121 ONLINE BRIDGE-TO-FINAL

Stuff that is covered between the last test and the Final, itself. See PDF of questions.

① $x = -1, y = 3$?

$4(-1)+3 = -4+3 = -1$ ✓

$3(-1)+4(3) = -3+12 = 9$ ✓

Yes

② $x+6y=6 \Rightarrow x = 6-6y$

$7x-8y=-8$

$7(6-6y)-8y=-8$

$42-42y-8y=-8$

$-50y+42=-8$

$-50y=-50$

$y=1 \Rightarrow$

$x=6-6(1) = 0 = x$

Check:

$\begin{bmatrix} 1 & 6 \\ 7 & -8 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \end{bmatrix}$ ✓

③ $x + y = -9$

$x - y = 12$

$2x = 3$

$x = \frac{3}{2} = 1.5 = x$

$\left(\frac{3}{2} + y = -9\right)(2)$

$3 + 2y = -18$

$2y = -21$

$y = -\frac{21}{2} = -10.5 = y$

Check:

$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} \frac{3}{2} \\ -\frac{21}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{2}-\frac{21}{2} \\ \frac{3}{2}+\frac{21}{2} \end{bmatrix} = \begin{bmatrix} -\frac{18}{2} \\ \frac{24}{2} \end{bmatrix} = \begin{bmatrix} -9 \\ 12 \end{bmatrix}$ ✓

121 Bridge to Final

(4) Let x = the # of adults who paid the Center
y = " " " seniors " " " "

$$x + y = 495 \quad \text{Total Attendance}$$
$$21x + 11y = 6805 \quad \text{Total Income}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 495 \\ 21 & 11 & 6805 \end{array}\right] \xrightarrow{-21R1+R2} \left[\begin{array}{cc|c} 1 & 1 & 495 \\ 0 & -10 & -3590 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{10}R2} \left[\begin{array}{cc|c} 1 & 1 & 495 \\ 0 & 1 & 359 \end{array}\right] \Rightarrow \boxed{y = 359}$$

On tests, I usually just ask for the setups (Circled in red)

(5) Let x = Amt of $ invested in CDs
y = " " " " " AA Bonds

$$x + y = 150{,}000 \quad \text{TOTAL INVESTED}$$
$$.07x + .12y = 15{,}500 \quad \text{TOTAL RETURN}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 150\,000 \\ .07 & .12 & 15500 \end{array}\right] \xrightarrow{100R2} \left[\begin{array}{cc|c} 1 & 1 & 150\,000 \\ 7 & 12 & 1550000 \end{array}\right]$$

$$\xrightarrow{-7R1+R2} \left[\begin{array}{cc|c} 1 & 1 & 150000 \\ 0 & 5 & 500{,}000 \end{array}\right] \xrightarrow{\frac{1}{5}R2} \left[\begin{array}{cc|c} 1 & 1 & 150000 \\ 0 & 1 & 100000 \end{array}\right]$$

$$\boxed{y = \$100{,}000 \Rightarrow x = \$50{,}000}$$

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(6) $4x - 2y = 7$
$4x - 2y = 9$

$$\left[\begin{array}{cc|c} 4 & -2 & 7 \\ 4 & -2 & 9 \end{array}\right] \xrightarrow{-R1+R2} \left[\begin{array}{cc|c} 4 & -2 & 7 \\ 0 & 0 & 2 \end{array}\right]$$

Bottom row says $0 = 2$!? **No Solution**

(7) $x + y + z = -4$
$x - y + 2z = -9$
$4x + y + z = -19$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -4 \\ 1 & -1 & 2 & -9 \\ 4 & 1 & 1 & -19 \end{array}\right] \begin{array}{l} -R1+R2 \\ -4R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & -4 \\ 0 & -2 & 1 & -5 \\ 0 & -3 & -3 & -3 \end{array}\right]$$

$$\begin{array}{l} -\frac{1}{3}R3 \\ R2 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & -4 \\ 0 & 1 & 1 & 1 \\ 0 & -2 & 1 & -5 \end{array}\right] \begin{array}{l} -R2+R1 \\ 2R2+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & -3 \end{array}\right]$$

$$\begin{array}{l} \\ \frac{1}{3}R3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right] -R3+R2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$x = -5$
$y = 2$
$z = -1$

Check?

$$\left[\begin{array}{ccc} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 1 \end{array}\right]\left[\begin{array}{c} -5 \\ 2 \\ -1 \end{array}\right] = \left[\begin{array}{c} -5+2-1 \\ -5-2-2 \\ -20+2-1 \end{array}\right] = \left[\begin{array}{c} -4 \\ -9 \\ -19 \end{array}\right] \checkmark$$

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(8)
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & -1 & 5 & -21 \\ 4 & 4 & 4 & 7 \end{array}\right] \begin{array}{l} \\ -R1+R2 \\ -4R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & 4 & -22 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

Bottom row says $0=3$!? **No Solution**

(9)
$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 3 \\ 1 & 5 & -2 & 5 \\ 3 & 12 & -3 & 9 \end{array}\right] \begin{array}{l} \\ -R1+R2 \\ -3R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 4 & -1 & 3 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$-4R2+R1 \left[\begin{array}{ccc|c} 1 & 0 & 3 & -5 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x+3z=-5$ ⟶
$y-z=2$

$x=-3z-5$
$y=z+2$
$z=$ Any real #

(10)
$$\left[\begin{array}{ccc|c} 3 & 8 & 3 & 35 \\ 6 & 9 & 8 & 73 \\ 4 & 7 & -2 & 13 \end{array}\right]$$

(11)
$x=5$
$y=7$
$z=5$

(12)
$7x+9y+7z=-2$
$9x+9z=4$
$3x+4y=2$

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(13) $2x + 3y = 0$
$4x + 8y = 4$

$$\left[\begin{array}{cc|c} 2 & 3 & 0 \\ 4 & 8 & 4 \end{array}\right] \; R1 \leftrightarrow R2$$

$$\left[\begin{array}{cc|c} 4 & 8 & 4 \\ 2 & 3 & 0 \end{array}\right] \; \tfrac{1}{4}R2 \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 3 & 0 \end{array}\right] \; -2R1+R2 \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -1 & -2 \end{array}\right]$$

$$2R2+R1 \; -1R2 \left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & 2 \end{array}\right]$$

$\boxed{\begin{array}{l} x = -3 \\ y = 2 \end{array}}$

Check:

$$\begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}\begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -6+6 \\ -12+16 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix} \checkmark$$

(14) $3x - y + 9z = 36$
$-8x + 6z = -44$
$2y + z = 8$

Because the #s turn ugly, I'm just going to Eliminate to triangular form & then back-substitute.

$$\left[\begin{array}{ccc|c} 3 & -1 & 9 & 36 \\ -8 & 0 & 6 & -44 \\ 0 & 2 & 1 & 8 \end{array}\right] \; \tfrac{1}{3}R1 \left[\begin{array}{ccc|c} 1 & -\frac{1}{3} & 3 & 12 \\ -8 & 0 & 6 & -44 \\ 0 & 2 & 1 & 8 \end{array}\right]$$

$$8R1+R2 \left[\begin{array}{ccc|c} 1 & -\frac{1}{3} & 3 & 12 \\ 0 & -\frac{8}{3} & 30 & 52 \\ 0 & 2 & 1 & 8 \end{array}\right] \; \tfrac{1}{2}R2 \; 3R3 \left[\begin{array}{ccc|c} 1 & -\frac{1}{3} & 3 & 12 \\ 0 & 1 & \frac{1}{2} & 4 \\ 0 & -8 & 90 & 156 \end{array}\right]$$

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$$\xrightarrow{8R2+R3}\left[\begin{array}{ccc|c} 1 & -\frac{1}{3} & 3 & 12 \\ 0 & 1 & \frac{1}{2} & 4 \\ 0 & 0 & 94 & 188 \end{array}\right] \xrightarrow{\frac{1}{94}R3}\left[\begin{array}{ccc|c} 1 & -\frac{1}{3} & 3 & 12 \\ 0 & 1 & \frac{1}{2} & 4 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$z = 2$

$y + \frac{1}{2}(2) = 4$

$\Rightarrow y + 1 = 4$

$y = 3$

$x - \frac{1}{3}(3) + 3(2) = 12$

$x - 1 + 6 = 12$

$x + 5 = 12$

$x = 7$

Sick

$x = 7$

$y = 3$

$z = 2$

Check:

$$\begin{bmatrix} 3 & -1 & 9 \\ -8 & 0 & 6 \\ 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 21 - 3 + 18 \\ -56 + 12 \\ + 2 \end{bmatrix} = \begin{bmatrix} 36 \\ -44 \\ 8 \end{bmatrix}$$ ✓

(15) $A - B = \begin{bmatrix} 7 & -4 & 8 \\ -6 & 5 & -1 \\ 0 & 6 & -3 \end{bmatrix} - \begin{bmatrix} -2 & -6 & -1 \\ -7 & -4 & 3 \\ -3 & -9 & -5 \end{bmatrix}$

$$= \begin{bmatrix} 9 & 2 & 9 \\ 1 & 9 & -4 \\ 3 & 15 & 2 \end{bmatrix}$$

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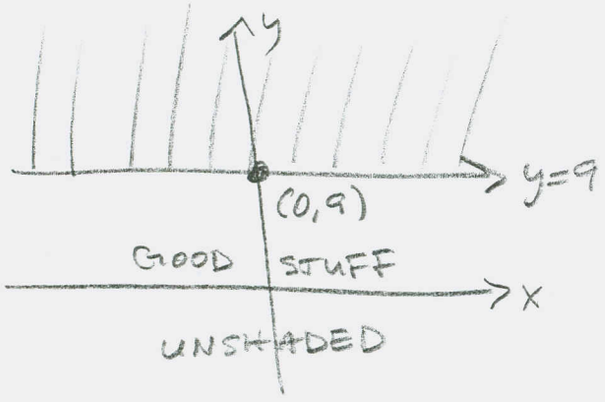
(16) $AB = \begin{bmatrix} 0 & -3 & 1 \\ 5 & -1 & 0 \end{bmatrix}_{2\times 3} \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}_{3\times 2} = \begin{bmatrix} 0+0+1 & 0-3-1 \\ 5+0+0 & 10-1+0 \end{bmatrix}_{2\times 2}$

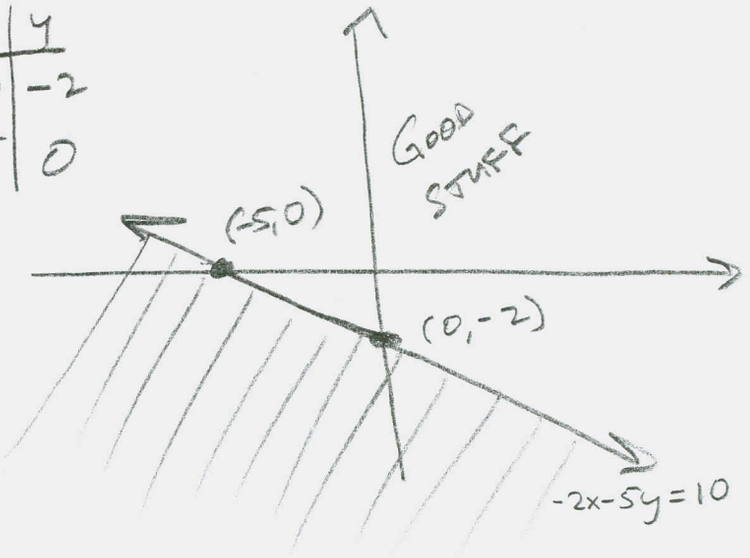
$$= \begin{bmatrix} 1 & -4 \\ 5 & 9 \end{bmatrix}$$

(17) $y \leq -9$



(18) $-2x - 5y \leq 10$

x	y
0	-2
-5	0



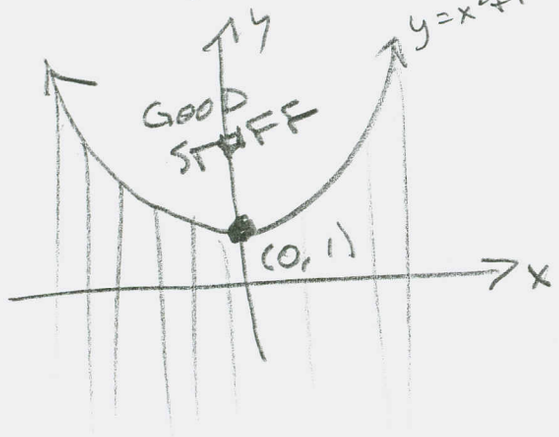
Test (0,0):

$-2(0) - 5(0) \leq 10$?

$0 \leq 10$? Yes. (0,0) GOOD

(19) $y > x^2 + 1$

want y-values above x^2+1



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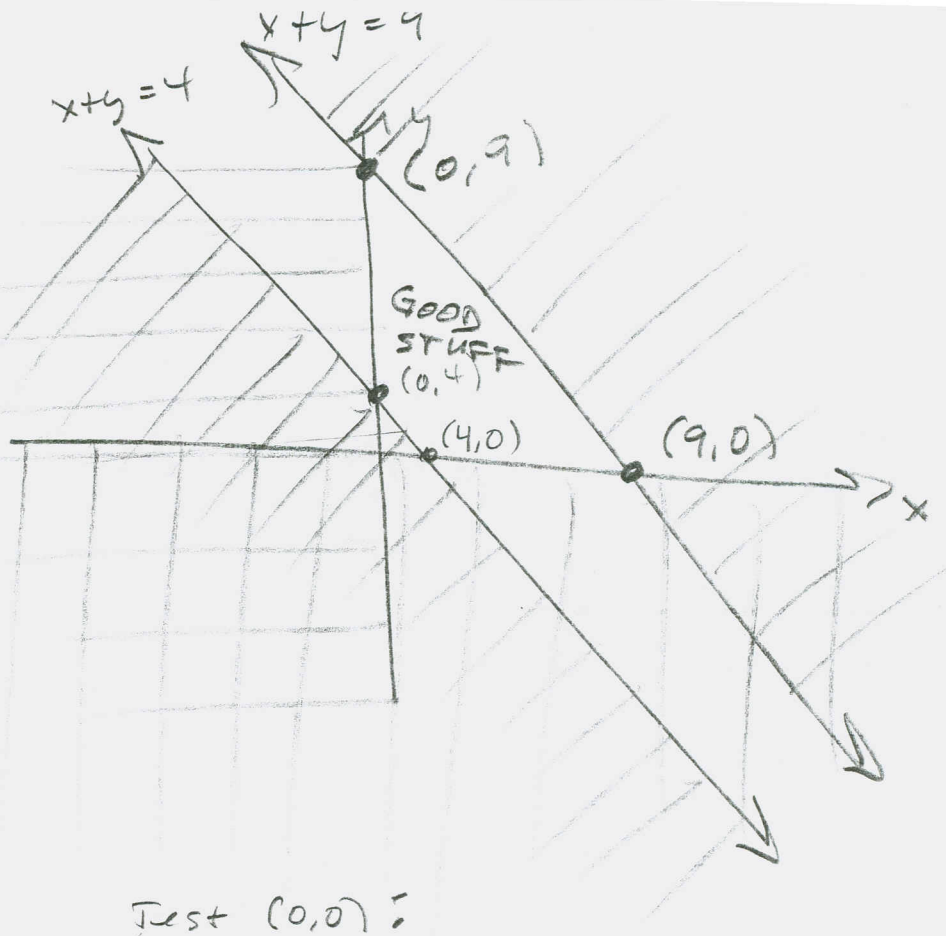
(20) $x \geq 0$

$y \geq 0$

$x + y \leq 9$

$x + y \geq 4$



GOOD STUFF is unshaded trapezoid in the middle. It's bounded.

Test (0,0):

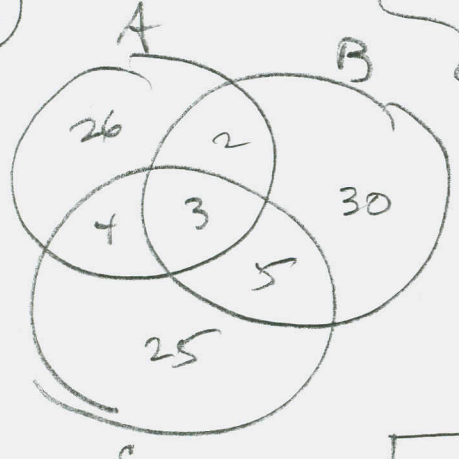
$x + y \leq 9$

$0 \leq 9$? Yes (0,0) good!

$x + y \geq 4$

$0 \geq 4$? No (0,0) Bad

(22)



$n(a) = 26+4+3+2 = \boxed{35}$

(23) Sorry out of order!

$n(A) = 25$, $n(B) = 18$,

$n(A \cup B) = 35$ ⟶

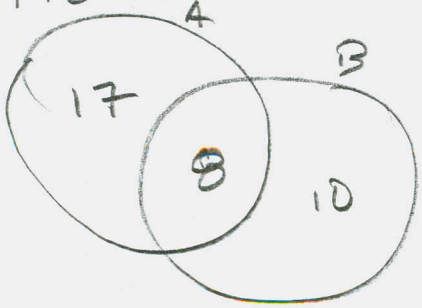
$n(A \cap B) = ?$

$n(A) + n(B) - n(A \cap B) = n(A \cup B)$

⟶ $n(A) + n(B) - n(A \cup B) = n(A \cap B)$

$25 + 18 - 35 = \boxed{8 = n(A \cap B)}$

Picture for #23:



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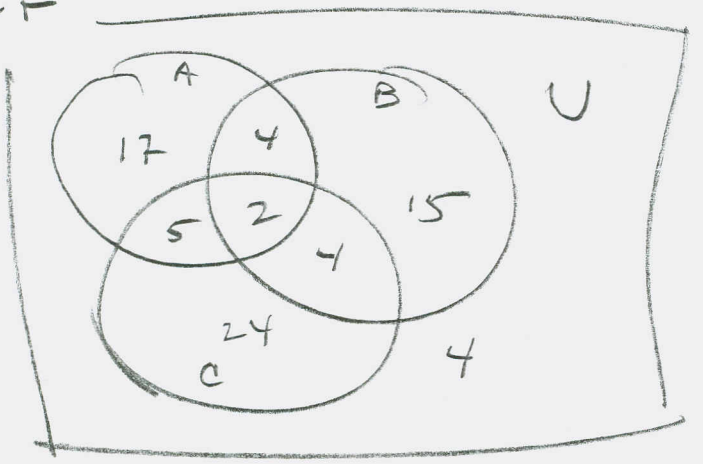
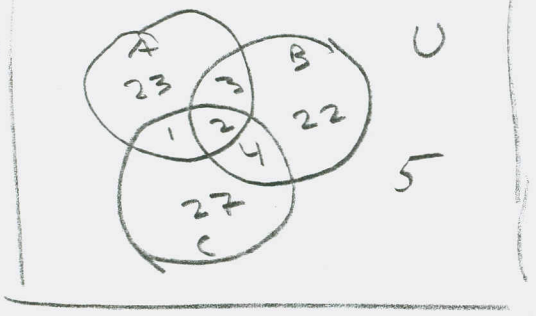
(23)



$n(B \text{ or } C) = n(B \cup C) = 4+2+4+15+5+24$

$= \boxed{54}$

(24)



$n(\text{not } C) =$

$23+3+22+5$

$= \boxed{53}$

(25) N = those surveyed who were happy with Nursing

M = " " " " " Medical

$n(U) = 50$

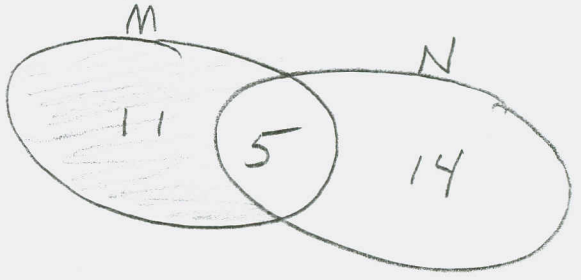
$n(N) = 19$

$n(M) = 16$

$n(N \cap M) = 5$

Find $n(M \text{ and not } N)$

$= n(M \cap N') = \boxed{11}$



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(26) S = students who speak Spanish
F = " " " French

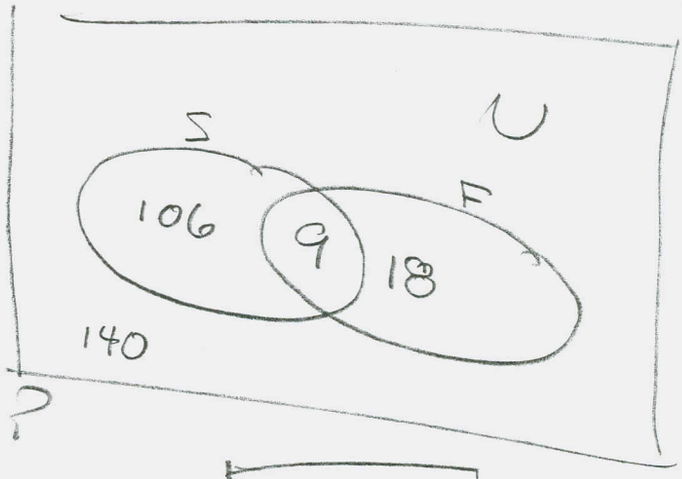
S' means "Not S", F' means "Not F"

$n(S) = 115$

$n(F) = 27$

$n(S \cap F) = 9$

$n(S' \cap F') = 140$



How many in survey?

$n(U) = 106 + 9 + 18 + 140 = \boxed{273}$

(27) $P(9,4) = \frac{9!}{(9-4)!} = \frac{9!}{5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6}{1} = \boxed{3024}$

(28) $P(4,3)$ on $\{1,2,3,4\}$: $6 \times 4 = \boxed{24}$

123, 132, 124, 142, 134, 143
213, 231, 214, 241, 234, 243
312, 321, 314, 341, 324, 342
412, 421, 413, 431, 423, 432

$P(4,3) = \frac{4!}{(4-3)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1!} = \boxed{24}$

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(29) How many ways can 4 people have different birth months?

$$P(12,4) = \frac{12!}{8!} = 12 \cdot 11 \cdot 10 \cdot 9 = \boxed{11,880}$$

(30) $C(13,8) = \binom{13}{8} = \frac{13!}{5!\,8!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{5 \cdot 4 \cdot 3 \cdot 2}$

$= 13 \cdot 3 \cdot 11 \cdot 3 = \boxed{1287}$

(31) All combos on $\{1,2,3,4\}$ taken 3 @ a time.

123, 124, 134, 234

$$\binom{4}{3} = \frac{4!}{1!\,3!} = \boxed{4}$$

(32) 9 multiple-choice, 6 essay. Answer 7 multiple choice & 4 essay.

$$\binom{9}{7}\binom{6}{4} = \frac{9!}{7!\,2!} \cdot \frac{6!}{4!\,2!} = \frac{9 \cdot 8}{2} \cdot \frac{6 \cdot 5}{2}$$

$= (9 \cdot 4)(15) = \boxed{540 \text{ ways}}$

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(33) $0.2, \frac{1}{2}, \frac{5}{4}, 1$ are legit probabilities

(34) 2 6-sided dice.

2	3	4	5	6	7	8	9	10	11	12
$\frac{1}{36}$	$\frac{2}{36}$									
11	12, 21									

P(rolling a '3') is $\frac{2}{36} = \frac{1}{18}$

(35) $\binom{4}{2} = C(4,2) = \frac{4!}{2!2!} = \frac{4 \cdot 3}{2!} = \frac{4 \cdot 3}{2} = 6$

(36) $(x+1)^6 =$

$x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

(37) $(5x+2)^3$

$= 1(5x)^3(2)^0 + 3(5x)^2(2)^1 + 3(5x)^1(2)^2 + 1(5x)^0(2)^3$

$= 125x^3 + 150x^2 + 60x + 8$