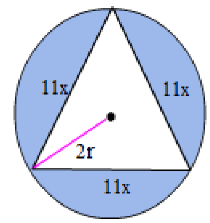
An equilateral triangle is inscribed in a circle of radius 2r. Express the area A within the circle but outside the triangle as a function of the length 11x of the side of the triangle.



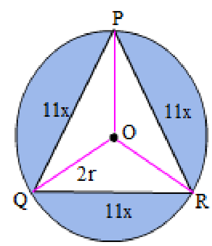
The area of the region within the circle but outside the triangle is obtained by subtracting the area of the equilateral triangle from the area of the circle.

First determine the expression for the area of a circle with radius 2r. The area of the circle is $4\pi r^2$.

Now determine the expression for the area of the equilateral triangle with side 11x. The area of the equilateral triangle is $\frac{121x^2\sqrt{3}}{4}$.

To express the area of the region A, within the circle but outside the triangle, as a function of the length 11x of a side of the triangle, first express the radius 2r in terms of x.

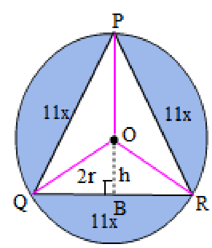
Note that the area of the equilateral triangle PQR is $\frac{121x^2\sqrt{3}}{4}$. Divide triangle PQR into three smaller congruent triangles, triangles POQ, QOR, and POR as shown in the figure to the right.



Note that the area of each of the smaller triangles is one-third the area of triangle PQR. Thus, the area of triangle OQR is $\frac{121x^2\sqrt{3}}{12}$.

Find the area of triangle OQR with base 11x and height h.

The area of triangle OQR with height h and base 11x is $\frac{11xh}{2}$.



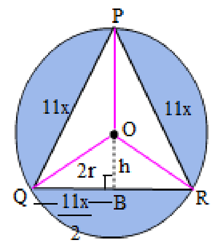
To find h in terms of x, equate the two expressions for the area of triangle OQR.

$$\frac{121x^2\sqrt{3}}{12} = \frac{11xh}{2}$$

$$\frac{11x\sqrt{3}}{6} = h$$

Multiply on both sides by $\frac{2}{11x}$ and cancel out the common terms..

Now use the Pythagorean theorem in triangle OBQ shown to the right.



$$OQ^2 = OB^2 + QB^2$$

$$(2r)^2 = h^2 + \left(\frac{11x}{2}\right)^2$$

$$4r^2 = h^2 + \frac{121x^2}{4}$$

$$= \left(\frac{11x\sqrt{3}}{6}\right)^2 + \frac{121x^2}{4} \qquad \text{Substitute } \frac{11x\sqrt{3}}{6} \text{ for h.}$$

$$= \frac{121x^2}{12} + \frac{121x^2}{4} \qquad \text{Square } \frac{11x\sqrt{3}}{6}.$$

$$4r^2 = \frac{121x^2}{3} \qquad \text{Add the terms on the right side.}$$

Now substitute the expression $\frac{121x^2}{3}$ for $4r^2$ in the area of the circle with radius 2r.

Now substitute the expression $\frac{121x^2}{3}$ for $4r^2$ in the area of the circle with radius 2r.

$$4\pi r^2 = \frac{121\pi x^2}{3}$$

Finally, determine the area within the circle but outside the triangle in terms of the length 11x of a side of the triangle by subtracting the area of the equilateral triangle from the area of the circle.

$$A(x) = \frac{121\pi x^2}{3} - \frac{121x^2\sqrt{3}}{4}$$

Therefore, the formula for the area for the region within the circle but outside the triangle as a function of the length 11x of a side of the triangle is $A(x) = \frac{121\pi x^2}{3} - \frac{121x^2\sqrt{3}}{4}$.