This assignment addresses concepts and skills that are lightly covered, but of fairly great importance. Completing the square for graphing and for solving equations are closely related, but subtly different processes.

We can find the vertex of $f(x) = ax^2 + bx + c$ by finding the point $(h, k) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$, by computing $-\frac{b}{2a}$ and $f\left(-\frac{b}{2a}\right)$ directly.

In this Supplementary Exercise, we use completing the square to write $f(x)$ in the form

$$f(x) = a(x-h)^2 + k$$

or

$$f(x) = a\left(x + \frac{b}{2a}\right)^2 + f\left(-\frac{b}{2a}\right)$$

I feel it is important that you be able to obtain this "completed square version" of $f(x)$ by actually completing the square, but the second way I wrote it will suggest a cheat/check using $-\frac{b}{2a}$. This cheat/check works the same as the quadratic formula works. But I insist that the student understand the completing-the-square process on which the quadratic formula is based.

Graph each of the following by completing the square. Features I expect to see:

a. y–intercept
b. x–intercepts (if any)
c. vertex
d. general "shape" (a smooth parabola)

I also expect to see the work you did in completing the square for each.

Grading Rubric:
5 points each problem
y–intercept – 1 point
x–intercepts – 1 point
Complete the square successfully – 1 point
vertex – 1 point
general shape of the parabola – 1 point

See the next page for a couple of examples of the algebraic manipulations involved.

Once you are done completing the square, you can see how to graph a quadratic function by transforming the graph of $f(x) = x^2$.

Example: Your first one:

$$g(x) = x^2 - 4x - 3$$
$$= x^2 - 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - 3$$
$$= x^2 - 4x + 2^2 - 2^2 - 3$$
$$= (x-2)^2 - 4 - 3$$
$$= (x-2)^2 - 7$$

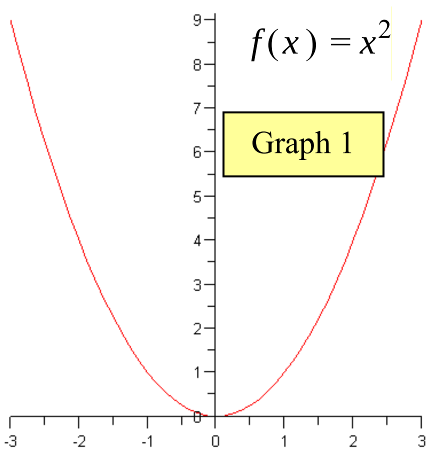
So, $(h, k) = (2, -7)$ is the vertex.

Find x-intercepts:

$$(x-2)^2 - 7 = 0$$
$$(x-2)^2 = 7$$
$$x - 2 = \pm\sqrt{7}$$
$$x = 2 \pm \sqrt{7}$$

So, $(2+\sqrt{7}, 0)$ and $(2-\sqrt{7}, 0)$ are x - intercepts.

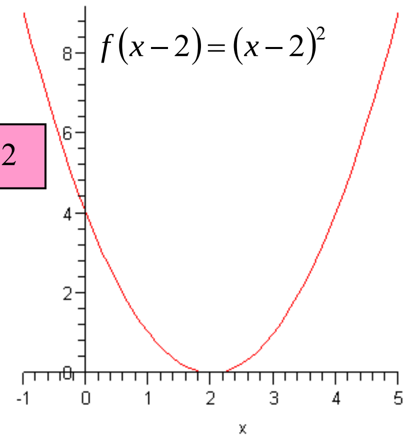
Graphing $(x-2)^2 - 7$ using Section 1.5:



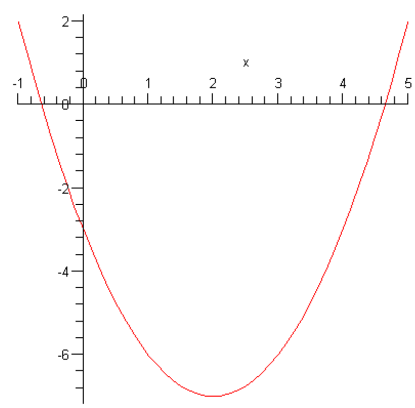
Points that should be labeled:
(-1, 1)
(0, 0)
(1, 1)

It's a big pain in the neck on a computer, so I did not finish labeling key points with ordered pairs on the graph, but I did list the key points below each graph.

Graph 2



Transformation: Right 2 from basic function.
(2, 0)
(0, 4)



$f(x-2) - 7 = (x-2)^2 - 7$

Transformation: Down 7 from the previous graph.

Points that should be labeled:
(2, - 7)
(0, - 3)
$(2-\sqrt{7}, 0)$ $(2+\sqrt{7}, 0)$

Another Example:

$$g(x) = 5x^2 + 3x - 8$$
$$= 5\left(x^2 + \frac{3}{5}x\right) - 8$$
$$= 5\left(x^2 + \frac{3}{5}x + \left(\frac{3}{10}\right)^2 - \left(\frac{3}{10}\right)^2\right) - 8$$
$$= 5\left(x^2 + \frac{3}{5}x + \left(\frac{3}{10}\right)^2\right) - 5\left(\frac{3}{10}\right)^2 - 8$$

SCRATCH :

$$-5\left(\frac{3}{10}\right)^2 - 8$$
$$= -5\left(\frac{9}{100}\right) - 8$$
$$= -\frac{9}{20} - 8$$
$$= -\frac{9}{20} - \frac{160}{20}$$
$$= -\frac{169}{20}$$

SCRATCH :

$$5\left(x^2 + \frac{3}{5}x + \left(\frac{3}{10}\right)^2\right)$$
$$= 5\left(x + \frac{3}{10}\right)^2$$

Therefore :

$$g(x) = 5\left(x + \frac{3}{10}\right)^2 - \frac{169}{20}$$

So, $(h,k) = \left(-\frac{3}{10}, -\frac{169}{20}\right)$ is the vertex.

Find x-intercepts:

$$5\left(x + \frac{3}{10}\right)^2 - \frac{169}{20} = 0$$
$$5\left(x + \frac{3}{10}\right)^2 = \frac{169}{20}$$
$$\left(x + \frac{3}{10}\right)^2 = \frac{169}{100}$$
$$x + \frac{3}{10} = \pm\sqrt{\frac{169}{100}} = \pm\frac{13}{10}$$
$$x = -\frac{3}{10} \pm \frac{13}{10}$$

So, x-intercepts are $(1,0)$ and $\left(-\frac{8}{5}, 0\right)$

To graph $g(x) = 5x^2 + 3x - 8 = 5\left(x + \frac{3}{10}\right)^2 - \frac{169}{20}$, we start with $f(x) = x^2$

Stretch vertically by a factor of 5 to obtain $5x^2 = 5f(x)$

Shift the previous left by 3/10 to obtain $5\left(x + \frac{3}{10}\right)^2 = 5f\left(x + \frac{3}{10}\right)$

Shift the previous down by 169/20 to obtain the final graph of

$$g(x) = 5\left(x + \frac{3}{10}\right)^2 - \frac{169}{20} = 5f\left(x + \frac{3}{10}\right) - \frac{169}{20}$$