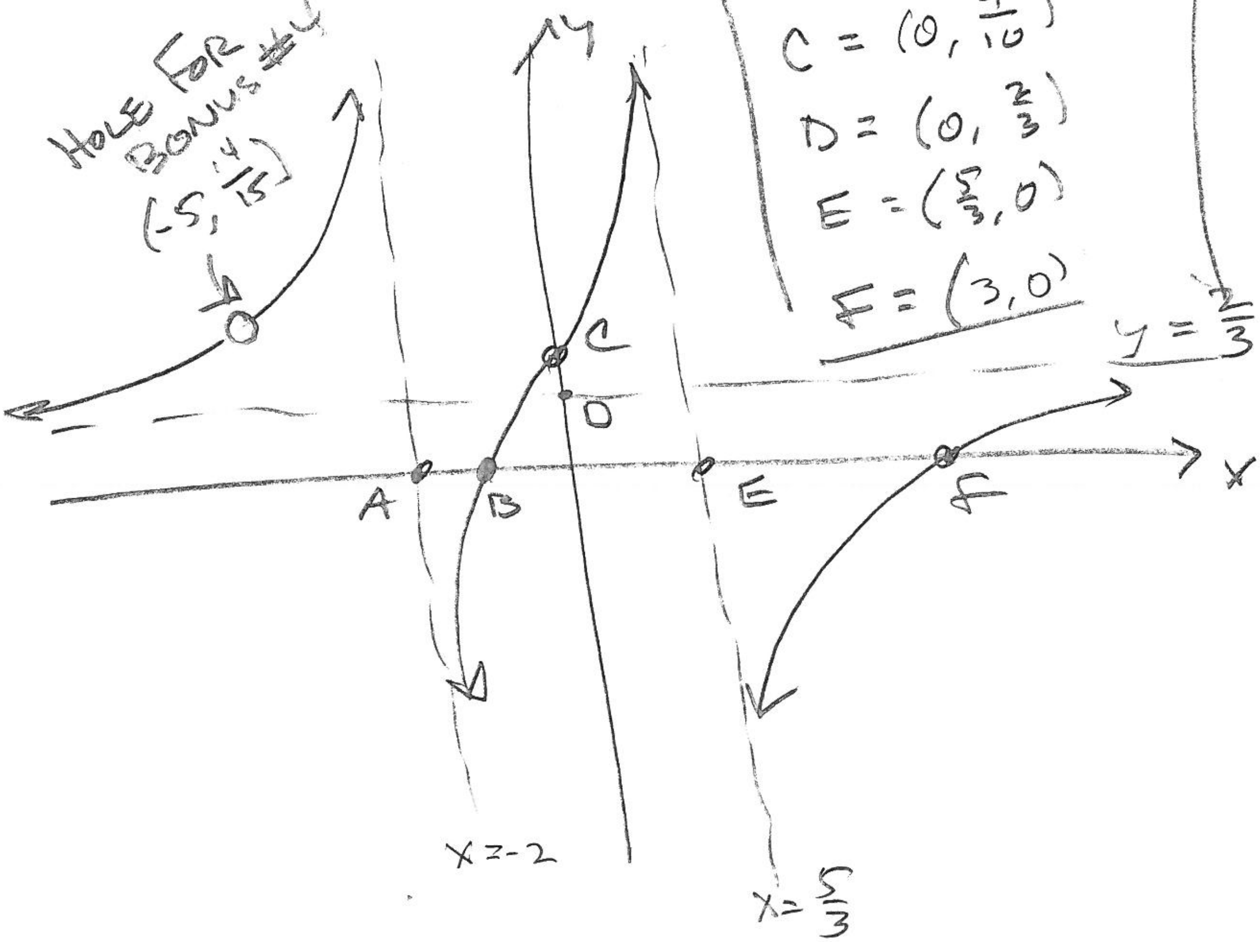
B = (−3/2, 0)

C = (0, 9/10)

D = (0, 2/3)

E = (5/3, 0)

F = (3, 0)



121 J3

B5 10 pts $f(x)=\sqrt{x+11}$, $g(x)=\frac{2}{x-6}$

$\Rightarrow f\circ g = f(g(x)) = \sqrt{g(x)+11}$

$= \sqrt{\frac{2}{x-6}+11}$

Need $x\neq 6$ for the $\frac{2}{x-6}$.

Need $\frac{2}{x-6}+11 \ge 0 \longrightarrow$

$\frac{2}{x-6}+\frac{11(x-6)}{x-6} \ge 0 \longrightarrow$

$\frac{2+11x-66}{x-6} = \frac{11x-64}{x-6} \ge 0$

$11x-64=0$

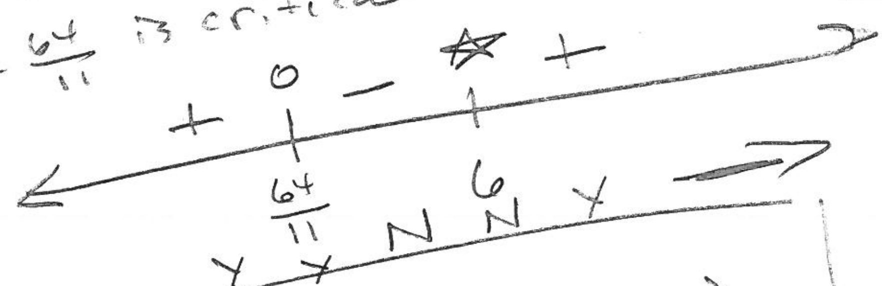
$11x=64$

$x=\frac{64}{11}$ is critical

$x-6=0$

$x=6$ is critical

Want ≥ 0



$\boxed{\mathcal{D}(f\circ g) = (-\infty, \frac{64}{11}] \cup (6,\infty)}$

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B1 10 pts $\boxed{(x-2+\sqrt{2})^2(x-2-\sqrt{2})^2(x-1+4i)(x-1-4i)(x-7)^5}$

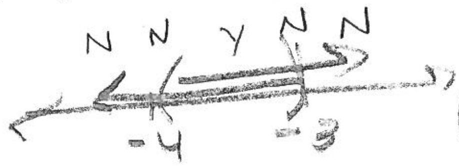
B2 a 5 pts $|2x+7|+8<9$

$|2x+7|<1$

$2x+7<1$ AND $2x+7>-1$

$2x<-6$ $2x>-8$

$\{x \mid x<-3$ AND $x>-4\}$



= AND

$= \boxed{(-4,3)}$

b 5 pts $|3x+11|+19>10$

$|3x+11|>-9$ ALWAYS

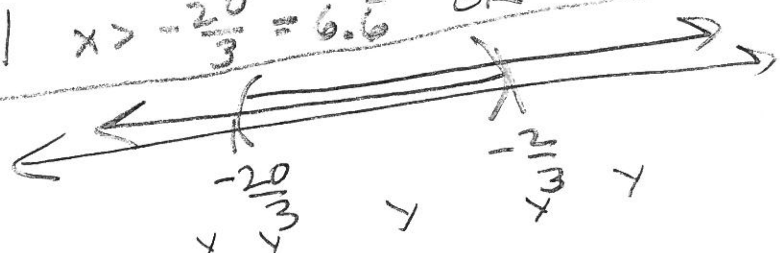
$\boxed{\begin{array}{l}(-\infty,\infty)\\ \{x \mid x\in\mathbb{R}\}\end{array}}$

$3x+11>-9$ OR $3x+11<9$

$3x>-20$ $3x<-2$

$\boxed{\{x \mid x>-\frac{20}{3}=6.\bar{6} \text{ OR } x<-\frac{2}{3}\}}$

OR



$= \boxed{(-\infty,\infty)}$

:12)   J3

B3 10 pts $R(x) = \dfrac{2x^3+7x^2-24x-45}{3x^2+x-10} = \dfrac{(2x+3)(x+5)(x-3)}{(3x-5)(x+2)}$

$\mathcal{D}: \mathbb{R} \setminus \{-2, \frac{5}{3}\}$

$-\frac{3}{2}, -5, 3$ $\frac{5}{3}, -2$

→ N.A.: $x=-2$, $x=\frac{5}{3}$

$x=3$ is a root:

$$\begin{array}{r|rrrr} 3 & 2 & 7 & -24 & -45 \\ & & 6 & 39 & 45 \\ \hline & 2 & 13 & 15 & 0 \end{array}$$

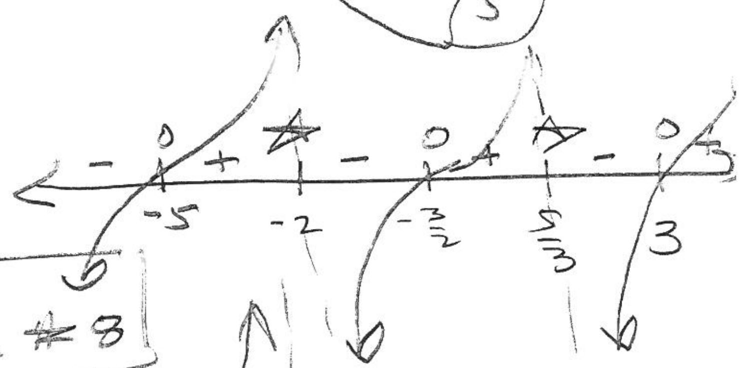
$\boxed{\text{y-int: } (0, \tfrac{9}{2})}$

Now, $2x^2+13x+15$ $15 \cdot 2 = 30$ $\begin{array}{r|l} 2 & 30 \\ 3 & 15 \\ & 5 \end{array}$

$= 2x^2+10x+3x+15$

$= 2x(x+5)+3(x+5)$

$= (x+5)(2x+3)$



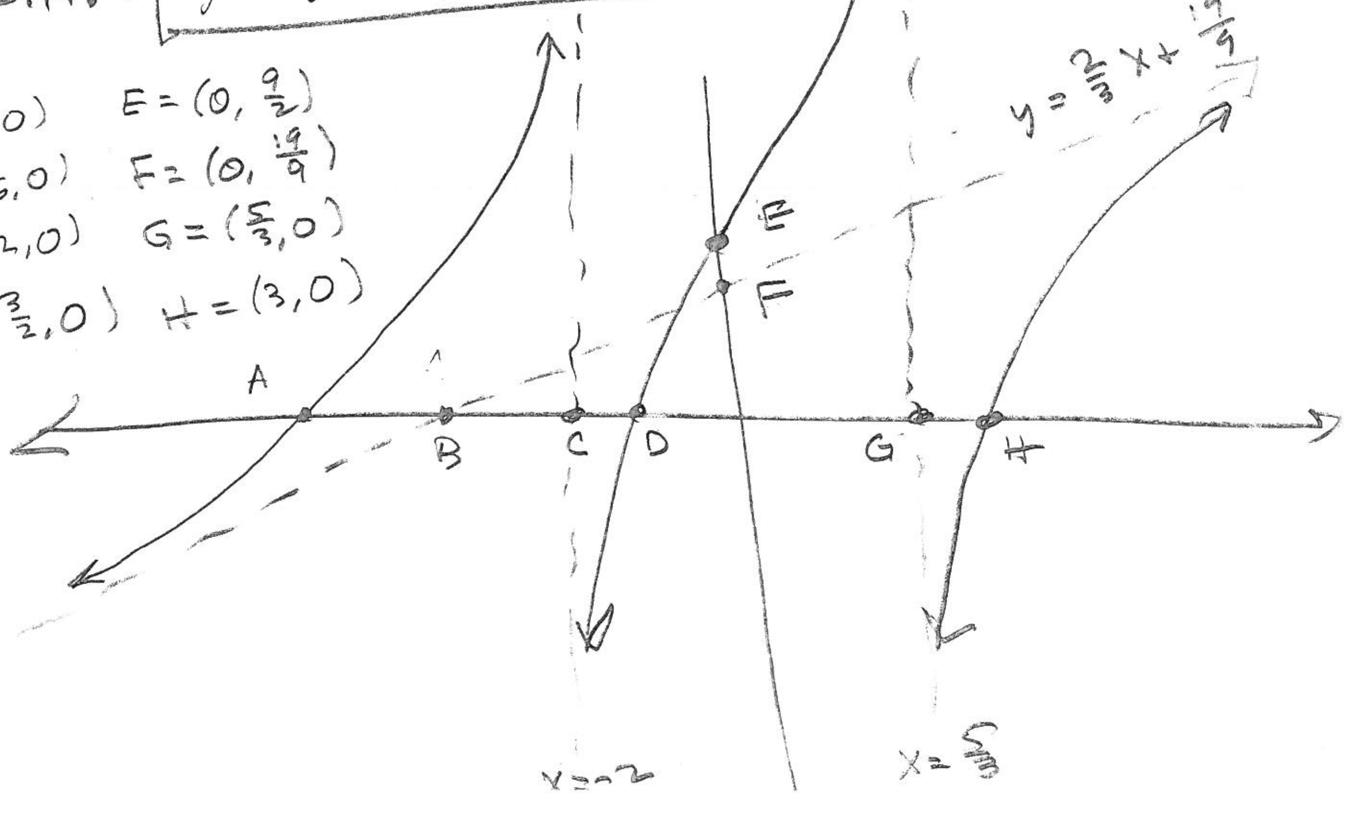
S.A.: $\boxed{y = \frac{2}{3}x + \frac{19}{9} \text{ from \#8}}$

$A = (-5, 0)$ $E = (0, \frac{9}{2})$

$B = (-\frac{19}{6}, 0)$ $F = (0, \frac{19}{9})$

$C = (-2, 0)$ $G = (\frac{5}{3}, 0)$

$D = (-\frac{3}{2}, 0)$ $H = (3, 0)$



121 T3

B4 10 pts From #9: $\dfrac{2x^2-3x-9}{3x^2+x-10} = \dfrac{(2x+3)(x-3)}{(3x-5)(x+2)}$

$$G(x) = \frac{2x^3+7x^2-24x-45}{3x^3+16x^2-5x-50} = R(x)\cdot\frac{(x-c)}{(x-c)}$$

Want to find c:

$$\begin{array}{r|rrrr} 3 & 2 & 7 & -24 & -45 \\ & & 6 & 39 & 45 \\ \hline -\frac{3}{2} & 2 & 13 & 15 & 0 \\ & & -3 & -15 & \\ \hline & 2 & 10 & & \end{array}$$

So $2x^3+7x^2-24x-45$

$= (x-3)(x+\frac{3}{2})(2x+10)$

$= 2(x-3)(x+\frac{3}{2})(x+5)$

$c=-5$

So $G(x) = \dfrac{(2x+3)(x-3)(x+5)}{(3x-5)(x+2)(x+5)}$ HOLE @ $x=-5$

And $F(-5) = \dfrac{(2(-5)+3)(-5-3)}{(3(-5)-5)(-5+2)} = \dfrac{(-7)\cancel{(-8)}^{2}}{\cancel{(-20)}_{5}(-3)}$

$= \dfrac{-14}{-15} = \dfrac{14}{15}$ ⤳ $\boxed{\left(-5, \frac{14}{15}\right) = \text{HOLE}}$

See #9 for HOLE