MAT 121 4.2 #4

Book Way: (Less satisfactory.)

$$\frac{\frac{1}{x}+2}{\frac{1}{x^2}-4}, \text{ where } x \neq 0, \pm\frac{1}{2}$$

$$\frac{\frac{1}{x}+2}{\frac{1}{x^2}-4}=\frac{x^2\left(\frac{1}{x}+2\right)}{x^2\left(\frac{1}{x^2}-4\right)}=\frac{\frac{x^2}{x}+2x^2}{\frac{x^2}{x^2}-4x^2}=\frac{x+2x^2}{1-4x^2}=\frac{x(1+2x)}{(1-2x)(1+2x)}=\frac{x}{1-2x}$$

It's fairly efficient, but it's one more thing to remember. I prefer just hammering away at skills with manipulating fractions, straight up:

My Way: (More satisfactory and 'generic,' without adding a whole 'nother thing to memorize.)

$$\frac{\frac{1}{x}+2}{\frac{1}{x^2}-4}=\frac{\frac{1}{x}+\frac{2}{1}\cdot\frac{x}{x}}{\frac{1}{x^2}-\frac{4}{1}\cdot\frac{x^2}{x^2}}=\frac{\frac{1+2x}{x}}{\frac{1-4x^2}{x^2}}=\frac{1+2x}{x}\cdot\frac{x^2}{1-4x^2}=\frac{(1+2x)x}{(1-4x^2)}=\frac{(1+2x)x}{(1-2x)(1+2x)}=\frac{x}{1-2x}$$

I'd do it in slightly fewer steps if I didn't want to make sure you could follow the moves, fairly easily. My method is still just the same old

1. Add Fractions to get one fraction upstairs and one fraction downstairs. (Steps 1 – 3)
2. Divide Fractions (Invert and Multiply in Step 4)

In practice, I'd probably do this in 4 steps, total. My way, you're just extending skills you should already have, and getting more practice with those skills, rather than learning something totally new and different. I made it all the way thru my PhD, without ever learning it the book's way. I just picked it up, when I saw students in MAT 099 trying to do it this way. Yes. It's a little quicker, that way, but it clutters your mind with more 'special cases junk.'