Simplify:

1. $\sqrt[5]{x^{14}y^7} = \sqrt[5]{x^{10}x^4y^5y^2} = \boxed{x^2y\sqrt[5]{x^4y^2}}$

$16\overline{)80}$
5

$\sqrt{80} = \sqrt{16}\sqrt{5}$
$= 4\sqrt{5}$

2. $6\sqrt{5} - 3\sqrt{4} - 7\sqrt{80} = 6\sqrt{5} - 3\cdot2 - 7\cdot4\sqrt{5}$
$= 6\sqrt{5} - 6 - 28\sqrt{5}$
$= \boxed{-22\sqrt{5} - 6}$

3. Multiply: $(x+1)^3 = \boxed{x^3 + 3x^2 + 3x + 1}$

$(12)(20) = 240$

$2\underline{|12}$ $\quad 2\underline{|20}$
$2\underline{|6}$ $\quad 2\underline{|10}$
3 $\quad 5$

Factors of
$240 = 2\cdot2\cdot2\cdot2\cdot3\cdot5$
whose sum is -31
Ah: $(-16)(-15) = 240$
$-16 - 15 = -31$

Factor:

4. $12x^2 + 20 - 31x = 12x^2 - 31x + 20$
$= 12x^2 - 16x - 15x + 20$
$= 4x(3x-4) - 5(3x-4) = \boxed{(3x-4)(4x-5)}$

5. $16x^2 + 12x - 18 = 2(8x^2 + 6x - 9)$
$= 2[8x^2 - 6x + 12x - 9]$
$= 2[2x(4x-3) + 3(4x-3)] = 2[(4x-3)(2x+3)]$

$(-9)(8) = -72$
$-(3)(3)(2)(2)(2)$
want a DIFFERENCE of $+6$:
$((-3)(2))((3)(2)(2))$
$(-6)(12) = -72$
$-6 + 12 = 6$ ✓

6. $-4x^4y^4 + 256 = -4[x^4y^4 - 64]$
$= \boxed{-4[(x^2y^2 - 8)(x^2y^2 + 8)]}$ DONE (STOP)
$= -4[(xy - \sqrt{8})(xy + \sqrt{8})(x^2y^2 + 8)]$

Factor:

7. $2x^3y^3 + 54x^3 = 2x^3(y^3 + 27)$
$\boxed{= 2x^3[(y+3)(y^2 + 3y + 9)]}$

Solve:

8. $(2x+4)^2 = 25$

$2x + 4 = \pm 5$

$2x = -4 \pm 5$

$\boxed{x = \dfrac{-4 \pm 5}{2}}$

9. $4x^2 + 7x = 15$

$4x^2 + 7x - 15 = 0$

$a = 4,\ b = 7,\ c = -15$

$b^2 - 4ac = 7^2 - 4(4)(-15)$

$= 49 + 240$

$= 289 = 17^2$

$\Rightarrow$ FACTORS!

$(4)(-15) = -60$

$(2)(2)(3)(5)$

want difference of 7

$(2)(2)(3) = 12$

$5 = 5$

$12 - 5 = 7$

$\Rightarrow 4x^2 + 12x - 5x - 15 = 0$

$4x(x+3) - 5(x+3) = 0$

$(x+3)(4x-5) = 0 \longrightarrow \boxed{x = -3, \frac{5}{4}}$

10. Solve by completing the square: $x^2 + 4x - 21 = 0$

$x^2 + 4x = 21$

$x^2 + 4x + 2^2 = 21 + 4$

$(x+2)^2 = 25$

$x + 2 = \pm 5$

$x = -2 \pm 5$

$\boxed{x = 3, -7}$

11. Write the expression in the form $a + bi$: $(9 - 5i)(-9 + 3i)$

$= -81 + 27i + 45i - 15i^2$

$= -81 + 72i + 15$

$= \boxed{-66 + 72i}$

12. Simplify: $\frac{8 - 3i}{5 + i}$

$\frac{(8-3i)}{(5+i)} \cdot \frac{(5-i)}{(5-i)}$

$= \frac{40 - 8i - 15i + 3i^2}{5^2 + 1^2} = \frac{40 - 23i - 3}{26} = \boxed{\frac{37}{26} - \frac{23}{26}i}$

13. Write the standard form of the equation of the line passing through the point (1, –5) and perpendicular to the line $4x - 3y = -6$.

$Ax + By = C \rightarrow m = -\frac{A}{B} = -\frac{4}{-3}$

$= \frac{4}{3}$

$m = \frac{4}{3} \Longrightarrow m_\perp = -\frac{3}{4}$

$y - y_1 = m_\perp(x - x_1)$

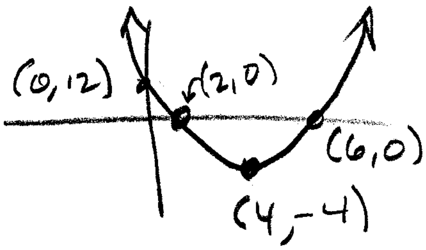
$y + 5 = -\frac{3}{4}(x - 1) \longrightarrow 4y + 20 = -3x + 3$

$\longrightarrow \boxed{3x + 4y = -17}$

14. Determine the domain and range: $y = \sqrt{x+4} + 9$

$\mathcal{D} = [-4, \infty)$, $R = [9, \infty)$

15. Graph the parabola and find the vertex: $f(x) = x^2 - 8x + 12 = (x-6)(x-2)$



$\frac{2+6}{2} = 4$

$f(4) = 4^2 - 8(4) + 12$

$= 16 - 32 + 12$

$= 28 - 32 = -4$

16. Determine the domain of the function $h(x) = \dfrac{7x}{x(x^2-36)}$.

$x \neq 0$, $x^2 - 36 \neq 0$

$x^2 \neq 36$

$x \neq \pm 6$

$\mathcal{D} = \{x \mid x \neq 0 \text{ and } x \neq \pm 6\}$

17. If $f(x) = x^4$ and $g(x) = -2 - 4x$, find $g(f(x))$.

$g(f(x)) = g(x^4) = -2 - 4(x^4) = -2 - 4x^4$

SOLVE:

18. $\sqrt{x-2} = x - 4$

$x - 2 = (x-4)^2$

$x - 2 = x^2 - 8x + 16$

$\Rightarrow x^2 - 9x + 18 = 0$

$\Rightarrow (x-3)(x-6) = 0$

$\Rightarrow x = 3, 6$

No

Check

$\sqrt{3-2} = 3 - 4$

$1 = -1$ No

$\sqrt{6-2} = 6 - 4$?

$\sqrt{4} = 2$ ✓ Yes

$\boxed{x = 6}$

19. $x^2+7x \geq 18$ $\Longrightarrow$ $x^2+7x-18 \geq 0 \Longrightarrow (x+9)(x-2) \geq 0$

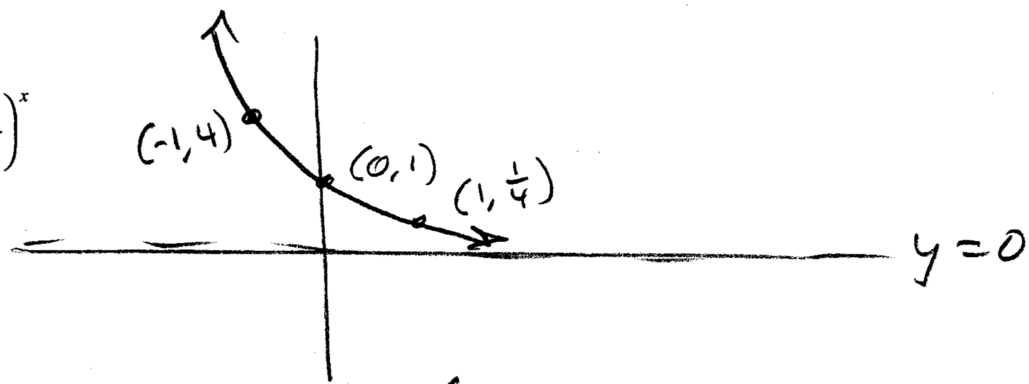
\+ − +

Yes −9 No 2 Yes

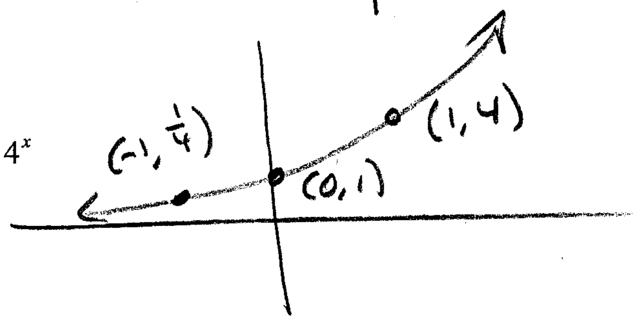
$\boxed{(-\infty,-9] \cup [2,\infty)}$ OR $\{x \mid x \leq -9 \text{ OR } x \geq 2\}$

Graph:

20. $f(x)=\left(\frac{1}{4}\right)^x$



21. $f(x)=4^x$



22. Solve: $\frac{1}{27} = 9^{9x-5}$ $\Longrightarrow$ $\frac{1}{3^3} = (3^2)^{9x-5} \Longrightarrow 3^{-3} = 3^{18x-10}$

$\Longrightarrow -3 = 18x-10 \Longrightarrow 7 = 18x \Longrightarrow \boxed{\frac{7}{18} = x}$

23. Write the equation $6^3 = 216$ in logarithmic form. $\boxed{\log_6(216)=3}$

24. Write the equation $\log_4 8 = \frac{3}{2}$ in exponential form.

$\boxed{8^{\frac{3}{2}} = 4}$

25. Evaluate: $\log_2\left(\frac{1}{16}\right)$ $= \log_2\left(\frac{1}{2^4}\right) = \log_2\left(2^{-4}\right) = -4$

26. Use the rules for logarithms of products, quotients, and powers to write as the sum or difference of logarithms: $\log_b \sqrt[5]{\frac{x^9 y^2}{z^7}}$

$$= \log_b\left(\left(\frac{x^9 y^2}{z^7}\right)^{\frac{1}{5}}\right) = \frac{1}{5}\left[\log_b\left(\frac{x^9 y^2}{z^7}\right)\right]$$

$$= \frac{1}{5}\left[9\log_b x + 2\log_b y - 7\log_b z\right]$$

27. Solve: $\log_{27} x = \frac{4}{3}$

$$27^{\frac{4}{3}} = x \Longrightarrow \left(27^{\frac{1}{3}}\right)^4 = x \Longrightarrow 3^4 = \boxed{x = 81}$$

28. $f(x) = x^3$ and $g(x) = 1 - 8x$, find $f(g(x))$.

REPEAT CONCEPT CUT.

29. A radioactive substance decays so that the amount A present at time t (years) is $A = A_0 e^{-1.9t}$. Find the half-life (time for half to decay) of this substance. ($\ln .5 = -0.69315$)

Want t when $A(t) = \frac{1}{2}$ of what we started w/.

$$A_0 e^{-1.9t} = \frac{1}{2}A_0 \Longrightarrow e^{-1.9t} = \frac{1}{2} \Longrightarrow$$

$$-1.9t = \ln\left(\frac{1}{2}\right) \Longrightarrow t = \frac{\ln\left(\frac{1}{2}\right)}{-1.9} \approx \frac{-.69315}{-1.9}$$

$$\approx \boxed{.3648 \text{ years}}$$

30. Find all real solutions of the following equation: $\log_2(x+2)+\log_2(x+4)=3$

$\Rightarrow \log_2((x+2)(x+4)) = 3$

$\Rightarrow (x+2)(x+4) = 2^3$

$\Rightarrow x^2+6x+8 = 8$

$\Rightarrow x^2+6x = 0$

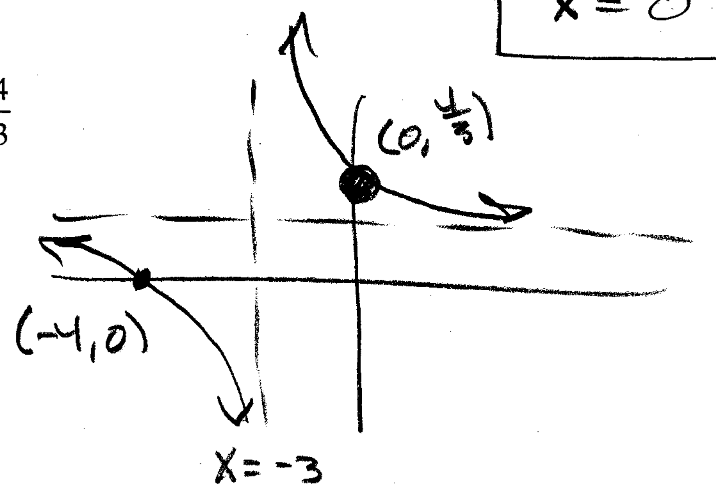
$\Rightarrow x(x+6) = 0$

$\Rightarrow x = 0, -6$ only

-6 is illegal, so

$\boxed{x = 0}$

31. Graph $f(x) = \dfrac{x+4}{x+3}$



32. Use synthetic division to find $f(-5)$ if $f(x) = 2x^6-49x^4+3x^3-14x^2+2x+10$.

-5	2	0	-49	3	-14	2	10
		-10	50	-5	10	20	-110
	2	-10	1	-2	-4	22	$\boxed{-100 = f(-5)}$

33. Use the Remainder Theorem to find $P(-4)$ if $P(x) = x^6+3x^5+3x^3-7x^2-34$.

Repeat of # 32

Ditch.

34. List all of the potential rational zeros of the polynomial function. Do not attempt to find the zeros. $f(x) = 2x^3-2x^2+8x-6$

$\pm 6, \pm 3, \pm 2, \pm 1, \pm \frac{1}{2}$

35. Use the Intermediate Value Theorem to show that the graph of the function has an x-intercept in the given interval. Approximate the x-intercept correct to two places.
$f(x) = x^3 + x^2 - 8x - 11$; $[-3, -2]$

$$\begin{array}{r|rrrr} -3 & 1 & 1 & -8 & -11 \\ & & -3 & 6 & 6 \\ \hline & 1 & -2 & -2 & -5 \end{array} = f(-3) \qquad \begin{array}{r|rrrr} -2 & 1 & 1 & -8 & -11 \\ & & -2 & 2 & 12 \\ \hline & 1 & -1 & -6 & 1 \end{array} = f(-2)$$

Since $f(-3) < 0$ & $f(-2) > 0$, IVT says $f(c) = 0$ for some c in $(-3, -2)$

36. Find a third-degree polynomial with real coefficients and with zeros -2 and $4 + i$.

$$(x+2)(x-(4+i))(x-(4-i))$$
$$= (x+2)(x^2 - (4-i)x - (4+i)x + (4+i)(4-i))$$
$$= (x+2)(x^2 - 8x + 17) = x^3 - 8x^2 + 17x + 2x^2 - 16x + 34$$
$$= \boxed{x^3 - 6x^2 + x + 34}$$

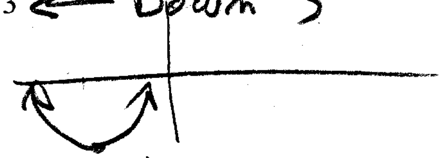
37. Find all the real and complex zeros of the polynomial $x^4 + 6x^3 + x^2 - 54x - 90$.

Too LONG DITCH.

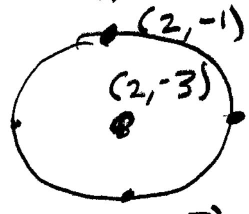
38. Find the rational roots of f. List any irrational roots correct to two decimal places.
$f(x) = x^4 - 9x^3 + 15x^2 + 45x - 100$ DITCH

39. Graph the parabola: $y = (x+3)^2 - 3$ ← Down 3

left 3



40. Graph: $(x-2)^2 + (y+3)^2 = 4$



circle of radius 2, centered @ (2,-3)

41. Find the equation of the circle with center (5, –2) and radius of 2.

$$(x-5)^2 + (x+2)^2 = 2^2$$

Method 2: $-\frac{b}{2a} = \frac{5}{2} = h$

42. Find the vertex of the parabola $y = x^2 - 5x - 5$.

Method 1:

$\boxed{\text{Vertex is } (h,k) = \left(\frac{5}{2}, -\frac{45}{4}\right)}$

$x^2 - 5x - 5$

$= x^2 - 5x + \left(\frac{5}{2}\right)^2 - \frac{25}{4} - \frac{20}{4} = \left(x - \frac{5}{2}\right)^2 - \frac{45}{4}$

$f\left(\frac{5}{2}\right) = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) - 5$

$= \frac{25}{4} - \frac{25}{2} - 5$

$= \frac{25 - 50 - 20}{4}$

$= -\frac{45}{4} = k$

43. Identify the following curve: $9x^2 = 64 - 64y^2$

$9x^2 + 64y^2 = 64$

Ellipse

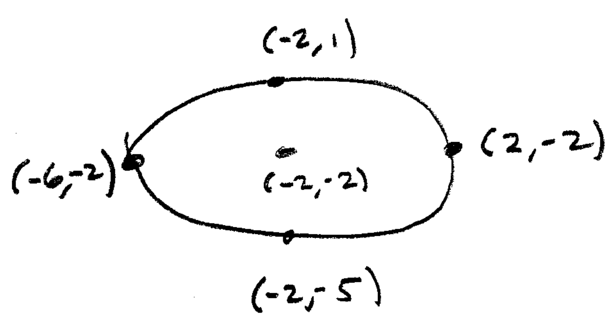
44. Find the center and radius of the circle with the following equation:
$4x^2 + 4y^2 - 24x - 32y + 84 = 0 \rightarrow 4(x^2 + y^2 - 6x - 8y + 21) = 0$

$x^2 - 6x + 3^2 + y^2 - 8y + 4^2 = -21 + 9 + 16$

$(x-3)^2 + (y-4)^2 = 4 = 2^2$

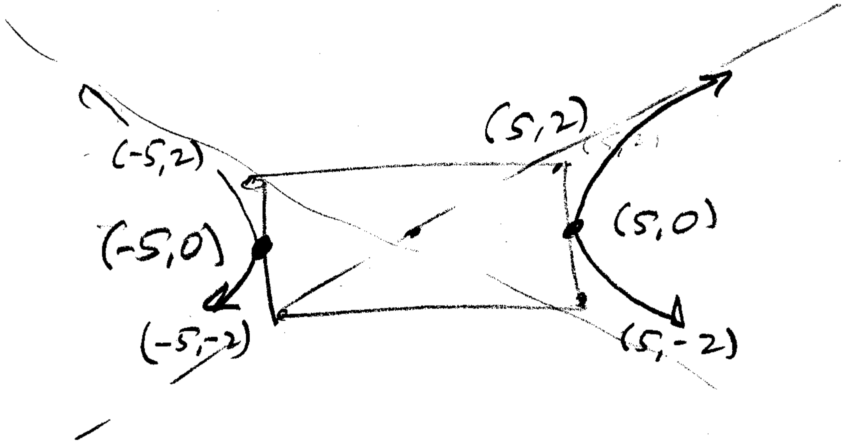
$\boxed{(h,k) = (3,4),\ r = 2}$

45. Graph: 57. $\frac{(x+2)^2}{16} + \frac{(y+2)^2}{9} = 1$



46. $4x^2 - 25y^2 = 100$

$$\frac{x^2}{25} - \frac{y^2}{4} = 1$$



47. If $A = \begin{bmatrix} 1 & 3 & -1 \\ -4 & 2 & -3 \\ 5 & 5 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 & -5 \\ 4 & -5 & -1 \\ 3 & -1 & -4 \end{bmatrix}$, find BA.

$$\begin{bmatrix} 2 & -2 & -5 \\ 4 & -5 & -1 \\ 3 & -1 & -4 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 \\ -4 & 2 & -3 \\ 5 & 5 & -4 \end{bmatrix} = \begin{bmatrix} -15 & -23 & 24 \\ 19 & -3 & 15 \\ -13 & -13 & 16 \end{bmatrix}$$

48. Find the inverse of the matrix (if it exists) $\begin{bmatrix} 5 & -2 \\ 3 & 3 \end{bmatrix}$.

INVERSE: $\begin{bmatrix} \frac{1}{7} & \frac{2}{21} \\ -\frac{1}{7} & \frac{5}{21} \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 5 & -2 & 1 & 0 \\ 3 & 3 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 3 & 3 & 0 & 1 \\ 5 & -2 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 1 & 0 & \frac{1}{3} \\ 5 & -2 & 1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 1 & 1 & 0 & \frac{1}{3} \\ 0 & -7 & 1 & -\frac{5}{3} \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 1 & 0 & \frac{1}{3} \\ 0 & 1 & -\frac{1}{7} & \frac{5}{21} \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{7} & \frac{2}{21} \\ 0 & 1 & -\frac{1}{7} & \frac{5}{21} \end{array}\right]$$

$$-\frac{5}{21} + \frac{1}{3} = \frac{-5+7}{21} = \frac{2}{21}$$

49. Write the augmented matrix for the system of equations.

$$\begin{cases} x - 2y + z = 6 \\ 3x - 5y - 6z = 4 \\ 2x - 6y + 21z = 1 \end{cases}$$

50. Use matrices to solve the following system
$$\begin{aligned} 2x+3z &= 11 \\ x-3y-5z &= -18 \\ -4x+y-z &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 3 & 11 \\ 1 & -3 & -5 & -18 \\ -4 & 1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & -5 & -18 \\ 2 & 0 & 3 & 11 \\ -4 & 1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & -5 & -18 \\ 0 & 6 & 13 & 47 \\ 0 & -11 & -21 & -72 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -3 & -5 & -18 \\ 0 & 1 & \frac{13}{6} & \frac{47}{6} \\ 0 & -11 & -21 & -72 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & \frac{3}{2} & \frac{11}{2} \\ 0 & 1 & \frac{13}{6} & \frac{47}{6} \\ 0 & 0 & \frac{17}{6} & \frac{85}{6} \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ 5 \end{bmatrix}$$

Check: $\begin{bmatrix} 2 & 0 & 3 \\ 1 & -3 & -5 \\ -4 & 1 & -1 \end{bmatrix}\begin{bmatrix} -2 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} -4+0+15 \\ -2+9-25 \\ 8-3-5 \end{bmatrix} = \begin{bmatrix} 11 \\ -18 \\ 0 \end{bmatrix}$ ✓

51. Find the common difference for the arithmetic sequence: –8, 21, 50, ...

$50-21=29$ ✓
$21-(-8)=29$

$\boxed{29}$

52. Give the first four terms of the arithmetic sequence for which $a_1 = -38$ and $d=3$.

$-38, -35, -32, -29$

53. Write in summation notation: $d=6$, $a_1=21$
$27+33+39+45+51+57+63$

$\sum_{k=1}^{7}(21+6k)$ is one answer

54. $\frac{x^2}{7} - \frac{x^3}{8} + \frac{x^4}{9} - \frac{x^5}{10} + \frac{x^6}{11}$

$\sum_{k=2}^{6}(-1)^k \frac{x^k}{k+5}$ is one answer

55. Write as an indicated sum: $\sum_{i=1}^{4}(-3i^2-2)$

$$= (-3(1)^2-2) + (-3(2)^2-2) + (-3(3)^2-2) + (-3(4)^2-2)$$

56. Find the common ratio: $-1, -\frac{3}{4}, -\frac{9}{16}, \ldots$

$$\frac{-\frac{3}{4}}{-1} = \frac{3}{4} \qquad r = \frac{3}{4}$$

$$-\frac{9}{16} \div -\frac{3}{4} = -\frac{\cancel{9}^{3}}{\cancel{16}_{4}} \cdot -\frac{\cancel{4}^{1}}{\cancel{3}_{1}} = \frac{3}{4}$$

57. Find the sum of the geometric series: $28, -7, \frac{7}{4}, -\frac{7}{16}, \ldots$

$a = 28$

$r = -\frac{1}{4}$

Check: $(28)(-\frac{1}{4}) = -7$

$(-7)(-\frac{1}{4}) = +\frac{7}{4}$

$$S = a\left(\frac{1}{1-r}\right) = 28\left(\frac{1}{1+\frac{1}{4}}\right)$$

$$= 28\left(\frac{1}{\frac{5}{4}}\right)$$

$$= (28)\left(\frac{4}{5}\right) = \boxed{\frac{112}{5} = S}$$

58. Let $P(n)$ represent the statement:

$4 + 12 + 20 + \cdots + (8n-4) = 4n^2$

Use the Principle of Mathematical Induction to show that $P(n)$ is true for all integers n, $n \geq 1$.

$P(1) = 4 = 4(1)^2$. Now $\mathscr{S}$ $P(K)$ holds for some K.

Then $4+12+20+\cdots+(8K-4) = 4K^2$ and so

$4+12+20+\cdots+(8K-4) + (8(K+1)-4)$

$= 4K^2 + 8(K+1) - 4$

$= 4K^2+8K+8-4$

$= 4K^2+8K+4$

$= 4(K^2+2K+1) = 4(K+1)^2 \Rightarrow P(K+1)$ holds and we are done!

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

59. Multiply: $(u-t)^6$

$$= \boxed{u^6 - 6u^5t + 15u^4t^2 - 20u^3t^3 + 15u^2t^4 - 6ut^5 + t^6}$$

60. Evaluate: 7! $= 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2 = \boxed{5040}$

61. Prestige Builders has a development of new homes. There are five different floor plans, seven exterior colors, and an option of either a one-car or a two-car garage. How many choices are there for one home?

$$(5)(7)(2) = \boxed{70}$$

62. Eleven people are entered in a race. If there are no ties, in how many ways can the first two places come out?

$${}_{11}P_2 = \frac{11!}{(11-2)!} = \frac{11!}{9!} = 11\cdot 10 = \boxed{110}$$

63. How many subsets of two elements are contained in the set $\{a, b, c, d\}$?

$${}_4C_2 = \frac{4!}{(4-2)!\,2!} = \frac{4!}{2!\,2!} = \frac{4\cdot 3}{2!} = \frac{4\cdot 3}{2} = \boxed{6}$$

64. How many different ways can 9 different runners finish in first, second, and third places in a race?

REPEAT OF 62 $\quad {}_9P_3 = \frac{9!}{6!} = 9\cdot 8\cdot 7 = \boxed{504}$

65. The probability of getting an A in Mrs. Ritchie's class in any semester is 19%. What is the probability of *not* getting an A?

$$1 - .19 = \boxed{.81}$$

66. Two urns each contain yellow balls and green balls. Urn I contains three yellow balls and four green balls and Urn II contains two yellow balls and four green balls. A ball is drawn from each urn. What is the probability that both balls are yellow?

$$\frac{{}_3C_1}{{}_7C_1} \cdot \frac{{}_2C_1}{{}_6C_1} = \frac{3}{7}\cdot\frac{2}{6} = \boxed{\frac{6}{42}} = \boxed{\frac{1}{7}}$$