121 TEST 2 FALL, 2018 KEY

(2) $R = \{(1,3), (2,4), (3,5), (1,7)\}$

(1) Kindness

(a) No. (1,3) & (1,7) have different outputs for the same input, $x = 1$

(b) $D = \{1, 2, 3\}$

(c) $R = \{3, 4, 5, 7\}$

(d) R isn't a function, so 1-to-1 property is out of the question

(3) $f(x) = \frac{1}{x-5}$ & $g(x) = \sqrt{x+4}$

(a) $D(f) = \mathbb{R} \setminus \{5\} = (-\infty, 5) \cup (5, \infty)$
(NEED $x - 5 \neq 0$)

(b) $D(g) = [-4, \infty)$
(NEED $x + 4 \geq 0$)

(c) $\boxed{\frac{f}{g} = \frac{\frac{1}{x-5}}{\sqrt{x+4}}}$ OR $\frac{1}{(x-5)\sqrt{x+4}}$ if they don't follow instructions

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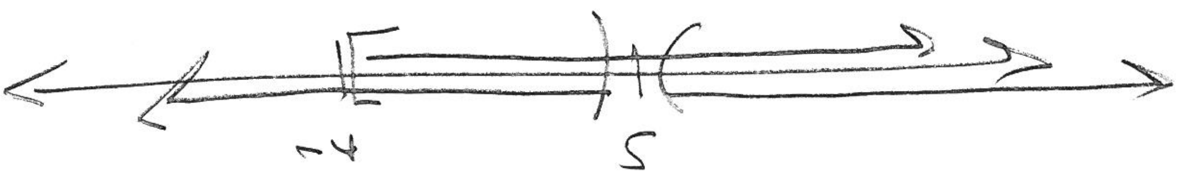
~~(2e) $f \circ g = \frac{1}{\sqrt{x+4} - 5}$~~

(2d) $\mathcal{D}\left(\frac{f}{g}\right) = \{x \mid x \in \mathcal{D}(f) \cap \mathcal{D}(g) \text{ and } g(x) \neq 0\}$



AND

$= \boxed{(-4,5) \cup (5,\infty)}$ we threw out $x=-4$

b/c $g(-4) = 0$

(2e) $f \circ g = \frac{1}{\sqrt{x+4} - 5}$

(2f) $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \geq -4 \text{ and } \sqrt{x+4} \neq 5\}$

$\sqrt{x+4} \neq 5$

$x+4 \neq 25$

$x \neq 21$

$= [-4,\infty) \cap (\mathbb{R} \setminus \{21\})$

AND

$=$ 

$= \boxed{[-4,21) \cup (21,\infty)}$

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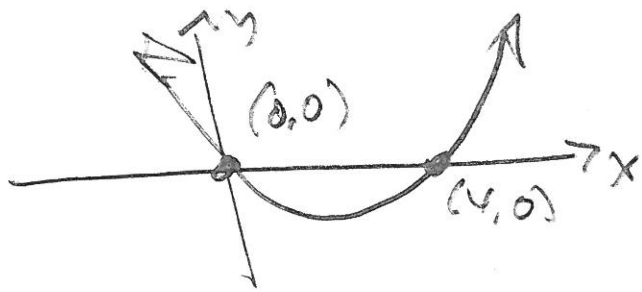
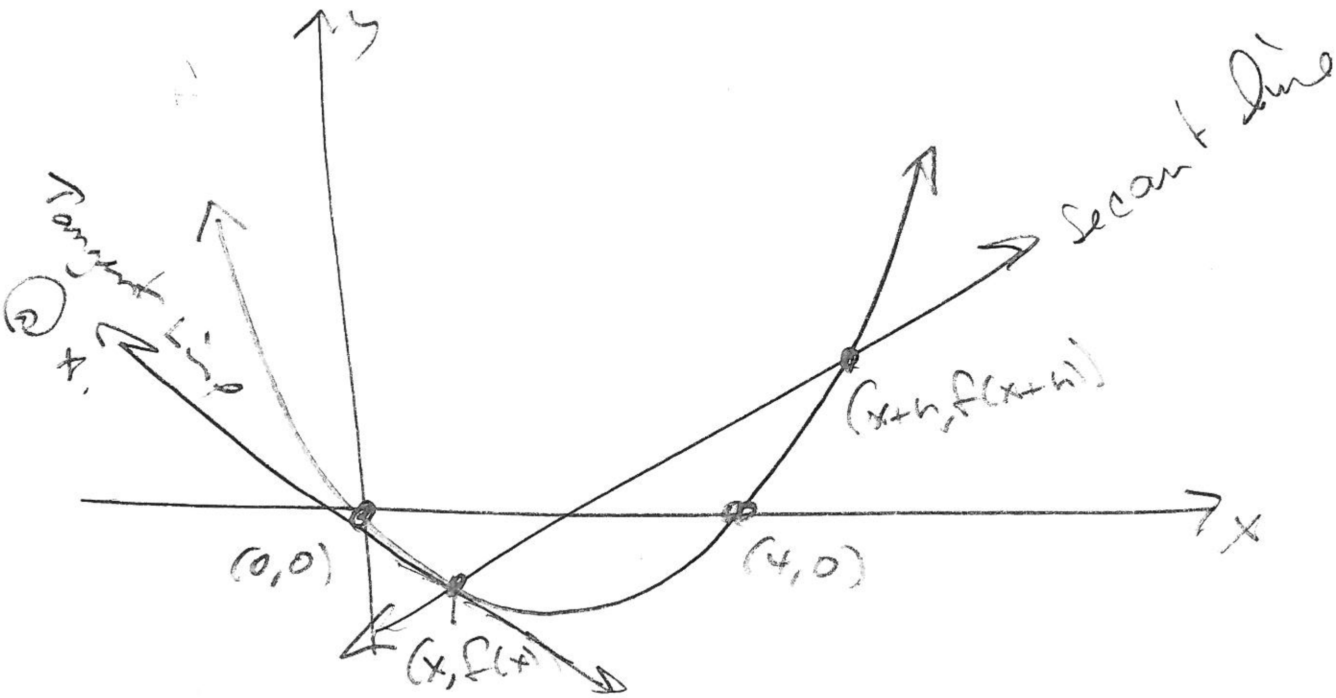
(4) $f(x) = x^2 - 4x$





$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - 4(x+h) - (x^2 - 4x)}{h} = m_{sec}$$

The difference quotient represents the slope of the secant line.

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5 $$\frac{(x+h)^2-4(x+h)-(x^2-4x)}{h}$$

$$= \frac{x^2+2xh+h^2-4x-4h-x^2+4x}{h}$$

$$= \frac{2xh+h^2-4h}{h} = \frac{h(2x+h-4)}{h} = \boxed{2x+h-4}$$

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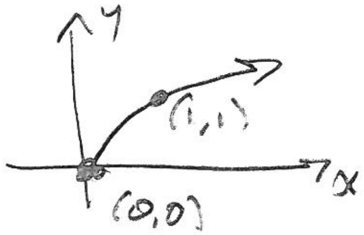
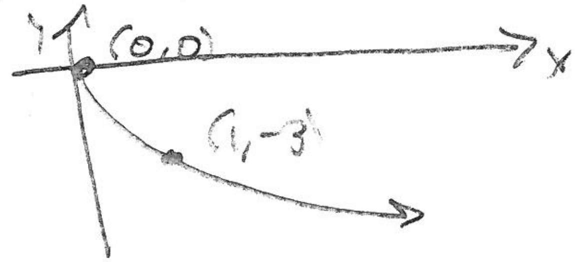
6 @ 20 pts $\quad g(x) = -3\sqrt{2x+8} + 7$

0

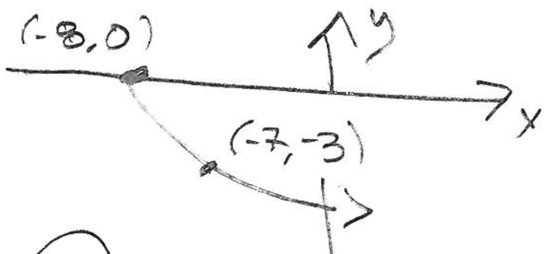


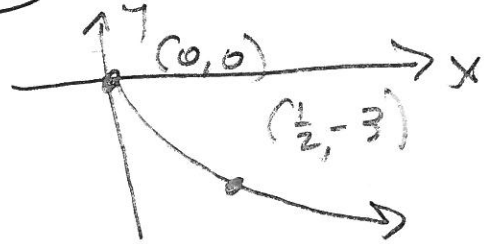
$f(x) = \sqrt{x}$

1 $\quad -3f(x) = -3\sqrt{x}$

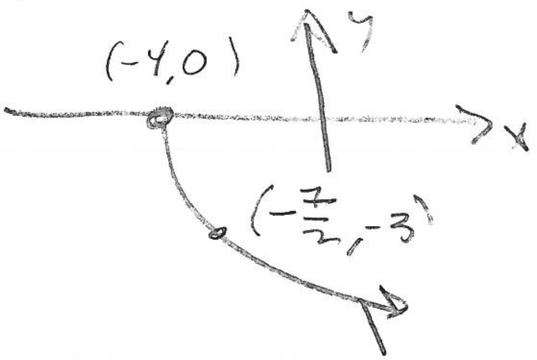


2 M1 $\quad -3\sqrt{x+8} = -3f(x+8)$ $\quad$ M2 $\quad -3\sqrt{2x} = -3f(2x)$





3 $\quad -3f(2x+8) = -3f(2(x+4)) = -3\sqrt{2x+8}$ $\quad$ OR $-3\sqrt{2(x+4)}$



6b

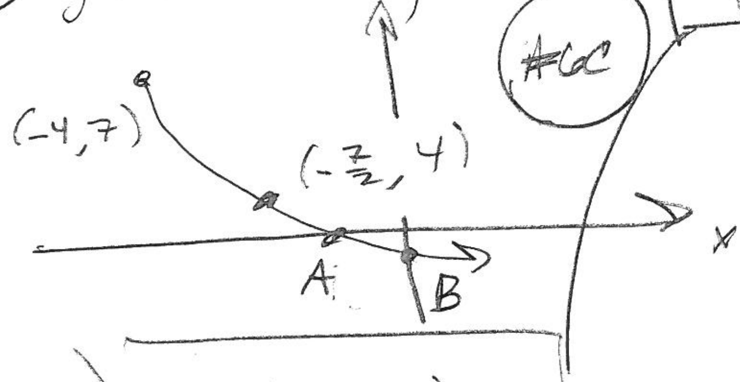
$$\boxed{\begin{array}{l} D = [-4, \infty) \\ R = (-\infty, 7] \end{array}}$$

$g(0) = -3\sqrt{8} + 7$

$= -6\sqrt{2} + 7 \approx -1.485$

$\boxed{B = (0, -1.485)}$

4 $\quad g(x)$



#GC

$\boxed{A = \left(-\frac{23}{18}, 0\right)}$

$g(x) = 0$

$-3\sqrt{2x+8} = -7$

$\sqrt{2x+8} = \frac{7}{3}$

$2x+8 = \frac{49}{9}$

$2x = \frac{49}{9} - \frac{72}{9} = \frac{-23}{9}$

$x = -\frac{23}{18}$

$-1.485 \approx -0.81374$

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7 5pB $f(x) = \frac{x+7}{x-7}$ is 1-to-1

$f(x_1) = f(x_2)$. Then

$$\frac{x_1+7}{x_1-7} = \frac{x_2+7}{x_2-7} \Longrightarrow$$

$$(x_1+7)(x_2-7) = (x_2+7)(x_1-7) \longrightarrow$$

$$x_1x_2 - 7x_1 + 7x_2 - 49 = x_2x_1 - 7x_2 + 7x_1 - 49 \Longrightarrow$$

$$-7x_1 + 7x_2 = -7x_2 + 7x_1 \Longrightarrow$$

$$-14x_1 = -14x_2 \Longrightarrow$$

$x_1 = x_2 \Longrightarrow f$ is 1-to-1 ∎

8 $\boxed{F = \frac{G m_1 m_2}{r^2}}$

B1 $2x + h - 4 \xrightarrow{h \to 0} \boxed{2x - 4 = f'(x)}$

B2

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B2 $f(x) = \sqrt{2x} \Longrightarrow$

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{2(x+h)}-\sqrt{2x}}{h} = \cancel{\left(\sqrt{2(x+h)}-\sqrt{2x}\right)(1}$$

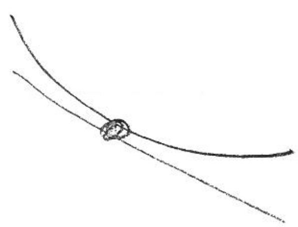
$$= \frac{\sqrt{2(x+h)}-\sqrt{2x}}{h} \cdot \frac{\sqrt{2(x+h)}+\sqrt{2x}}{\sqrt{2(x+h)}+\sqrt{2x}} = \frac{2(x+h)-2x}{h\left(\sqrt{2(x+h)}+\sqrt{2x}\right)}$$

$$= \frac{2x+2h-2x}{h\left(\sqrt{2(x+h)}+\sqrt{2x}\right)} = \frac{2\cancel{h}}{\cancel{h}\left(2(x+h)+\sqrt{2x}\right)}$$

$$= \frac{2}{\sqrt{2(x+h)}+\sqrt{2x}} \xrightarrow{h\to 0} \frac{2}{\sqrt{2x}+\sqrt{2x}} = \frac{2}{2\sqrt{2x}}$$

$$\boxed{= \frac{1}{\sqrt{2x}}}$$

B3



Tan line. See previous

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B4 $h(x) = 3x^2 + 2x - 11$

$= 3\left(x^2 + \frac{2}{3}x \qquad \right) - 11$

$= 3\left(x^2 + \frac{2}{3}x + \left(\frac{1}{3}\right)^2\right) - 11 - 3\left(\frac{1}{4}\right)$

$= 3\left(x + \frac{1}{3}\right)^2 - \frac{34}{3} \Longrightarrow (h,k) = \left(-\frac{1}{3}, -\frac{34}{3}\right)$

Scratch:

$-11 - \frac{1}{3} = \frac{-33-1}{3}$

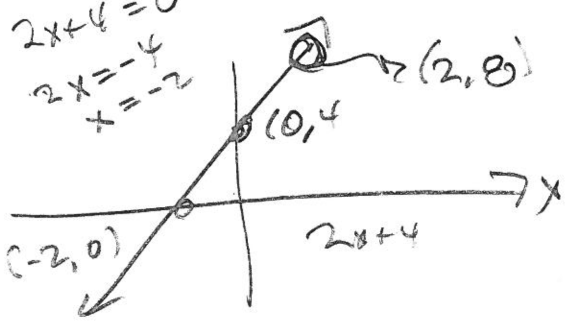
$= -\frac{34}{3}$

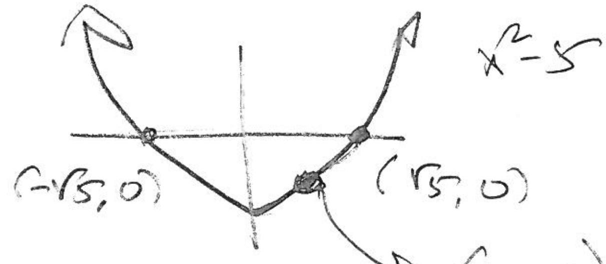
B5 $h(x) = \begin{cases} 2x+4 & \text{if } x < 2 \\ x^2-5 & \text{if } x \geq 2 \end{cases}$

$2x+4=0$

$2x=-4$

$x=-2$

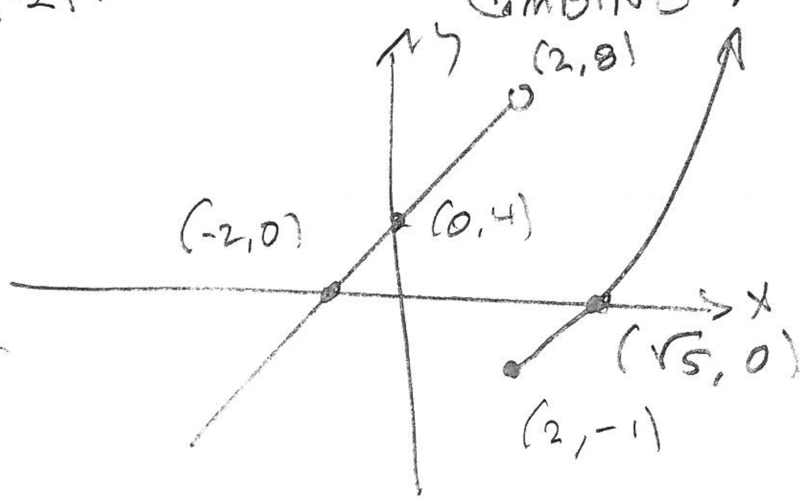




$2^2-5=-1$

$2(2)+4=8$

COMBINE:



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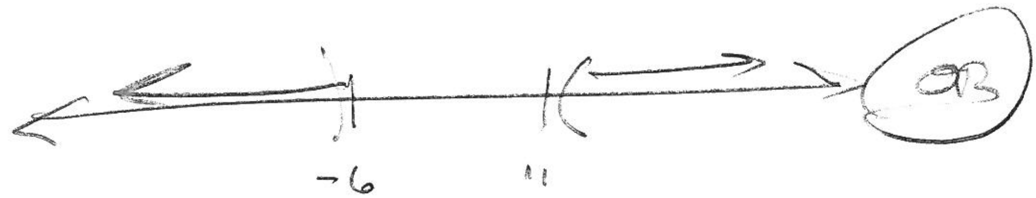
36 $|5-2x| > 17$

$5-2x > 17$ OR $5-2x < -17$

$-2x > 12$ OR $-2x < -22$

$\{x \mid x < -6$ OR $x > 11\}$

$=$ 

$= \boxed{(-\infty,-6) \cup (11,\infty)}$