MAT 121 Test 1

100 Points Name KEY

1. (5 pts) State whether the relation below represents a function (Yes/No). If not, why or why not? What is the domain and what is the range?

$$\{(-3,6), (6, 5), (-3, 8), (4, 5)\}$$

No. -3 corresponds to $y = 6$ AND $y = 8$

$\mathcal{D} = \{-3, 4, 6\}$, $\mathcal{R} = \{5, 6, 8\}$

2. (5 pts) Determine whether the equation $y^2 = x + 11$ defines y as a function of x. If it does *not*, show/explain why not.

No.

$y = \pm\sqrt{x+11}$ means there are two y-values associated with a single x-value, e.g., $x = 5 \Rightarrow y = \pm\sqrt{16} = \pm 4$, so $(5,-4)$ & $(5,4)$ are pairs in the relation.

3. (5 pts) Find the domain of $f(x) = \dfrac{x^2 - 4}{\sqrt{x+2}}$.

$x + 2 > 0 \Rightarrow$

$x > -2 \Rightarrow \mathcal{D} = \{x \mid x > -2\}$

4. (5 pts) Find the domain of $g(x) = \dfrac{x^2 + 5x + 17}{x^2 + x - 12}$.

$x^2 + x - 12 \neq 0$

$\Rightarrow (x+4)(x-3) \neq 0$

$\Rightarrow x \neq -4$ and $x \neq 3$

$\mathcal{D} = \mathbb{R} \setminus \{-4, 3\}$

5. (5 pts) Let $f(x) = x^2 + 1$. Find the average rate of change of f from $x = -1$ to $x = 1$.

$$\frac{f(1) - f(-1)}{1 - (-1)} = \frac{1^2 + 1 - ((-1)^2 + 1)}{2}$$

$$= \frac{2 - (2)}{2} =$$

$$= \frac{0}{2}$$

$$\boxed{= 0}$$

KEY

6. Let $f(x)=\sqrt{x+6}$ and $g(x)=5x+2$.

   a. Determine each of the following functions.

      i. (5 pts) $(f+g)(x) = \sqrt{x+6} + 5x + 2$

      ii. (5 pts) $(f \cdot g)(x) = \left(\sqrt{x+6}\right)(5x+2)$

      iii. (5 pts) $\left(\frac{g}{f}\right)(x) = \dfrac{5x+2}{\sqrt{x+6}}$

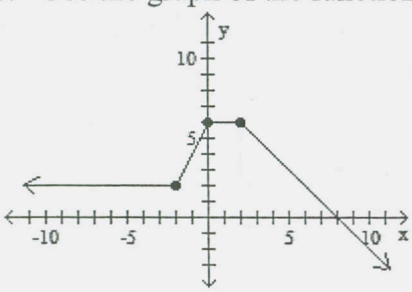
   b. (5 pts) What is the domain of $\left(\frac{g}{f}\right)(x)$?

   $\{x \mid x > -6\}$

7. (5 pts) Determine algebraically whether $h(x)=4x^2-2$ is even, odd, or neither.

$h(-x) = 4(-x)^2 - 2 = 4x^2 - 2 = h(x) \implies$ EVEN

8. Use the graph of the function *f*, below, to find:



   a. (5 pts) The intercepts (Express answers in ordered pairs.)

   $(0,6)$, $(8,0)$

   b. (5 pts) The domain and range.

   $D = \mathbb{R}$, $R = (-\infty, 6]$

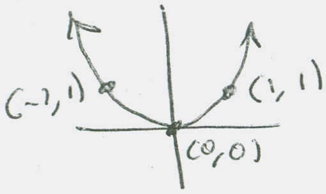
   c. (2 pts) *f* is increasing on $(-2, 0)$

   d. (2 pts) *f* is decreasing on $(2, \infty)$

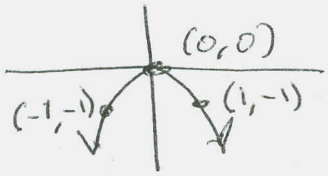
   e. (1 pts) *f* is constant on $(0, 2) \cup (-\infty, -2)$

9. Graph each of the following functions using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all stages.

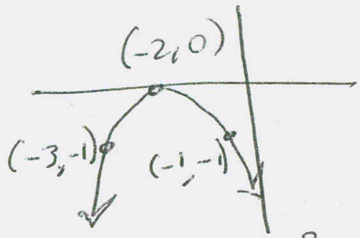
a. (5 pts) $g(x) = -(x+2)^2 + 11$



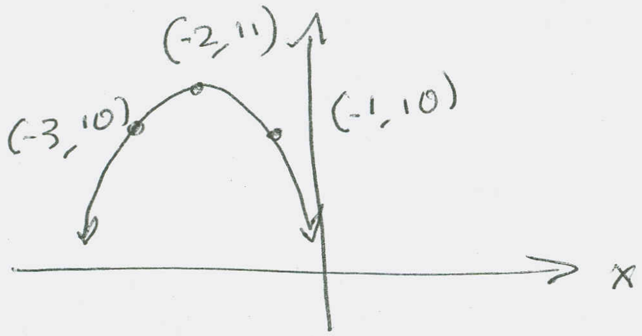
$x^2 = f(x)$



$-x^2 = -f(x)$

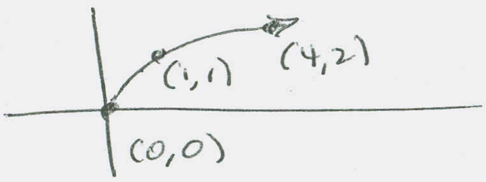


$-(x+2)^2 = -f(x+2)$

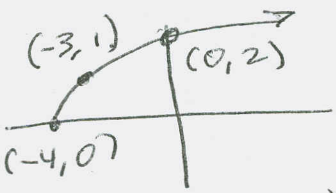


$g(x) = -(x+2)^2 + 11 = -f(x+2) + 11$

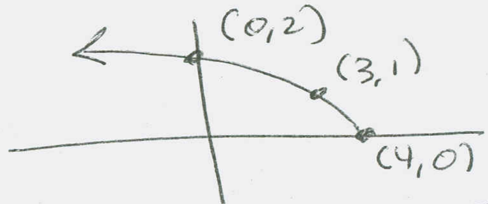
b. (5 pts) $g(x) = \sqrt{4-x} + 5$



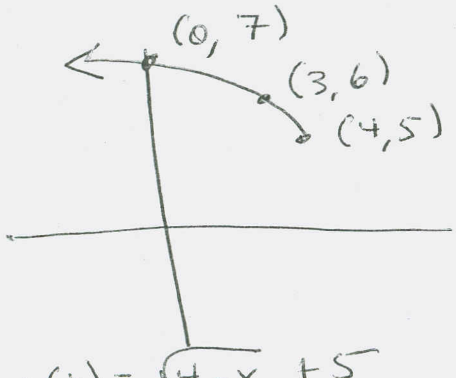
$f(x) = \sqrt{x}$



$\sqrt{x+4} = f(x+4)$



$\sqrt{-x+4} = \cancel{\sqrt{4+x}}$
$= \sqrt{4-x} = f(4-x)$



Alternatives:

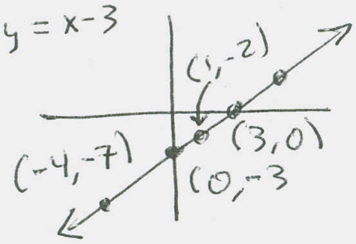
$\sqrt{x} \to \sqrt{-x} \to \sqrt{-(x-4)}$
$\to \sqrt{-(x-4)} + 5 = f(-(x-4)) + 5$

$g(x) = \sqrt{4-x} + 5$
$= f(4-x) + 5$

10. (5 pts) Sketch the graph of $f(x) = \begin{cases} x-3 & \text{if } -4 \le x < 1 \\ 4 & \text{if } x = 1 \\ 3x & \text{if } 1 < x \le 4 \end{cases}$. Include all intercepts.

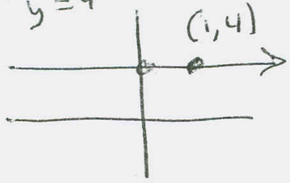
State the domain and range.

$y = x-3$

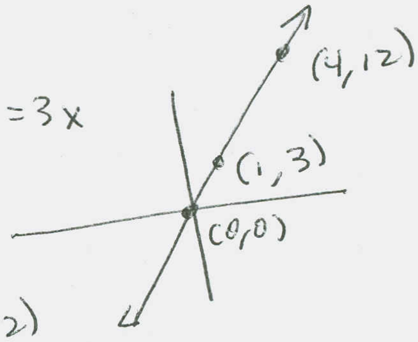


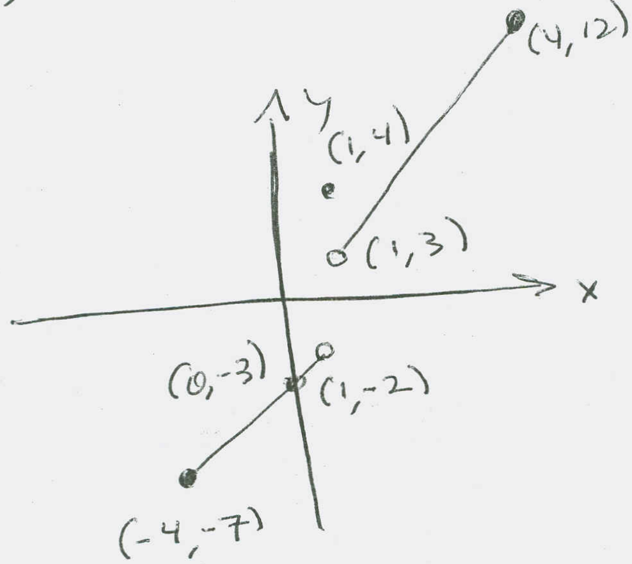
x	x-3
-4	-7
1	-2

$y=4$



$y=3x$



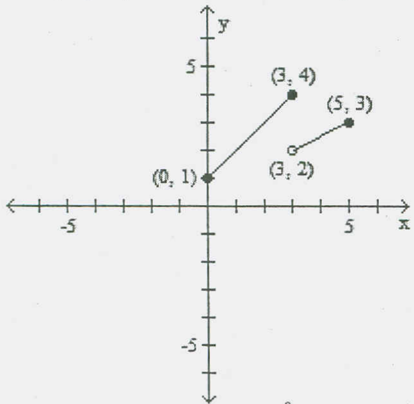


$D = [-4,4]$

$R = [-7,-2) \cup (3,12]$

$\cup \{4\}$ → Redundant.

11. (5 pts) Determine the piecewise-defined function g from its graph, below.



1st: $(0,1)$ & $(3,4)$ $\quad \underline{0 \le x \le 3}$

$m = \frac{4-1}{3-0} = \frac{3}{3} = 1$

$y = x+1$

$$\boxed{g(x) = \begin{cases} x+1 & \text{if } 0 \le x \le 3 \\ \frac{1}{2}x + \frac{1}{2} & \text{if } 3 < x \le 5 \end{cases}}$$

2nd: $(3,2)$ & $(5,3)$ $\quad 3 < x \le 5$

$m = \frac{3-2}{5-3} = \frac{1}{2}$

$y = m(x-x_1) + y_1$

$= \frac{1}{2}(x-3) + 2 = \frac{1}{2}x - \frac{3}{2} + 2 = \frac{1}{2}x + \frac{1}{2}$

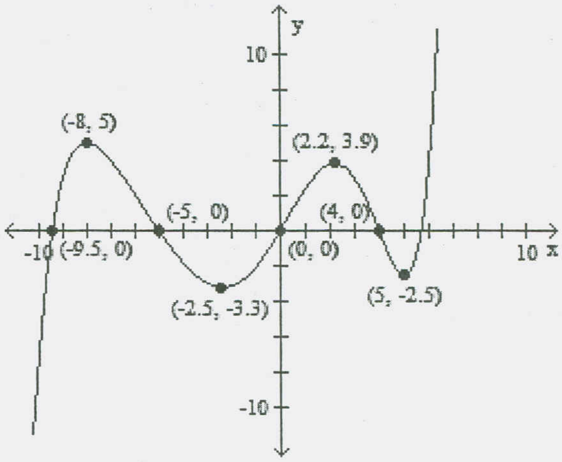
12. (5 pts) Let $f(x) = x^2 - 2x$. Compute the difference quotient $\frac{f(x+h) - f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - 2(x+h) - (x^2-2x)}{h}$$

$$= \frac{x^2+2xh+h^2-2x-2h-x^2+2x}{h}$$

$$= \frac{2xh+h^2-2h}{h} = \frac{h(2x+h-2)}{h} = \boxed{2x+h-2}$$

13. (5 pts) Determine the numbers, if any, at which the function shown on the right has local minima. What are the minima?



min of ...

... -3.3 @ $x = -2.5$ $(-2.5, -3.3)$

... -2.5 @ $x = 5$ $(5, -2.5)$

14. (5 pts) Determine the numbers, if any, at which the function shown on the right has local maxima. What are the maxima?

max of ...

... 5 @ $x = -8$ $(-8, 5)$

... 3.9 @ $x = 2.2$ $(2.2, 3.9)$