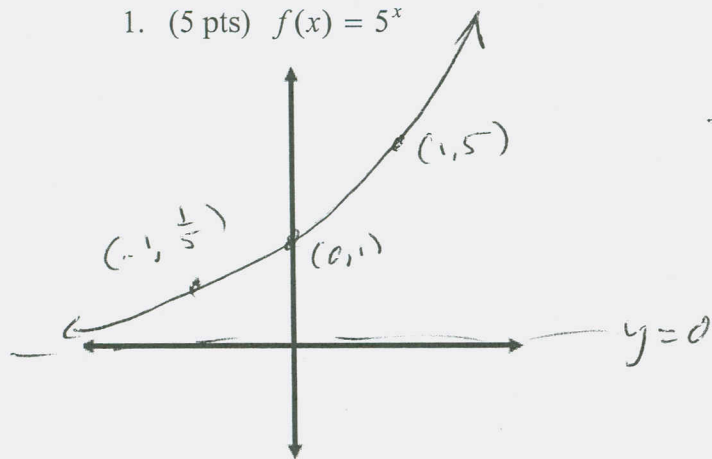


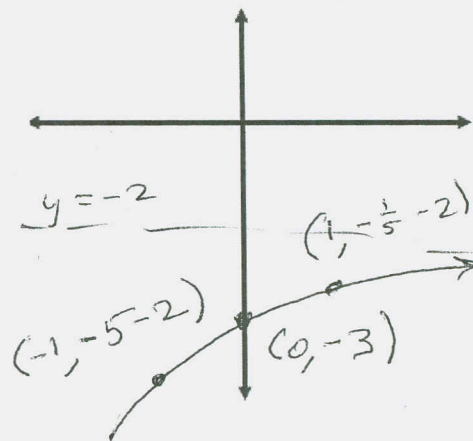
Graph:

1. (5 pts) $f(x) = 5^x$

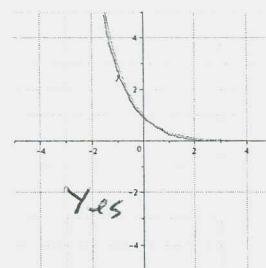
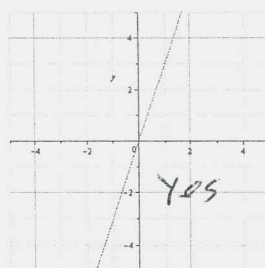
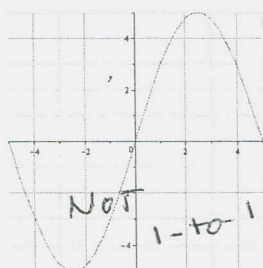
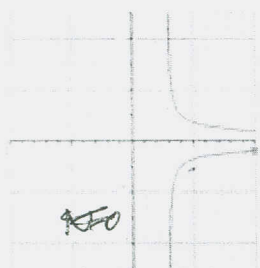


2. (5 pts) $f(x) = -\left(\frac{1}{5}\right)^x - 2$

~~X~~
~~X~~



3. (5 pts) Determine which of the following are one-to-one. Indicate by writing "Yes" or "No" on the graphs. Find the one that's not a function for 4 bonus points.



~~NO~~
NOT
FUNC.

4. (10 pts) Solve $25 = \left(\frac{1}{5}\right)^{-2x+3}$

$$5^{-2} = (5^{-1})^{-2x+3}$$

$$5^{-2} = 5^{2x-3}$$

$$-2 = 2x - 3$$

$$5 = 2x$$

$$\frac{5}{2} = x$$

$$x \in \left\{ \frac{5}{2} \right\}$$

5. (10 pts) Write an exponential function to model the situation. Tell what each variable represents. A fast-growing city is growing exponentially with a growth rate of 10%. The population was 25,000 in 2005.

$$P(t) = 25000 e^{.1t}, \quad t = \text{time in years, after 2005}$$

$P = \text{population}$

6. (5 pts) Evaluate $\log_4\left(\frac{1}{1024}\right)$ without a calculator !!

$$\begin{array}{l} 2 \overline{)1024} \\ 2 \overline{)512} \\ 2 \overline{)256} \\ 2 \overline{)128} \\ \quad 64 \end{array}$$

$$\begin{array}{l} 2 \overline{)64} \\ 2 \overline{)32} \\ 2 \overline{)16} \\ 2 \overline{)8} \\ 2 \overline{)4} \\ \quad 2 \end{array}$$

$$1024 = 2^{10} = 2^{2 \cdot 5} = (2^2)^5 = 4^5$$

$$\log_4(4^{-5}) = -5$$

7. (10 pts) Express $\log_3\left(\sqrt[7]{\frac{23x^5y^{-5}}{z^4}}\right)$ in terms of logarithms of x , y , and z .

$$= \frac{1}{7} \left[\log_3 23 + 5 \log_3 x - 5 \log_3 y - 4 \log_3 z \right]$$

8. (10 pts) Solve correct to four decimal places: $3^{2x} = 2^{-3x+4}$. The final decimal answer is worth 2 points. I'm more interested in a correct symbolic answer and symbolic manipulations you perform to get there.

$$(\ln 3)(2x) = (\ln 2)(-3x+4)$$

$$a = \ln 3, b = \ln 2$$

$$(a)(2x) = (b)(-3x+4)$$

$$2ax = -3bx + 4b$$

$$(2a+3b)x = 4b \implies x = \frac{4b}{2a+3b}$$

$$= \frac{4 \ln 2}{2 \ln 3 + 3 \ln 2}$$

$$\approx .6483060976$$

9. (10 pts) ^{→ 5 pts} Solve: $\log(x+5) - \log(x-3) = \log 3$

$$\log\left(\frac{x+5}{x-3}\right) = \log 3$$

$$\frac{x+5}{x-3} = 3$$

$$x+5 = 3(x-3)$$

$$x+5 = 3x-9$$

$$-2x = -14$$

$$x = 7$$

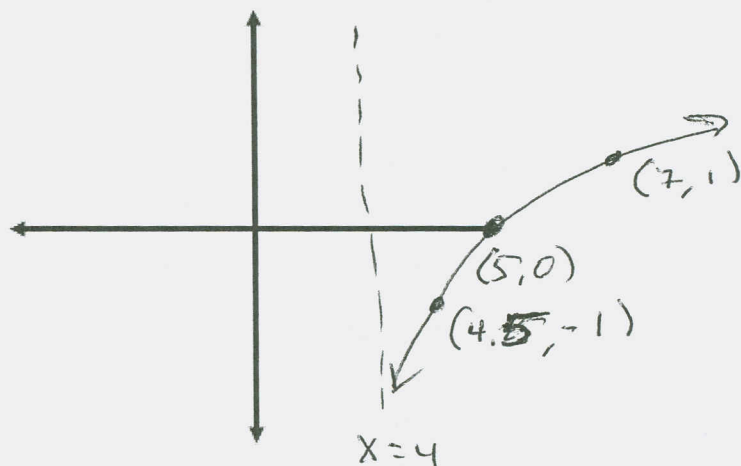
$$x \in \{7\}$$

$$\log\left(\frac{12}{4}\right) = \log 3 \quad \checkmark$$

10. (5 pts) Find the value of the expression: $27^{\log_3 2} = (3^3)^{\log_3 2}$

$$= 3^{3 \log_3 2} = 3^{\log_3 (2^3)} = 2^3 = 8$$

11. (5 pts) Graph: $\log_2(x-4)$



12. (5 pts) Write the following as the logarithm of a single expression. Assume that variables represent positive numbers. $3 \log_5(x+2) - 4 \log_5(x-7) + \log_5 9$

$$= \log_5 \left(\frac{9(x+2)^3}{(x-7)^4} \right)$$

13. (5 pts) Genevum-48 is a radioactive substance that decays according to the model $A(t) = A_0 e^{-0.025t}$, where $A = A(t)$ is the amount of Genevum-48 present at time t (in years). Find the half-life of Genevum-48, to the nearest 10^{th} of a year.

$$A_0 e^{-0.025t} = \frac{1}{2} A_0$$

$$e^{-0.025t} = \frac{1}{2}$$

$$-0.025t = \ln\left(\frac{1}{2}\right)$$

$$t = \frac{\ln\left(\frac{1}{2}\right)}{-0.025} \approx 27.72588722$$

$$\approx 27.7 \text{ yr.}$$

14. The half-life of carbon-14 is (approximately) 5700 years. (I think it's 5600 years in the textbook, but let's roll with 5700.)

- a. (5 pts) Find an exponential model $A(t) = A_0 e^{kt}$ that gives the amount of radioactive carbon-14 present in a charcoal sample after t years.

$$A_0 e^{-kt} = \frac{1}{2} A_0 \text{ when } t = 5700, \text{ i.e.,}$$

$$A_0 e^{-5700k} = \frac{1}{2} A_0$$

$$A(t) = A_0 e^{-\frac{\ln\left(\frac{1}{2}\right)}{5700} t}$$

$$e^{-5700k} = \frac{1}{2}$$

$$\approx A_0 e^{-0.0001216 t}$$

$$-5700k = \ln\left(\frac{1}{2}\right)$$

$$\Rightarrow k = \frac{\ln\left(\frac{1}{2}\right)}{-5700} \approx 1.216047685 \times 10^{-4} \approx .0001216$$

- b. (5 pts) How old is a sample from a neolithic fire pit if it is found that 27% of naturally-occurring carbon-14 is present in the sample?

$$A(t) = A_0 e^{-kt} = .27 A_0$$

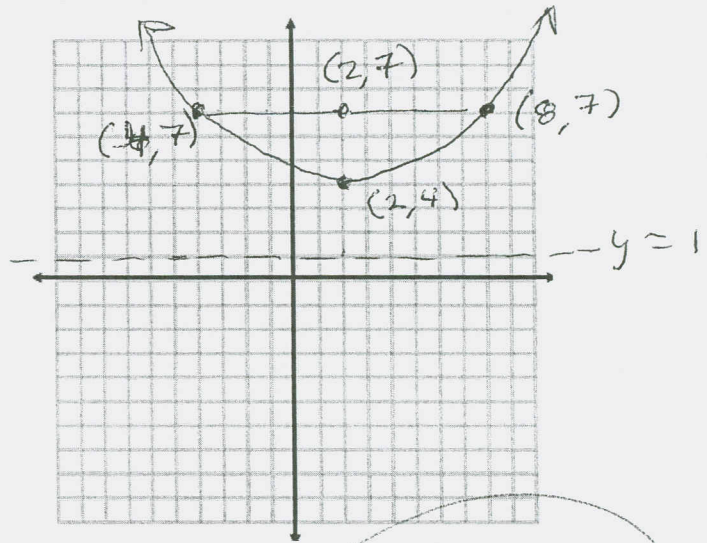
$$e^{-kt} = .27$$

$$-kt = \ln(.27) \Rightarrow t = \frac{\ln(.27)}{-k} = -\frac{\ln(.27)}{\frac{\ln\left(\frac{1}{2}\right)}{5700}}$$

$$\approx 10,767.12152 \text{ yrs.}$$

15. (5 pts bonus) Sketch the graph of the equation $(x-2)^2 = 12(y-4)$. Clearly label the following: Vertex, focus, directrix, and endpoints of the latus rectum.

$$4a = 12 \Rightarrow a = 3$$



16. (5 pts bonus) Complete the square and write $x^2 - 6x = 4y - 37$ in standard form.

$$x^2 - 6x + 3^2 = 4y - 37 + 9$$

$$(x-3)^2 = 4y - 28$$

$$(x-3)^2 = 4(y-7)$$

$$(h,k) = (3, 7)$$

$$\text{Focus} = (3, 8)$$

Identify vertex
& focus

17. (5 pts bonus) Sketch the graph of the ellipse $\frac{(x+4)^2}{25} + \frac{(y-2)^2}{9} = 1$. Clearly label the following: Center, foci, and endpoints of major and minor axes.

$$(h,k) = (-4, 2)$$

$$a = 5, b = 3$$

$$c^2 = a^2 - b^2 = 25 - 9 = 16$$

$$\Rightarrow c = 4$$

