

This test is due first thing Monday morning, 8:10 sharp. It is hoped that this test will be a learning experience for you. :o)
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find an equation of the parabola described.

- 1) Focus at (4, 0); vertex at (0, 0)

$a = 4$, on its side, $(h, k) = (0, 0)$
 $4a = 16$, $y^2 = 4ax = 16x$, $y^2 = 16x$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

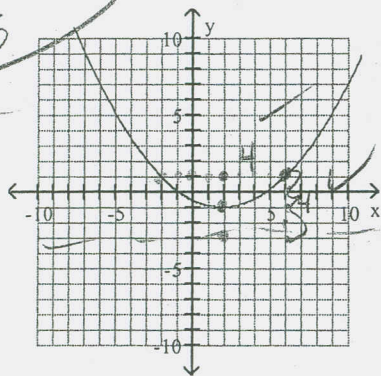
Match the equation to the graph.

2) $(x - 2)^2 = 8(y + 1)$

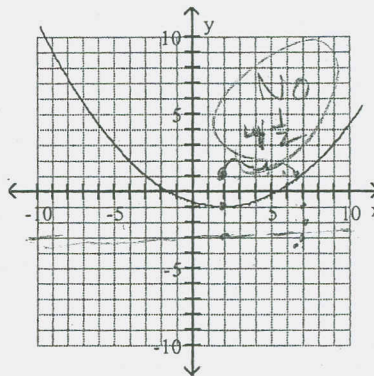
$(h, k) = (2, -1)$
 $4a = 8 \Rightarrow a = 2$

A)

Yes

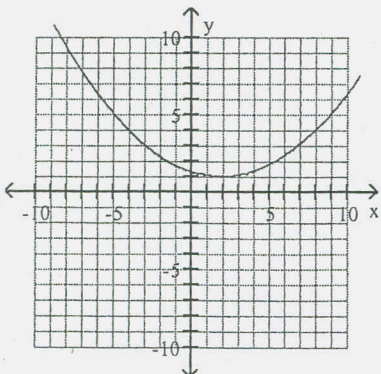


B)

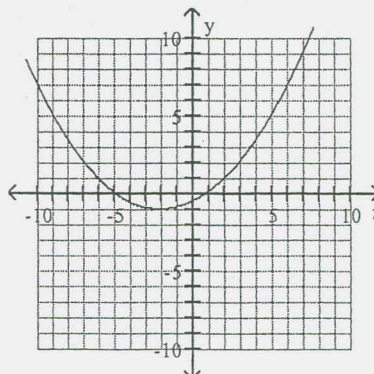


Wrong focal length

C)



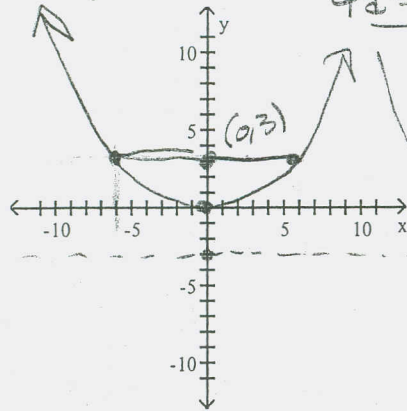
D)



SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find the vertex, focus, and directrix of the parabola.

3) $x^2 = 12y$



$$4a = 12 \Rightarrow a = 3$$

$$(h, k) = (0, 0)$$

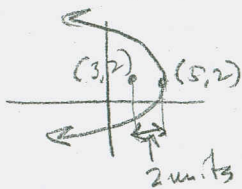
$$\text{Focus} = (0, 3)$$

$$\text{Directrix: } y = -3$$

$$y = -3$$

Find an equation for the parabola described.

4) Vertex at (5, 2); focus at (3, 2)



$$y^2 = -4(2)x$$

$$y^2 = -8x$$

$$(h, k) = (5, 2)$$

$$(y-2)^2 = -8(x-5)$$

Find the vertex, focus, and directrix of the parabola with the given equation.

5) $(y-3)^2 = 12(x-1)$

$$V = (h, k) = (1, 3)$$

$$4a = 12 \Rightarrow a = 3$$

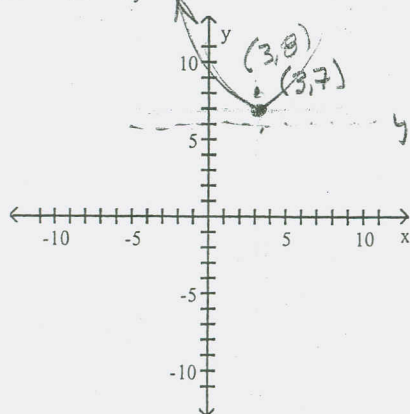
$$(-2, 3) \quad (1, 3) \quad (4, 3)$$

$$\text{Focus} = (4, 3)$$

$$\text{Directrix} = x = -2$$

Find the vertex, focus, and directrix of the parabola. Graph the equation.

6) $x^2 - 6x = 4y - 37$



$$x^2 - 6x + 3^2 = 4y - 37 + 9$$

$$(x-3)^2 = 4y - 28 = 4(y-7)$$

$$4a = 4 \Rightarrow a = 1$$

upright

$$\text{Vert: } (3, 7)$$

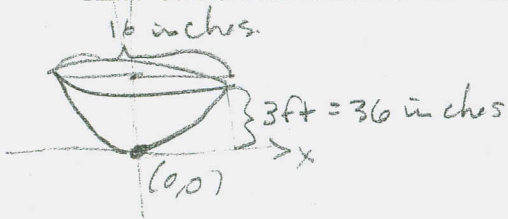
$$\text{Focus: } (3, 8)$$

$$\text{Directrix: } y = 6$$

Solve the problem.

- 7) A reflecting telescope contains a mirror shaped like a paraboloid of revolution. If the mirror is 16 inches across at its opening and is 3 feet deep, where will the light be concentrated?

Hint: Convert feet to inches. We're asking you where the focus is.



$(0,0)$ & $(8,36)$ are on it

$$x^2 = 4ay$$

$$8^2 = 4a(36)$$

$$64 = 144a$$

$$\begin{array}{r} .44 \\ 9 \overline{) 4.0000} \\ \underline{3.6} \\ .40 \end{array}$$

$$a = \frac{4}{9} = \frac{4 \cdot 2 \cdot 2}{9 \cdot 3 \cdot 3} = \frac{88}{1212} - \frac{64}{144} = 2$$

Verify that the values of the variables listed are solutions of the system of equations.

8)
$$\begin{cases} 3x + y = -1 \\ 4x + 3y = 7 \\ x = -2, y = 5 \end{cases}$$

$3(-2) + 5 = -6 + 5 = -1$ ✓
 $4(-2) + 3(5) = -8 + 15 = 7$ ✓

Solve the system of equations by using substitution.

9)
$$\begin{cases} 3x + y = 13 \\ 2x + 9y = -8 \end{cases} \Rightarrow y = 13 - 3x$$

$$2x + 9(13 - 3x) = -8$$

$$2x + 117 - 27x = -8$$

$$\begin{aligned} -25x &= -125 \\ \boxed{x = 5} \end{aligned}$$

$$y = 13 - 3x$$

$$= 13 - 3(5)$$

$$= 13 - 15$$

$$= -2 = y$$

$(5, -2) = (x, y)$

Use the elimination method to solve the system.

10)
$$\begin{cases} 2x + 10y = -64 \\ 11x + 2y = 72 \end{cases}$$

Hint: Divide the first equation by 2 in your first step.

$$\begin{array}{r} x + 5y = -32 \quad \text{TIMES } -11 \Rightarrow -11x - 55y = 352 \\ 11x + 2y = 72 \\ \hline -53y = 424 \end{array}$$

$$\begin{array}{r} 32 \\ 11 \\ \hline 32 \\ 32 \\ \hline 64 \end{array}$$

$(x, y) = (8, -8)$

$$\begin{array}{r} 253 \\ 8 \\ \hline 424 \end{array} \checkmark$$

$$y = -\frac{424}{53} = \boxed{-8 = y}$$

$$\Rightarrow x + 5(-8) = -32$$

$$\Rightarrow x = -32 + 40 = \boxed{8 = x}$$

Identify variables in words and units. Write the system of equations.

- 11) The Family Fine Arts Center charges \$24 per adult and \$10 per senior citizen for its performances. On a recent weekend evening when 455 people paid admission, the total receipts were \$6496. How many who paid were senior citizens?

$x = \# \text{ of seniors}$	$x + y = 455$
$y = \# \text{ of adults}$	$24x + 10y = 6496$

The following system has no solution. Explain how you can tell. What's the geometry of this situation?

$$12) \begin{cases} 9x - 8y = 1 \\ 9x - 8y = -8 \end{cases} \quad \text{Two parallel lines}$$

$$E1 - E2 = 0 = 9 \quad \text{etc.}$$

Solve the system of equations.

$$13) \begin{cases} x + y + z = 0 \\ x - y + 5z = 26 \\ 4x + y + z = 3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -1 & 5 & 26 \\ 4 & 1 & 1 & 3 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -2 & 4 & 26 \\ 0 & -3 & -3 & 3 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & -13 \\ 0 & 1 & 1 & -1 \end{array} \right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & -13 \\ 0 & 0 & 3 & 12 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & -13 \\ 0 & 0 & 1 & 4 \end{array} \right]$$

$$\rightarrow \boxed{z = 4}$$

$$\rightarrow y - 2z = -13$$

$$y - 2(4) = -13$$

$$\boxed{y = -5}$$

$$x + y + z = 0$$

$$x - 5 + 4 = 0$$

$$x - 1 = 0$$

$$\boxed{x = 1}$$

$$\boxed{(x, y, z) \in \{(1, -5, 4)\}}$$

Identify variables and write the system of equations for the following problem.

- 14) Lexie wants to have an income of \$9000 per year from investments. To that end she is going to invest \$90,000 in three different accounts. These accounts pay 7%, 10%, and 14% simple interest. If she wants to have \$10,000 more in the account paying 7% simple interest than she has in the account paying 14% simple interest, how much should go into each account?

$$\text{Income} \quad : \quad .07x + .10y + .14z = 9000$$

$$\text{Total} \quad = \quad x + y + z = 90000$$

\$10000 more in 7% than in 14% acct.

Amnt invested @ 7% is 10000 plus Amnt invested @ 14%

$$= 10000 + z$$

$$7\% - \text{vs} - 14\% : \quad x - z = 10000$$

x = Amnt invested @ 7% (\$)
 y = " " " 10% (\$)
 z = " " " 14% (\$)

Explain how you can tell that #15 has no solution.

$$15) \begin{cases} x + y + z = 1 \\ x - y + 5z = -1 \\ 4x + 4y + 4z = -8 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & -1 & 5 & -1 \\ 4 & 4 & 4 & -8 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & 0 & -12 \end{array} \right]$$

$\rightarrow 0 = -12$ ~~×~~ No Sol'n

This system has infinitely many solutions. Solve it and state the general solution, for instance, $x = 3z - 2$, $y = 2z + 7$, $z = \text{any real number}$.

$$16) \begin{cases} x + 4y - z = 3 \\ x + 5y - 2z = 5 \\ 3x + 12y - 3z = 9 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 3 \\ 1 & 5 & -2 & 5 \\ 3 & 12 & -3 & 9 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 4 & -1 & 3 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\Rightarrow \begin{cases} x + 4y - z = 3 \\ y - z = 2 \\ 0 = 0 \end{cases} \Rightarrow \boxed{y = z + 2}$$

Check:

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 3 \\ 1 & 5 & -2 & 5 \\ 3 & 12 & -3 & 9 \end{array} \right] \begin{bmatrix} -5 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 9 \end{bmatrix} \checkmark$$

$$\Rightarrow x + 4(z + 2) - z = 3$$

$$x + 4z + 8 - z = 3$$

$$x + 3z = -5$$

$$\boxed{x = -3z - 5}$$

$$\boxed{z = \text{Any real } \neq}$$

Write the augmented matrix for the system. Do not solve.

$$17) \begin{cases} 8x + 6y = 34 \\ 7x + 8y = 49 \end{cases} \quad \left[\begin{array}{cc|c} 8 & 6 & 34 \\ 7 & 8 & 49 \end{array} \right]$$

Write the system of equations associated with the augmented matrix. Do not solve.

$$18) \left[\begin{array}{ccc|c} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 2 \end{array} \right] \quad \begin{cases} x = -10 \\ y = -7 \\ z = 2 \end{cases}$$

Perform the row operation(s) on the given augmented matrix.

$$19) R_1 = \frac{1}{3}r_1$$

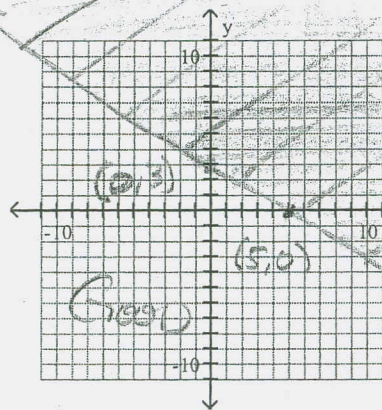
$$\left[\begin{array}{cc|c} 3 & -15 & -6 \\ 5 & 1 & -7 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & -5 & -2 \\ 5 & 1 & -7 \end{array} \right]$$

$$20) R_3 = 4r_1 + r_3$$

$$\left[\begin{array}{ccc|c} -7 & -5 & -1 & -10 \\ 6 & -2 & 9 & 5 \\ 28 & -6 & 6 & 18 \end{array} \right] \sim \left[\begin{array}{ccc|c} -7 & -5 & -1 & -10 \\ 6 & -2 & 9 & 5 \\ 0 & -26 & 2 & -22 \end{array} \right]$$

Graph the inequality.

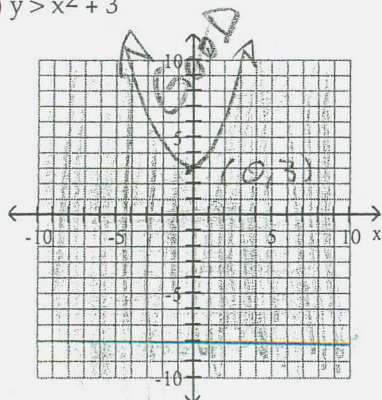
$$21) 3x + 5y \leq 15$$



$$\begin{array}{c|c} x & y \\ \hline 0 & 3 \\ 5 & 0 \end{array}$$

(0,0) Good,
b/c $0 \leq 15$

$$22) y > x^2 + 3$$

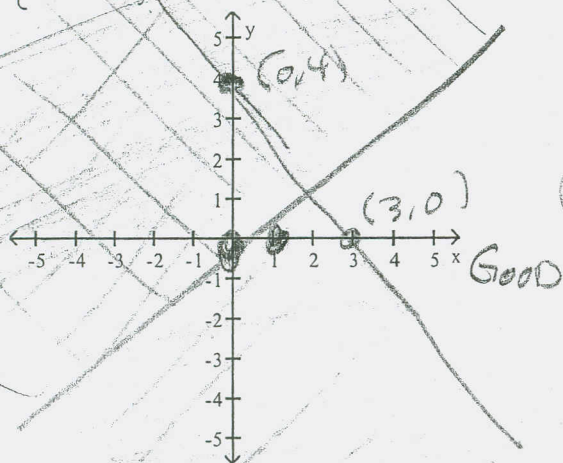


$0 > 3$? No

(0,0) BAD

Graph the system of inequalities.

$$23) \begin{cases} 4x + 3y \geq 12 \\ x \geq y \end{cases}$$



x	y
0	4
3	0

(0,0) BAD
 $0 \geq 12?$

$$x \geq y$$

$$0 \geq 0?$$

Yes

Try (1,0) =

$$1 \geq 0?$$

Yes

(1,0) Good

Bonus : $xy \leq 5$

$y \leq \frac{5}{x}$ if $x > 0$ Below curve

$y \geq \frac{5}{x}$ if $x < 0$ Above curve

