MAT 121-G81-G82 Test 3 – Fall, 2014 Name KEY

70 Points Covers Chapter 3 NO GRAPHING CALCULATORS!!!

1. (10 pts) Form a polynomial of *minimial degree* in *factored form* that has real coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong!

$x = 3$, multiplicity 2;
$x = 3 - 7i$, multiplicity 1;
$x = 2$, multiplicity 4.

$(x-3)^2(x-(3-7i))(x-(3+7i))(x-2)^4$

2. (10 pts) Use synthetic division to find $P(3)$ if $P(x) = 5x^5 - 2x^3 + 3x^2 - 4x + 3$.

$$\begin{array}{r|rrrrrr} 3 & 5 & 0 & -2 & 3 & -4 & 3 \\ & & 15 & 45 & 129 & 396 & 1176 \\ \hline & 5 & 15 & 43 & 132 & 392 & 1179 \end{array}$$

$\boxed{1179 = P(3)}$

$$\begin{array}{r} 392 \\ 3 \\ \hline 1176 \end{array}$$

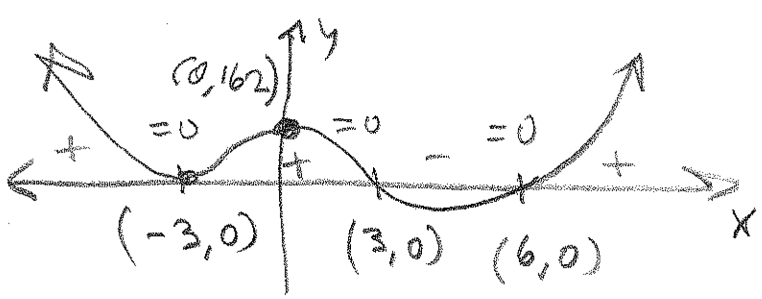
3. (5 pts) Represent the work you just did on the previous problem by writing $P(x)$ in the form *Dividend* = *Divisor* • *Quotient* + *Remainder*.

$P(x) = (x-3)(5x^4 + 15x^3 + 43x^2 + 89x + 263) + 792$

4. Suppose $f(x) = (x+3)^2(x-3)(x-6) = x^4 - 3x^3 - 27x^2 + 27x + 162$. I'm showing you both factored and expanded form to help you answer the following:

a. (10 pts) Provide a rough sketch of *f*, using its zeros, their respective multiplicities and its end behavior. Include *x*- and *y*-intercepts. Your graph should be smooth. Un-exaggerate the vertical for a better quality graph.

x^4



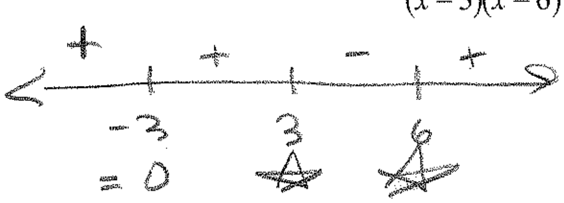
b. Solve the inequalities (You've done the work. Now, INTERPRET.):

i) (5 pts) $f(x) = (x+3)^2(x-3)(x-6) < 0$

$\boxed{(3,6)}$

ii) (5 pts) $f(x) = \dfrac{(x+3)^2}{(x-3)(x-6)} \geq 0$

$\boxed{(-\infty, 3) \cup (6, \infty)}$



5. (10 pts) Find the *real* zeros of $f(x)=2x^5-4x^4-11x^3+41x^2 \overset{-43x}{-16x}+15$. Then factor *f* over the set of **real numbers**. This should involve an irreducible quadratic factor.

-3	2	-4	-11	41	-43	15
		-6	30	-57	48	-15
1	2	-10	19	-16	5	0
		2	-8	11	-5	
1	2	-8	11	-5	0	
		2	-6	5		
	2	-6	5	0		

$2x^2-6x+5=0$

$a=2, b=-6, c=5$

$b^2-4ac=(-6)^2-4(2)(5)$

$=36-40=-4$

No real roots, so

$$x=\frac{6\pm\sqrt{-4}}{2(2)}=\frac{6\pm 2i}{4}$$

$$=\frac{3\pm i}{2}$$

$x=-3, m=1$

$x=1, m=2$

and $f(x)=(x+3)(x-1)^2(2x^2-6x+5)$

6. (5 pts) Find the remaining (nonreal) zeros of *f* and factor *f* over the set of **complex numbers**. This step requires breaking down the quadratic piece that's irreducible over the real numbers. The fundamental theorem tells us that *nothing* is irreducible over the complex numbers.

$x=\frac{2\pm i}{2}$

$f(x)=2(x+3)(x-1)^2\left(x-\left(\frac{3+i}{2}\right)\right)\left(x-\left(\frac{3-i}{2}\right)\right)$

7. (5 pts) You don't need to graph $R(x)=\dfrac{2x^3+6x^2+4x}{x^2-4}$, here, but I do want to see you graph its asymptotes.

$$= \frac{2x(x^2+3x+2)}{(x+2)(x-2)} = \frac{2x\cancel{(x+2)}(x+1)}{\cancel{(x+2)}(x-2)} = \frac{2x(x+1)}{x-2} = \frac{2x^2+2x}{x-2}$$

$x \neq -2$

$\mathcal{D} = \mathbb{R} \setminus \{\pm 2\}$

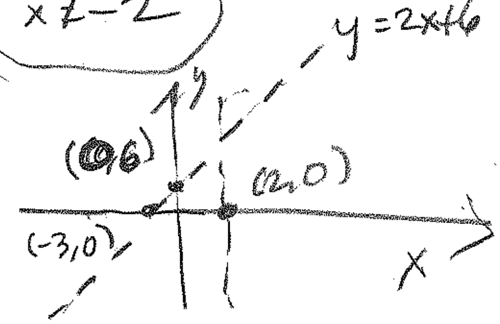
V.A.: $x=2$

HOLE: $(-2,-1)$

$\dfrac{2(-2)^2+2(-2)}{-2-2} = \dfrac{8-4}{-4} = \dfrac{4}{-4} = -1$

$$\begin{array}{r|rrr} 2 & 2 & 2 & 0 \\ & & 4 & \\ \hline & 2 & 6 & \end{array}$$

$\rightarrow y=2x+6$



$x=2$

8. (10 pts) Sketch the graph of $R(x)=\dfrac{3x^2-13x-4}{x^2-3x-10}$. Show all asymptotes and intercepts.

$$= \frac{(3x \quad)(x \quad)}{(x-5)(x+2)}$$ DNF !? What was I thinking? Tougher than intended, but still legit.

$b^2-4ac = (-13)^2-4(3)(-4) = 169+48 = 217$

$x = \dfrac{13 \pm \sqrt{217}}{2(3)}$

$\rightarrow \dfrac{13+\sqrt{217}}{6} \approx 4.621819977$

$\rightarrow \dfrac{13-\sqrt{217}}{6} \approx -0.2884866438$

zeros of numerator aren't clean.

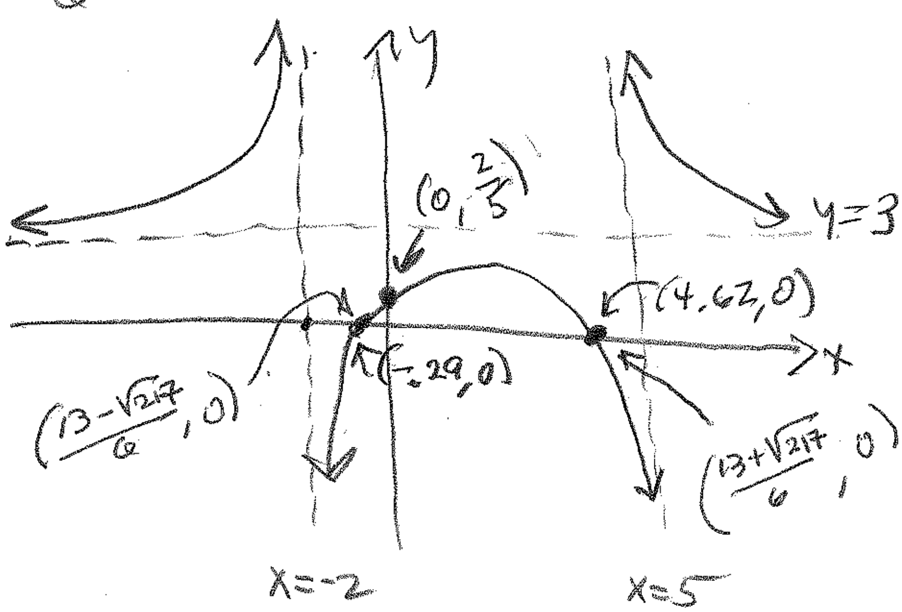
$\boxed{\text{V.A. } x=5,\ x=-2}$

H.A.: $y = \dfrac{3x^2}{x^2} = 3$

$y=3$

y-int: $\dfrac{-4}{-10} = \dfrac{2}{5}$

$\rightsquigarrow (0, \tfrac{2}{5})$



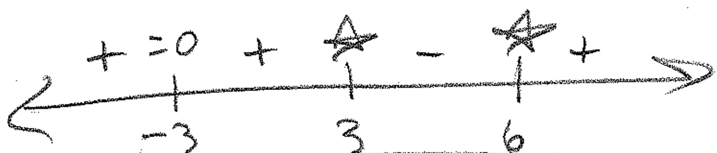
ANSWER ANY TWO (2) OF THE FOLLOWING.

Bonus: (5 pts) Form a polynomial of *minimial degree* in *factored form* that has ***rational*** coefficients (after expanding) and will have the given zeros. Do *not* expand your polynomial. Leave it factored! If you run out of room, you're doing it wrong!

Zeros: $x = 2+\sqrt{3}$, multiplicity 1; $x = 2+3i$, multiplicity 2; $x = -5$, multiplicity 17.

$(x-(2+\sqrt{3}))(x-(2-\sqrt{3}))(x-(2+3i))^2(x-(2-3i))^2(x+5)^{17}$

Bonus: (5 pts) What is the domain of $f(x)=\sqrt{\dfrac{(x+3)^2}{(x-3)(x-6)}}$? Hint: See previous work.



$\boxed{(-\infty, 3) \cup (6, \infty)}$

Bonus: (5 pts) Sketch the graph of $R(x)=\dfrac{2x^3+6x^2+4x}{x^2-4}$. Hint: See previous work.