121 FALL 15 TEST 2 121-G81

1. $f = \{(-2,3),(1,5),(2,8),(3,-2)\}$

a. is a function 5pts

b. $\mathcal{D} = \{-2,1,2,3\}$ 5pts

c. $\mathcal{R} = \{3,5,8,-2\}$ 5pts

d. yes, it's 1-to-1. 5pts

2. $f(x) = \dfrac{x+2}{x-3}$, $g(x) = \sqrt{x+4}$ $\longrightarrow$

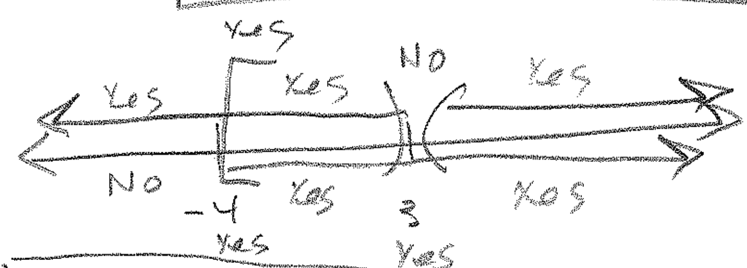
a. $\dfrac{f}{g} = \dfrac{\frac{x+2}{x-3}}{\sqrt{x+4}}$ 5pts

b. $\mathcal{D}(f) = \mathbb{R} \setminus \{3\}$, $\mathcal{D}(g) = \{x \mid x \geq -4\}$ 5pts

$= (-\infty,3) \cup (3,\infty)$ $\qquad = [-4,\infty)$

$\longrightarrow \mathcal{D}\left(\frac{f}{g}\right) = \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g) \text{ and } g(x) \neq 0\}$

$= \boxed{\{x \mid x \neq 3 \text{ and } \underbrace{x \geq -4 \text{ and } x \neq -4}_{x > -4}\}}$

= 

$= \boxed{[-4,3) \cup (3,\infty)}$

2 c $(f \circ g)(x) = f(g(x))$

$$= \boxed{\frac{\sqrt{x+4}+2}{\sqrt{x+4}-3}}$$

5 pts

d $\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \geq -4 \text{ and } \sqrt{x+4} \neq 3\}$

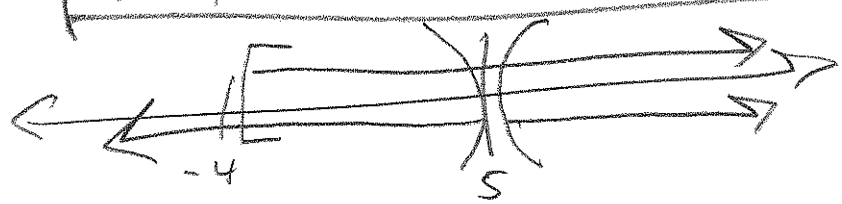
Scratch: $\sqrt{x+4} \neq 3$

$x+4 \neq 9$

$x \neq 5$

5 pts

$= \boxed{\{x \mid x \geq -4 \text{ and } x \neq 5\}}$

$=$ 

$= \boxed{[-4, 5) \cup (5, \infty)}$

(3) $f(x) = 3x^2 - 5x \longrightarrow$

$$\frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - 5(x+h) - (3x^2 - 5x)}{h}$$

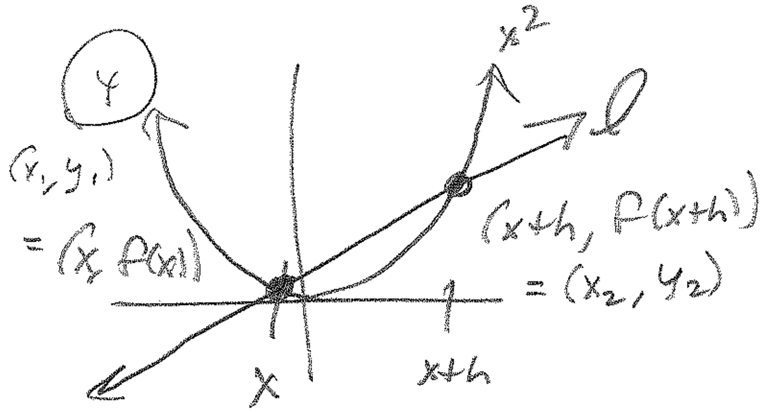
$$= \frac{3(x^2 + 2xh + h^2) - 5x - 5x - 3x + 5x}{h}$$

$$= \frac{3x^2 + 6xh + 3h^2 - 5x - 5h - 3x^2 + 5x}{h}$$

$$= \frac{6xh + 3h^2 - 5h}{h} = \frac{h(6x + 3h - 5)}{h}$$ 5pts

$$= \boxed{6x + 3h - 5}$$

BONUS: $\boxed{\xrightarrow{h \to 0} 6x - 5}$ 5pts



Slope of line l is $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{f(x+h) - f(x)}{(x+h) - x}$$

$$= \frac{f(x+h) - f(x)}{h} \; !$$

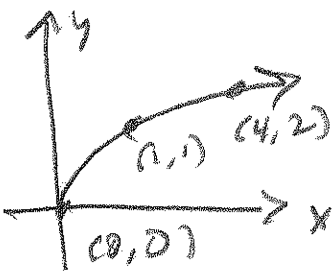
$$= \frac{(x+h)^2 - x^2}{h} \; !$$

5pts

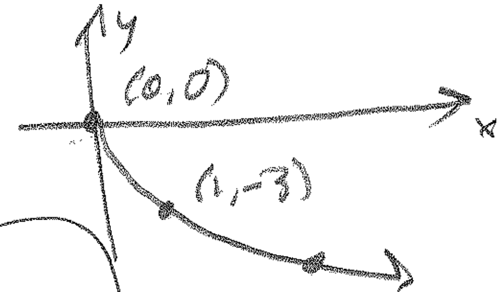
121 TEST 2

(5) $g(x) = -3\sqrt{-7x+14} + 5$ $\qquad -7x+14 = -7(x-2)$

(1) $f(x) = \sqrt{x}$

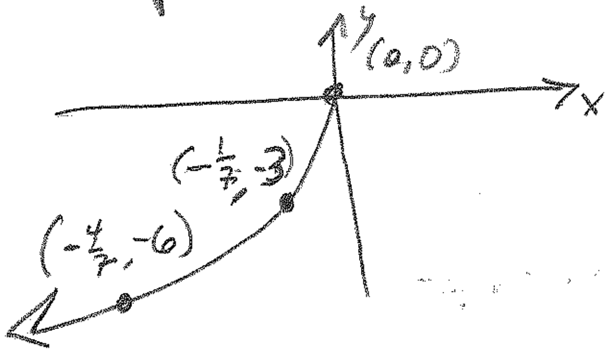


(2) $-3\sqrt{x} = -3f(x)$



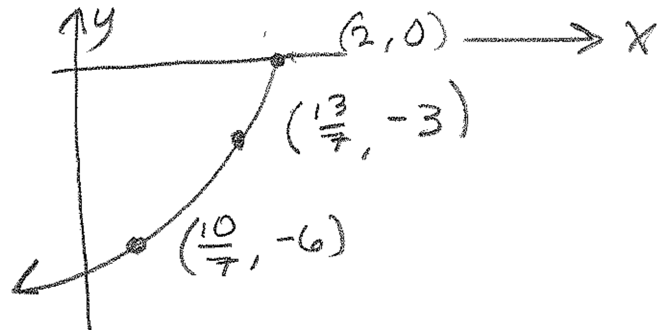
10 pts

(3) $-3\sqrt{-7x} = -3f(-7x)$



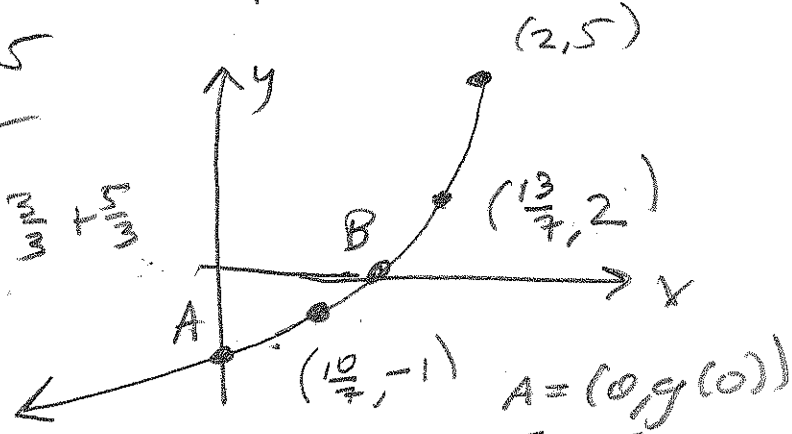
(4) $-3\sqrt{-7(x-2)}$

$= -3f(-7(x-2))$



(5) $g(x) = -3f(-7(x-2)) + 5$

$= -3\sqrt{-7(x-2)} + 5$



$-\frac{14}{1}\cdot\frac{3}{3} + \frac{5}{3}$

x-int:

$-3\sqrt{-7x+14} + 5 = 0$

$-3\sqrt{-7x+14} = -5$

$\sqrt{-7x+14} = \frac{5}{3}$

$-7x+14 = \frac{25}{9}$

$-7x = -\frac{101}{9} \rightarrow x = \frac{101}{63}$

$\frac{25}{9} - \frac{14}{1}\cdot\frac{9}{9} = -\frac{101}{9}$

$A = (0, g(0))$

$= -3\sqrt{14} + 5$

$\boxed{A = (0, -3\sqrt{14} + 5)}$

$\boxed{B = (\frac{101}{63}, 0)}$ 5 pts

$\boxed{A \approx (0, -6.224972) \quad B \approx (1.60317, 0)}$

121 TEST 2

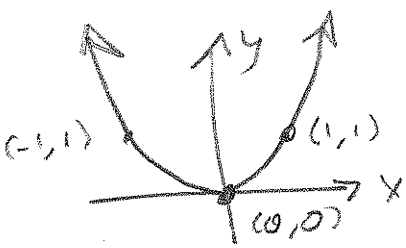
5c 5pts for intercepts

5b $D = (-\infty, 2]$ 5pts

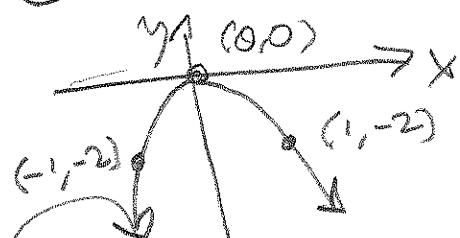
$R = (-\infty, 5]$

6 $r(x) = -2(x+6)^2 + 8$

1 $f(x) = x^2$

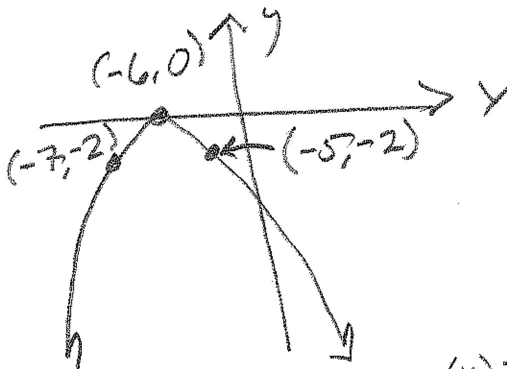


2 $-2f(x) = -2x^2$

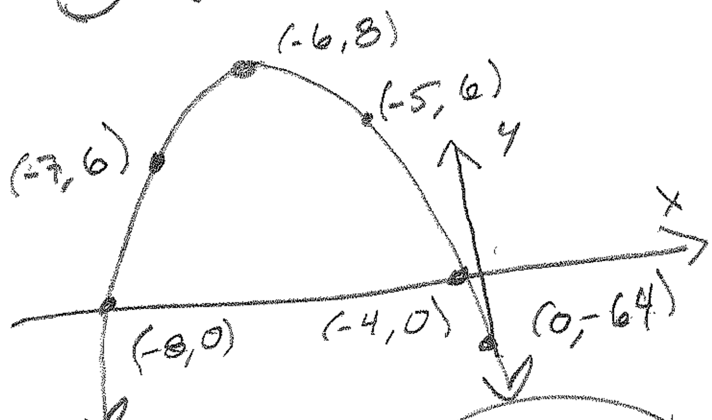


10pts

3 $-2f(x+6) = -2(x+6)^2$



4 $r(x) = -2f(x+6) + 8$



$r(x) = 0$

$-2(x+6)^2 + 8 = 0$

$-2(x+6)^2 = -8$

$(x+6)^2 = 4$

$x+6 = \pm 2$

$x = -6 \pm 2$

$-6+2 = -4$

$-6-2 = -8$

$r(0) = -2(6)^2 + 8$

$= -2(36) + 8$

$= -72 + 8$

$= -64 \rightarrow (0,-64)$

Intercepts #7 5pts

:121 TEST 2

8 $\quad \dfrac{x+7}{x-11} = f(x)$ is 1-to-1

$\boxed{\text{Pf}}$ $\quad \dfrac{y+7}{y-11} = x$

$y+7 = x(y-11) = xy - 11x$

$y - xy = -11x - 7$

$y(1-x) =$

This is finding f^{-1}. Want to prove f is 1-to-1.

8 5pts $\quad$ Pf $\quad f(x_2) = f(x_1) \Longrightarrow$

$$\frac{x_2+7}{x_2-11} = \frac{x_1+7}{x_1-11}$$

$$(x_2+7)(x_1-11) = (x_1+7)(x_2-11)$$

$\Longrightarrow x_2x_1 - 11x_2 + 7x_1 - 77 = x_1x_2 - 11x_1 + 7x_2 - 77$

$\Longrightarrow x_2x_1 - 11x_2 + 7x_1 = -11x_1 + 7x_2$

$\Longrightarrow -11x_2 + 7x_1 = -11x_1 + 7x_2$

$\Longrightarrow -18x_2 = -18x_1$

$\Longrightarrow x_2 = x_1$ ∎

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9 $y = k \dfrac{\sqrt[3]{x}\, z^3}{\sqrt{u}\, w^2}$ 5pts

10 $x = y^2 - 5 \Rightarrow$

$y^2 - 5 = x \Rightarrow$

$y^2 = x + 5 \Rightarrow$

$y = \pm\sqrt{x+5}$ 5pts

Observe $(0, \sqrt{5})$ & $(0, -\sqrt{5})$ are members of the relation. Not a func.!

5pts/Bonus

B1 $f(x) = \dfrac{1}{\sqrt{x}} \Rightarrow \dfrac{f(x+h) - f(x)}{h} = \dfrac{1}{h}\left[f(x+h) - f(x)\right]$

$= \dfrac{1}{h}\left[\dfrac{1}{\sqrt{x+h}} - \dfrac{1}{\sqrt{x}}\right] = \cancel{\dfrac{1}{h}\left[\dfrac{1}{x}\right]}$

$= \dfrac{1}{h}\left[\dfrac{1}{\sqrt{x+h}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} - \dfrac{1}{\sqrt{x}} \cdot \dfrac{\sqrt{x+h}}{\sqrt{x+h}}\right] = \dfrac{1}{h}\left[\dfrac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\,\sqrt{x+h}}\right]$

$= \dfrac{1}{h}\left[\dfrac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\,\sqrt{x+h}}\right]\left[\dfrac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}\right] = \dfrac{1}{h}\left[\dfrac{\sqrt{x}^2 - \sqrt{x+h}^2}{\sqrt{x}\,\sqrt{x+h}\,(\sqrt{x} + \sqrt{x+h})}\right]$

$= \dfrac{1}{h}\left[\dfrac{x - (x+h)}{\sqrt{x}\,\sqrt{x+h}\,(\sqrt{x} + \sqrt{x+h})}\right] = \dfrac{1}{h}\left[\dfrac{-h}{\sqrt{x}\,\sqrt{x+h}\,(\sqrt{x} + \sqrt{x+h})}\right]$

$= \dfrac{-1}{\sqrt{x}\,\sqrt{x+h}\,(\sqrt{x} + \sqrt{x+h})} \xrightarrow{h \to 0} \dfrac{-1}{\sqrt{x}\,\sqrt{x}\,(\sqrt{x} + \sqrt{x})}$

$= \boxed{-\dfrac{1}{2x\sqrt{x}}}$

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B2 $h(x) = 3x^2 + 5x - 5$

$= 3\left(x^2 + \frac{5}{3}x + \left(\frac{5}{6}\right)^2\right) - 5 - \frac{25}{12}$

$3\left(\frac{25}{36}\right) = \frac{25}{12}$ Added

$\frac{-60-25}{12}$

$= \boxed{3\left(x+\frac{5}{6}\right)^2 - \frac{85}{12}} \Rightarrow \boxed{(h,k) = \left(-\frac{5}{6}, -\frac{85}{12}\right)}$

B3 $r(x) = \dfrac{x-5}{x^2-5x-7}$

Need $x^2 - 5x - 7 \neq 0$

Solve $x^2 - 5x - 7 = 0$

$x^2 - 5x + \left(\frac{5}{2}\right)^2 = 7 + \frac{25}{4} = \frac{28+25}{4}$

$\left(x - \frac{5}{2}\right)^2 = \frac{53}{4}$

$x - \frac{5}{2} = \pm\sqrt{\frac{53}{4}} = \pm\frac{\sqrt{53}}{2}$

$x = \frac{5}{2} \pm \sqrt{\frac{53}{2}}$

$\boxed{D = \mathbb{R} \setminus \left\{ \frac{5}{2} \pm \frac{\sqrt{53}}{2} \right\}}$

$a=1, b=-5, c=-7$

$b^2 - 4ac = 25 - 4(1)(-7) = 53$

$= 25 + 28$

$x = \frac{5 \pm \sqrt{53}}{2}$

121 TEST 2

B4 $\quad w(x) = \dfrac{x^{77} - 5x^{12} + 17x}{\sqrt{-7x+14}}$

Need $-7x+14 \geq 0$ AND $\sqrt{-7x+14} \neq 0$

$\Rightarrow -7x+14 > 0$

$-7x > -14$

$\boxed{\{x \mid x < 2\}} = \boxed{(-\infty, 2)}$

B5 $\quad g(x) = -3\sqrt{-7x+14} + 5$

has $\mathcal{D} = (-\infty, 2] \longrightarrow \mathcal{D}(g^{-1}) = (-\infty, 5]$

$\mathcal{R} = (-\infty, 5] \longrightarrow \mathcal{R}(g^{-1}) = (-\infty, 2]$

B6

$-3\sqrt{-7y+14} + 5 = x$

$-3\sqrt{-7y+14} = x - 5$

$\sqrt{-7y+14} = \dfrac{x-5}{-3}$

$-7y + 14 = \left(\dfrac{x-5}{3}\right)^2$

$-7y = \left(\dfrac{x-5}{3}\right)^2 - 14$

$\boxed{y = -\frac{1}{7}\left(\dfrac{x-5}{3}\right)^2 + 2 = g^{-1}(x)}$

121 TEST 2

B6

$g(x) = -3\sqrt{-7x+14}+5$ $\qquad \mathcal{D} = (-\infty, 2]$

$\qquad\qquad\qquad\qquad\qquad\qquad \mathcal{R} = (-\infty, 5]$

We find $g^{-1}(x)$:

$$-3\sqrt{-7y+14}+5 = x$$

$$-3\sqrt{-7y+14} = x-5$$

$$\sqrt{-7y+14} = \frac{x-5}{-3}$$

$$-7y+14 = \frac{(x-5)^2}{9}$$

$$-7y = \frac{(x-5)^2}{9} - 14$$

$$y = -\frac{1}{63}(x-5)^2 + 2$$

B7

$$\boxed{\begin{array}{l}\text{(Restricted) } \mathcal{D} = (-\infty, 5] \\ \text{and } \mathcal{R} = (-\infty, 2]\end{array}}$$